A Bayesian Network Model to Improve Stock Market Trend Following Strategies

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Abstract. *Traditional trend-following models have been used for many years to invest in various asset classes. In this paper we propose a Bayesian Network architecture to enhance classical trend following performance applied to the US stock index. The results show that a Bayesian Network that considers other market variables outperforms both traditional trend following and buy and hold strategies.*

Keywords: *Bayesian Network, Stock Market, Trend Following Strategy.*

1. Introduction

A trend can be referred to as time-series momentum, meaning going long markets with positive returns and taking short positions on the ones showing negative returns. Nevertheless, predicting trends in financial time series is challenging due to all noise related to it. For example, a trend in stock can be formed by a repricing of company fundamentals. If financial analysts suddenly find out that the sustainable margins for a given company are higher than they first thought, the company stock price can show a trend up until it comes to a fair price. However, all conditions can quickly change if we get a situation where new information arrives and harms company fundamentals, suddenly reversing the uptrend in prices.

Trends exist in different markets and have been one of the most studied strategies as in [Hurst et al. 2017]. In their work, Hurst et al. tested a simple trend strategy for the last 120 years, and it has performed positively, which is going long markets with recent positive returns and shorting those with recent negative returns, all decades since 1880 on an annual average of 14.9%.

A famous trader, Jesse Livermore, once said, "big money was not in the individual fluctuations but in sizing up the entire market and its trend." [Lefèvre 1923], emphasizing some of the main focus for him, which are trends and proper sizing.

We can apply the trend following strategy to currencies, commodities, bonds, and stock indices worldwide by looking at different asset classes. Moreover, it is possible to see the aggregate results as in [Lemperiere et al. 2014], showing that stock indices and currencies have shown the best risk-return profile across these classes.

Regarding the latest developments in machine learning methods used to forecast time series, one can take into consideration [Fazelabdolabadi 2019] where a Bayesian Network was used to forecast oil prices, and its performance was outstanding compared to the other hybrid models. They tested several methods, such as Neural Networks, SVM, and Random Forest. The best results turned out to be the Bayesian Network method in terms of the chosen accuracy metrics.

Another exciting application of Bayesian Networks can be seen in [Wang et al. 2015]. In this application, Wang et al. used the Bayesian Network method in order to identify the macroeconomic variables that were affecting the US stock market and also the Chinese stock market in different periods, presenting a graphical way to analyze the main factors behind the price action of the financial assets and the essential factor in each moment for a specific asset.

The goal of this work is to look for the possibility to enhance a classical trend following strategy by applying a Bayesian Network that takes into consideration the output from the trend following strategy itself and some other financial assets as the daily price data of the US stock index, US treasury rates, and volatility index. For the work done, it was found that the method used did help to outperform the classical trend following model in all metrics chosen, and besides that, one can use the topological structure of the graph obtained to analyze and better understand the market behavior and better manage risk given how the other variables are behaving.

This paper is organized in the following sections: Introduction 1, Financial Assets and Trends 2, where a classical strategy is defined and used as the base and benchmark for the rest of the work, and the assets considered in this work are defined, Bayesian Networks 3, with a brief explanation of Bayesian Network and an example on how it could be used in practical terms, Data and Methods 4, where it is explained what was used as a dataset, the time series downloaded, and the method applied, Results and Discussion 5, where the results are discussed and some outputs shown, and finally in Conclusion 6.

2. Financial Assets and Trends

Financial assets can be traded worldwide, and there are many different contracts and asset classes that can be considered. Regarding this work, the focus will be on the US stock index ($SPX500$), a method to track the performance of a group of assets in a standardized way. Indexes typically measure the performance of a basket of securities intended to replicate a specific area of the market, as seen in [Chen 2021].

Another asset considered in this work is the US treasury rates for 10y ($T10y$), which is the government cost of debt for close to 10 years, and the US Department of the Treasury issues this debt.

An essential asset is the volatility index (VIX), which is the market's expectations for the relative strength of near-term price changes of the $SPX500$ as seen in [Kuepper 2021].

This work is focused on these assets and their link to each other. For more information about the mechanics of futures markets, one could read [Hull 2015].

In [Zhang et al. 2018b] trend is defined as the ratio of daily returns by its standard deviation. Zhang applies a Hidden Markov Model (HMM) to China and US stock market indices. In [Zhang et al. 2016] trend is defined simply as the signal or direction of the stock return for one day, labeling as uptrend if the return was positive or downtrend if it was negative on a given day. A probabilistic support vector machine (PSVM) is applied in the Chinese stock index and some stocks from a US stock exchange.

