Heuristic, Exact and Hybrid Approaches for Vehicle Routing Problems

Anand Subramanian[1]

[1]Doctoral thesis presented to the Computing Graduate Program
of the Universidade Federal Fluminense
anand@ct.ufpb.br

Advisor: Luiz Satoru Ochi
Instituto de Computação – Universidade Federal Fluminense
satoru@ic.uff.br
Co-Advisor: Eduardo Uchoa
Departamento de Engenharia de Produção – Universidade Federal Fluminense
uchoa@producao.uff.br

Abstract

This is an extended abstract of the author's Doctoral thesis supervised by Luiz Satoru Ochi and Eduardo Uchoa and defended on 05 March 2012 at the Universidade Federal Fluminense, Brazil. The thesis is written in English and is available online at `http://www2.ic.uff.br/PosGraduacao/Teses/532.pdf`.

Resumo

Este é um resumo expandido da tese de doutorado do autor orientada por Luiz Satoru Ochi e Eduardo Uchoa e defendida em 05 de março de 2012 na Universidade Federal Fluminense, Brasil. A tese está escrita em inglês e encontra-se disponível em `http://www2.ic.uff.br/PosGraduacao/Teses/532.pdf`.

1. Motivation

The Vehicle Routing Problem (VRP) is a classical Combinatorial Optimization (CO) problem that was proposed in the late 1950's and it is still one of the most studied in the field of Operations Research (OR). The great interest in the VRP is due to its practical importance, as well as the difficulty in solving it.

Although the VRP is $\mathcal{NP}$-hard, there has been lot of advances in the development of exact algorithms for dealing with this problem, particularly those based on mathematical programming techniques. However, up to date, there is no exact algorithm that consistently solves VRP instances with more than 150 customers. Nonetheless, in those cases where the optimal solution could not be determined, one can still make use of the the value of the dual bounds for evaluating the solution quality obtained by heuristics

Due to their ability of obtaining good solutions in an acceptable time, heuristic procedures are the most common method employed to solve CO problems. A special attention must be given to metaheuristics, which can be defined as general master processes that guide a subordinate heuristic in order to efficiently find high quality solutions.

Metaheuristics are the core of a huge number of successful heuristic algorithms for CO problems, including the VRP. Such popularity arises from the fact that metaheuristics are more flexible and easier to understand.

Combining (meta)heuristic and exact methods appears to be a very promising alternative in solving CO problems. The interest in hybrid approaches is rapidly growing especially due to several encouraging results obtained by the fusion of these two methods. The interaction between mathematical programming techniques and metaheuristics led to a new class of optimization algorithms called *matheuristics*. Nevertheless, the application of these kind of approaches have not received much attention yet from the VRP literature

Most VRP heuristics usually focus on a particular type of problem. A relatively small number of works have suggested unified heuristic procedures for dealing with several variants (see, for example, [Pisinger and Røpke 2007, Røpke and Pisinger 2006, Cordeau et al. 2011]). One of the interests of this work is to propose general heuristic and hybrid algorithms for solving different VRPs. However, because of the huge number of existing variations it becomes virtually impossible to tackle all of them here. Therefore, it was thought advisable to turn attention only to a subset of variants, namely the following ones: (i) Capacitated VRP (CVRP), (ii) Asymmetric CVRP (ACVRP), (iii) Open VRP (OVRP), (iv) VRP with Simultaneous Pickup and Delivery (VRPSPD), (v) VRP with Mixed Pickup and Delivery (VRPMPD), (vi) Traveling Salesman Problem with Mixed Pickup and Delivery (TSPMPD), (vii) Multi-Depot VRP (MDVRP), (viii) Multi-Depot Vehicle Routing Problem with Mixed Pickup and Delivery (MDVRPMPD) and (ix) Heterogeneous Fleet VRP (HFVRP).

Exact algorithms capable of solving instances with more than 50 customers can be found in the literature for problems (i)-(iii), (vi), (viii) and (ix). It was thus decided to develop exact approaches for problems (iv), (v) and (vi) in order to obtain new lower bounds and some optimal solutions that can be used as a reference for measuring the performance of the proposed heuristic and hybrid algorithms. Although there are more sophisticated exact algorithms for solving problems (i)-(iii) and (vi), the proposed exact strategies can still be used to solve these problems.

2. Relevance of the theme

The VRP plays an important role in the supply chain of several companies that are involved with the transportation of goods or people. This problem is regularly faced by the distribution systems of these corporations and its solution quality may have direct implications on the logistic performance. In addition, the VRP can arise in different contexts within the same company, whether by transporting raw-materials and/or finished goods between the production unit and its subsidiaries, delivering (collecting) products to (from) the customers, or even transporting employees from their homes to the company and vice-versa. Motivated by real-life situations, many VRP variants were proposed throughout the years. They may include additional constraints to satisfy customers' needs such as pickup and delivery services, or additional features such as route duration, multiple depots, etc.