Another definition is found in [Zhang et al. 2018a] where some predefined patterns describe the trend in China's stock price index. In this work, Zhang uses chart heuristics to define if the trend is up or down and an algorithm to perform unsupervised labeling and data splitting into four classes. After these steps, a random forest (RF) model is trained, and classification is performed.

An interest implementation is the one in [Fu and Wu 2017], where no trend is evaluated a priori, but an HMM is used in order to obtain excess returns in Chinese and US stock indices. Two states are defined, one of them as a profitable state and the other as unprofitable. In order to train the HMM, a set of features was chosen to predict the next day's state better.

Taking as a example any asset with prices represented by the time series $(p_1, ..., p_n)$, its exponential moving average with decay equal to n days is represented by $\langle p \rangle_{n,t}$ and σ_n represents the standard deviation of the time series, the trend can be quantified as in [Lemperiere et al. 2014] by the difference between the asset price and its moving average divided by the standard deviation of the series, coming up with a number that is similar to the z-score of the price time series. A more detailed formula to quantify a trend can be seen in Equation 1.

The signal ($s_n(t)$) for time t is constructed as in Equation 1 and is used to compute the $signal\_trend$ and then to simulate the classical trend following the model and as an input for the Bayesian Network.

$$s_n(t) = \frac{p(t-1) - \langle p \rangle_{n,t-1}}{\sigma_n(t-1)} \tag{1}$$

For the following equations the function $sign(x)$ will be used and it is defined as $+1$ if $x > 0$ and as -1 otherwise.

$$signal\_trend(t) = sign(s_n(t)) \tag{2}$$

The signal strength of the trend is measured by Equation 3 and an example of this equation applied to $SPX500$ can be seen in Figure 1.

$$Q_n(t) = \sum_{t'=0}^{t} sign[s_n(t')] * \frac{p(t'+1) - p(t')}{\sigma_n(t'-1)} \tag{3}$$

3. Bayesian Networks

A Bayesian Network [Jensen and Nielsen 2007] is a graph with a set of variables and a set of directed edges between variables. Each variable has a finite set of mutually exclusive states, and the variables, together with the directed edges, form an acyclic directed graph. To each variable with parents, a conditional probability table is attached.

Figure 1. SPX signal strength

According to [Heckerman 2008], a Bayesian Network is a graphical model that encodes probabilistic relationships among variables of interest. When used with statistical techniques, we can see that it can handle missing data and can be used to learn causal relationships, helping to gain an understanding of the problem domain and predict the consequences of intervention. It helps to combine prior knowledge with the data, and combining Bayesian statistical methods with Bayesian Networks offers an efficient and straightforward approach to avoiding the overfitting of data.

An example of Bayesian Network can be seen in Figure 2, where we can see that the $SPX500$ being Down is dependent on VIX Up and $T10y$ Up and whether VIX was Up or $T10y$ was Up depends on $SPX500$ Down yesterday. A general question for this type of problem is "*$SPX500$ was Down yesterday, what's the probability that the $SPX500$ is Down today?*", so we want to compute $P\left(SPX500\_Down\_today = True | SPX500\_Down\_yesterday = True\right)$

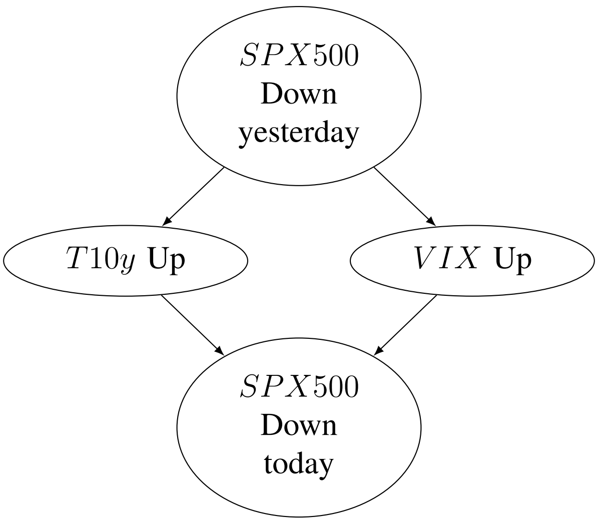

Figure 2. Bayesian network example

An important property for the chain rule in Bayesian Network is given by:

$$p\left(X_1, ..., X_n\right) = \prod_{i=1}^{n} p\left(X_i | pa\left(X_i\right)\right) \tag{4}$$

where $p\left(X_i | pa\left(X_i\right)\right)$ is a conditional probability distribution of X_i given the parent nodes $pa\left(X_i\right)$.

Applying the chain rule for the problem on the Figure 2, we have that the joint probability distribution can be represented by equation 5, making it possible to analyze various combinations of variables and its probability.

$$\begin{aligned} p\left(SPX500\_Down\_Yest, T10y\_Up, VIX\_Up, SPX500\_Down\_Today\right) = \\ p\left(SPX500\_Down\_Today | T10y\_Up, VIX\_Up\right) * \\ p\left(T10y\_Up | SPX500\_Down\_Yest\right) * \\ p\left(VIX\_Up | SPX500\_Down\_Yest\right) * \\ p\left(SPX500\_Down\_Yest\right) \end{aligned} \tag{5}$$

4. Data and Methods

The daily price data of $SPX500$, $T10y$ and VIX for the last 30 years were downloaded through the *yfinance* package [Bland 2020]. As for $SPX500$ prices, the standard deviation of its daily returns (volatility) was calculated to use as input for the Bayesian Network model. Also, looking into $SPX500$, the shifted return one day ahead was computed and used as a target variable. After getting the data it was labeled in uptrend or downtrend via Equations 1 and 3, defined in the Section 2.

As defined in Equation 2, we consider all these variables to compute the $signal\_trend$. The $signal\_trend$ is used to simulate the classical trend following the model and as an input for the Bayesian Network.

For implementing the Bayesian Network, we use the *blearn* package [Scutari 2010]. For each day, the probability of the next day's return being either positive or negative is assessed, and a decision to buy or sell the asset is made to backtest this strategy.

Given the Bayesian Network, a trading system is tested, simulating a portfolio trading $SPX500$ and going long whenever an uptrend is more likely or short when a downtrend is likely.

In order to examine the results, some metrics are calculated as defined in Equations 6, 7, 8 and 9.

Given a time series of returns $(r_1, ..., r_n)$:

$$\overline{R} = mean\left(r_1, ..., r_n\right) * 252 \tag{6}$$

$$\sigma = stdev\left(r_1, ..., r_n\right) * \sqrt{252} \tag{7}$$

$$Sharpe = \frac{\overline{R}}{\sigma} \tag{8}$$

$$WR = \frac{number\_of\_r_i \geq 0}{n} \tag{9}$$

where $\overline{R}$ is the annualized return, σ is the annualized volatility, $Sharpe$ is the Sharpe Ratio and WR is the win ratio.

4.1. Experiments

In this section, details about the models and framework used will be explored.

First, a dataset from *yahoo finance* [Yahoo 2022] for the last 30 years of daily closing prices from $SPX500$, $T10y$, and VIX was downloaded.

Data pre-processing was done by filling blank prices and guaranteeing that all dates were filled with the latest available data each day.

We use the method called backtesting to check how the strategy would have performed historically, which means that the strategy is simulated and parameters updated with all the data available up to that date.

After data pre-processing, the trend signal was calculated as described in 2 and just its sign is used, as in Equation 2, to backtest this model.

The variables used in our Bayesian Network are the next day return for the $SPX500$ (SPX500_ret_shift), and this one is used as the **target** variable. Other variables are $SPX500\_ret$, vol_ret, $T10y\_ret$ and VIX_ret, which are daily returns for the $SPX500$, $SPX500$ volatility, $T10y$ and VIX accordingly.

Table 1. Bayesian Network Variables

variable	description	values
SPX500_ret	SPX500 return sign	{0,1}
signal_trend	Signal from trend model	{-1,1}
vol_ret	Volatility return sign	{0,1}
T10y_ret	Treasury return sign	{0,1}
VIX_ret	VIX return sign	{0,1}
SPX500_ret_shift	SPX500 return sign for the next day	{0,1}

After discretizing the variables, the Bayesian Network is built. For each set of available data, the probability of $SPX500\_ret\_shift$ is computed, and a modified signal is evaluated ($signal\_trend\_mod$) using the Algorithm 1.