The computing revolution as well as the industrial boom had impulsed the application of OR methods in real-life CO problems such as the VRP in the last 50 years. In [Golden et al. 2002], the authors described a number of case studies in which the application of computerized vehicle routing systems in the solid waste, beverage, food, dairy and newspaper industries led to substantial cost reductions. Furthermore, the use of computerized procedures in distribution planning results in 5% to 10% savings in transportation costs.

Seen from a practical point of view, the development of unified solution approaches are highly relevant. For instance, VRP commercial packages must be prepared to face real-life problems of different classes. Therefore, the development of efficient general algorithms are crucial for achieving a satisfactory performance. When talking about attributes for good heuristics, one should take into account not only the solution quality (accuracy) and computational time (speed), but also the simplicity and flexibility factors [Cordeau et al. 2002]. Hence, a general VRP heuristic that contains these four attributes, is more likely to be successfully employed in practice.

3. Objectives

The objectives of the thesis are: (a) Review the VRPs (i)-(ix), describing some practical applications and solution methods proposed in the literature; (b) Develop exact approaches for problems (iv), (v) and (vii); (c) Develop a general heuristic framework capable of solving a large class of VRPs with emphasis on problems (i)-(ix); and (d) Develop a general hybrid algorithm capable of solving a large class of VRPs with emphasis on problems (i)-(v) and (vii)-(ix).

4. Contributions and results

Chapter 2 presents an extensive literature review of the VRPs considered in this thesis, focusing on describing the main contributions of each work. By observing the substantial number of publications, one can verify that this is indeed an area of intense and continuous research in the fields of CO and OR.

Chapter 3 dealt with Mixed Integer Programming (MIP) flow formulations for the VRPSPD/VRPMPD. Two versions of two-commodity flow formulations (an undirected and a directed) were tested within a Branch-and-Cut (BC) scheme, using cuts from the CVRPSEP library, and their results were compared with the one-commodity flow formulation from the literature. The optimal solutions of 30 VRPSPD open problems were proved. The three formulations were also tested in benchmark instances of the VRPMPD, which is a particular case of the VRPSPD, and were able to prove the optimality of 7 open problems. Furthermore, new lower bounds were produced for both VRPSPD and VRPMPD instances with up to 200 customers. In addition, although it has been shown that the one-commodity flow formulation produces a stronger linear relaxation, the two-commodity flow formulations have found, on average, better lower bounds in the VRPSPD instances. As for the VRPMPD, the lower bounds were, on average, quite similar, but with a slight superiority of the one-commodity formulation.

In Chapter 4, a BC algorithm with a lazy separation scheme was developed for the VRPSPD, VRPMPD and MDVRPMPD. This approach relies on a formulation only over the edge variables and the constraints that ensure that the capacity is not exceeded in the middle of the route and those that ensure that a route starts and ends at the same depot are separated in a lazy fashion. The results obtained in the VRPSPD/VRPMPD instances using this BC outperformed those found using the flow formulations in most cases, where a total of 59 optimal solutions are found for instances with up to 200 customers. As for the MDVRPMPD, the first LBs are presented for this variant and 4 optimal solutions are found for instances with up to 100 customers.

Chapter 5 presents the third exact approach proposed for the VRPSPD/VRPMPD which consists of a Branch-Cut-and-Price (BCP) algorithm that combines the CVRP cuts used in the BCs (rounded capacity, multistar and comb) with column generation. The BCP is mostly based on the one developed in [Fukasawa et al. 2006] for the CVRP. The original column generation module was replaced by a new dynamic programming based algorithm, that includes an efficient state-elimination scheme, in order to consider both delivery and pickup demands. This approach is a key contribution of this approach since it allowed for either solving or improving the LBs of open instances with up to 200 customers in a reasonable time. The BCP algorithm found capable of proving the optimality of 4 problems for the first time, where three of them are VRPSPD instances containing 100 customers and one of them corresponds to a VRPMPD instance containing 75 customers. The best known LBs of other 10 instances involving 150-200 customers were improved. Moreover, the BCP clearly outperformed previous methods in a subset of instances.

Chapter 6 presents a heuristic algorithm, called ILS-RVND, for a large class of VRPs. The developed algorithm has a simple structure and relies on relatively very few parameters. The ILS-RVND heuristic is a multi-start procedure that uses insertion heuristics in the constructive phase, a Variable Neighborhood Descent with Random neighborhood ordering (RVND) in the local search phase and simple moves as perturbation mechanisms. The RVND is composed of well-known VRP inter-route neighborhood structures, namely those based on λ-interchanges and Cross-exchange; and also by specific ones, namely ShiftDepot and SwapDepot. The best improvement strategy was adopted and the neighborhoods are explored exhaustively. Every time a route is modified due to an inter-route move an intra-route local search is performed using classical Traveling Salesman Problem neighborhood structures, more precisely, Reinsertion, Or-opt2, Or-opt3, 2-opt and Exchange. The perturbation mechanisms consist of performing multiple Swap(1,1) or Shift(1,1) moves. Extensive computational experiments on 634 instances proved the efficiency of the proposed solution approach, especially in terms of flexibility and solution quality. ILS-RVND was found capable of improving the result of 98 instances and to equal the result of another 464. Furthermore, for every set of instances, the average gap between the solutions found by ILS-RVND and the best known solutions, was always smaller than 0.66%, except for one set, where the average gap was 1.03%. Finally, it is important to mention that this method served as a base for several recent works published by the author in high quality journals. More precisely, state-of-the-art results were found for the Minimum Latency Problem [Silva et al. 2012], Traveling Salesman Problem with Mixed Pickup and Delivery [Subramanian and Battarra 2013], Large Scale Capacitated Arc Routing Problem [Martinelli et al. 2013] and Split-Delivery Vehicle Routing Problem [Silva et al. 2013].