The probability of $SPX500\_ret\_shift$ is computed by using the Bayesian Network output:

$$P(SPX500\_ret\_shift = 1|SPX500\_ret, signal\_trend, vol\_ret, T10y\_ret, VIX\_ret).$$

Figure 3 shows the Bayesian Network used, its variables, and the relationship of each variable.

Algorithm 1: Computing modified signal

Result: signal_trend_mod
if $P(SPX500\_ret\_shift > 0) > 50\%$ **then**
  signal_trend_mod = 1;
else
  signal_trend_mod = -1;
end

Considering $signal\_trend$ and $signal\_trend\_mod$, both signals were backtested based on some simple rules and analyzed as in Section 5. The main difference between both signals is that $signal\_trend$ is computed using traditional trend following calculations as explained in Section 2. The output of the Bayesian Network (probability of $SPX500$ given the other input variables) represents the modified signal $signal\_trend\_mod$.

For the backtesting, each model was trained considering all data available up to date, as in Figure 4, therefore every new day, one day of data is added to the train set and tested for the next day of trading.

Trading rules considered for this experiment were given by the following rules:

- If signal≥0; Buy $SPX500$
- If signal<0; Sell $SPX500$

For example, if we had $signal = 1$, the strategy would Buy $SPX500$ to follow the rules defined.

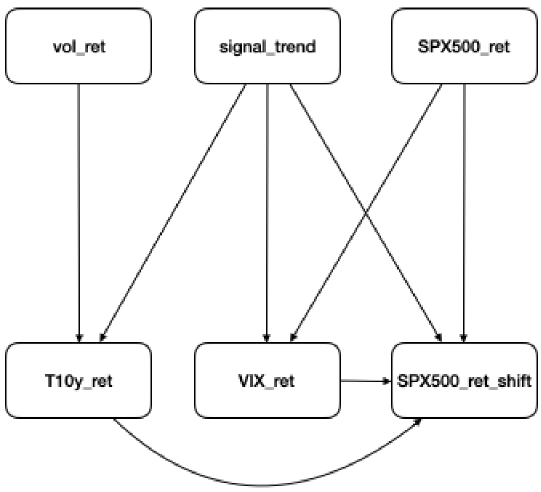

Figure 3. Bayesian Network Model

5. Results and Discussion

For testing the performance of the models, a fixed start window was used. As time went by, the amount of data grew, and the test was made for the next day [Brownlee 2019], running from 30Mar91 to 9Jun21.

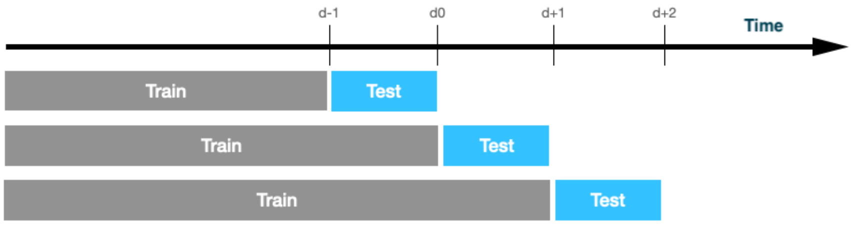

Figure 4. Backtest framework

For the trend following model, the signal we got can be seen in Figure 5 represented by the green line, where we see that for the last two years. The model just pointed to short the $SPX500$ during the last COVID crisis, but the signal is an entirely consistent point to a trend higher.

Signals generated by the trend following and Bayesian Network can seen in the Figure 5. This chart is divided in 3 subdivisions with the first being the original signal, the second is the modified signal and the last one is the $SPX500$. It is possible to see that the modified signal trades more often than the original and during the last 2 years changed its recommendation from long to short in higher frequency, but most of the pointed trend was higher than lower.