In Chapter 7, a hybrid algorithm, called ILS-RVND-SP, was proposed to solve a large class of VRPs where an exact procedure, based on a Set Partitioning (SP) formulation, was incorporated into the ILS-RVND heuristic. This strategy is quite similar to the classical two-phase petal algorithm (see [Laporte and Semet 2002]). The idea is to store a pool of routes generated during the heuristic execution and then solve a SP problem in order to extract the best combination of routes. However, unlike traditional petal algorithms and other SP based approaches to VRPs, the proposed hybrid algorithm includes some enhanced strategies. The first one is the cooperation between a MIP solver and the ILS heuristic (while solving the SP problem). The second strategy consists of a reactive mechanism that dynamically controls the dimension of the SP models when dealing with large size instances, that still allows for taking advantage of the exact procedure. These ideas can be employed to efficiently solve a large class of combinatorial optimization problems. While ILS-RVND seeks an equilibrium among solution quality, speed, simplicity and flexibility, ILS-RVND-SP gives preference to the solution quality at the expense of the speed, but still holding all the flexibility. Computational tests were performed on 381 instances, where ILS-RVND-SP found 52 new best solutions and equaled the result of another 301. For every set of instances, the average gap was always smaller than 0.55%. It is worth mentioning that ILS-RVND-SP was cited as one of the best algorithms ever proposed for VRPs in the recent survey of [Vidal et al. 2013]. Moreover, to the best of the author's knowledge, ILS-RVND-SP is to date the best unified matheuristic approach available for VRPs.

5. Publications

The relevance of the contributions of the thesis can be clearly verified by the considerable number of papers published in highly-ranked journals in the field of Optimization.

5.1. Journals

1. Subramanian, A., Uchoa, E., Ochi, L. S. A hybrid algorithm for a class of vehicle routing problems. *Computers & Operations Research* (2013). Accepted for publication. **(Qualis A1)**. From Chapter 7.
2. Subramanian, A., Uchoa, E., Pessoa, A. , Ochi, L. S. Branch-cut-and-price for the vehicle routing problem with simultaneous pickup and delivery. *Optimization Letters* (2012). Accepted for publication. **(Qualis B1)**. From Chapter 5.
3. Penna, P. H. V., Subramanian, A., Ochi, L. S. An iterated local search heuristic for the heterogeneous fleet vehicle routing problem. *Journal of Heuristics* (2013), 19: 201–232. **(Qualis A2)**. From Chapter 6.
4. Subramanian, A., Penna, P. H. V., Uchoa, E., Ochi, L. S. A hybrid algorithm for the heterogeneous fleet vehicle routing problem. *European Journal of Operational Research* (2012), 221: 285–295. **(Qualis A1)**. From Chapter 7.
5. Subramanian, A., Uchoa, E., Pessoa, A., Ochi, L. S. Branch-and-cut with lazy separation for the vehicle routing problem with simultaneous pickup and delivery. *Operations Research Letters* (2011), 39: 338–341. **(Qualis B1)**. From Chapter 4.

5.2. Proceedings of International Conferences

1. Subramanian, A., Penna, P. H. V., Uchoa, E., Ochi, L. S. A hybrid algorithm for the fleet size and mix vehicle routing problem. In: *Proceedings of the International Conference on Industrial Engineering and Systems Management* (2011), 1214–1223, Metz, France. From Chapter 7.
2. Subramanian, A., Uchoa, E., Ochi, L. S. New lower bounds for the vehicle routing problem with simultaneous pickup and delivery. In: *Proceedings of the 9th International Symposium on Experimental Algorithms (SEA), Lecture Notes in Computer Science 6049* (2010), 276–287, Ischia Islands, Italy. **(Qualis B1)** From Chapter 3.

5.3. Book Chapter

1. Subramanian, A., Penna, P. H. V., Ochi, L. S., Souza, M. J. F. *Meta-heurísticas em Pesquisa Operacional*. Omnipax editora (2013). Chapter: Um Algoritmo Heurístico Baseado em Iterated Local Search para Problemas de Roteamento de Veículos. ISBN: 978-85-64619-10-4. *In Portuguese*. From Chapter 6.

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Subramanian, A. and Battarra, M. (2013). An iterated local search algorithm for the travelling salesman problem with pickups and deliveries. *Journal of Operational Research Society*, 64:402–409.

Vidal, T., Crainic, T., Gendreau, M., and Prins, C. (2013). Heuristics for multi-attribute vehicle routing problems: A survey and synthesis. *European Journal of Operational Research*. To appear.