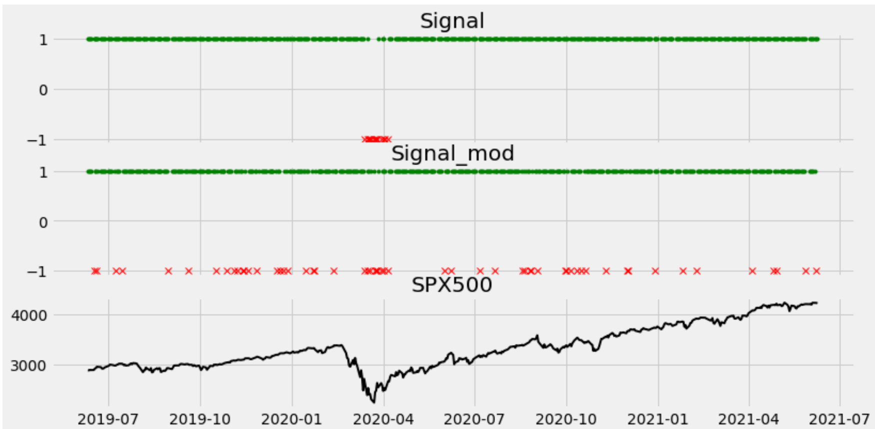

Figure 5. Model position and $SPX500$

The accumulated results curve was generated for each strategy based on the signals above, in order to better to understand its differences and the source of possible improvements. For the original model in the Figure 6 one can see the result of following the signal described in the Section 2 and the results from the modified model can be seen in the Figure 7 represented by the orange line.

The results are resumed on the Table 2, where we can see three different metrics for three different strategies. The first one is the Sharpe Ratio, as defined by 8. Financial market players use this metric because it gives an intuition about how dislocated the generated returns by zero are. The second metric is the average return per year, as defined by Equation 6, and the third one is the win rate, as defined by Equation 9. The fourth metric is Trades per year which is the average number of trades in a year, the fifth is the average return per year minus the transaction costs and the last is the Sharpe Ratio calculated taking into consideration the transaction costs. These metrics were chosen in order to have a benchmark that was independent on other works due to the specificity of each work, in general it is very hard to find a perfect comparison, because there are many methods, many assets to test and many metrics to assess the quality of the results, given this one possible framework is look at the investment in the asset itself then look at the classical model and then check the results on the enhanced model.

Taking a look at Table 2 it is possible to check that the Sharpe for the $SPX500$ without any strategy applied to it is 0.34, applying the classical trend following it improved to 0.56 and applying the Bayesian Network to the signal generated by the classical model we get 0.99, which is a significant improvement. We also see this improvement for the returns having 6.3%, 10.5%, and 18.4%. For the win rate, we have 54%, 53.0%, and 54.5%, showing that both the annual return per year and the win rate improved from the classical model to the modified model.

One crucial point is to take into consideration transaction costs for final results. In order to estimate transaction costs, it is assumed the market impact is going to be zero as the $SPX500$ is one of the most liquid assets in the world and the commission cost used is 0.0025% per contract as seen in [CME Group 2016]. Looking at the Table 2 we can see that the main difference in terms of return is the Model_mod return that shows a return of 18.4% and after costs of 18.14%.

Another critical analysis looks at the model results split by $SPX500$ daily returns. On Table 3, one can see that the classical model presented 28.4% of the whole dataset with positive returns on days that $SPX500$ was up and 24.6% of positive returns on $SPX500$ down, this compares to the modified model as 28.7% of positive returns when $SPX500$ was up and 25.8% on $SPX500$ down.

Figure 6. Trend following result

6. Conclusion

In this paper, a Bayesian Network was proposed and developed to improve the performance of a classical trend following model. The proposed architecture for the Bayesian Network was based on experts' knowledge, and its results were tested via a backtest framework where it has proven itself to be better than both the simple buy and hold and trend

Figure 7. Results from both models

Table 2. Metrics for different strategies

	SPX500	Model	Model_mod
Sharpe Ratio	0.34	0.56	0.99
Return (p.y)	6.3%	10.5%	18.4%
Win rate	54.0%	53.0%	54.5%
Trades per year	0	4	103
Return (p.y) adj	6.3%	10.49%	18.14%
Sharpe Ratio adj	0.34	0.56	0.98

following strategy. The hybrid model performed better in all metrics chosen for comparing the results; it had outstanding results in terms of Sharpe ratio, and also, the win rate showed some excellent improvements. Other than that, one could use the Bayesian Network results to see that the signal trend from the classical model has some influential characteristics on $T10y$, VIX, and $SPX500$ returns. This model could also be used to understand the market dynamics better and help decision-making in a fundamental analysis environment, given its simplicity to check the Bayesian Network graph and the variables' relationships. Given the above, it is possible to conclude that Bayesian Network can help identify trends in the US stock market index, and it has performed very well in all sets of tests done.

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Table 3. Models vs SPX500 returns

	Model		Model Mod	
SPX500	ret >= 0	ret <0	ret >= 0	ret <0
ret >= 0	28.4%	25.3%	28.7%	25.0%
ret <0	24.6%	21.7%	25.8%	20.5%

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