A Multilevel Thresholding Approach Based on Improved Particle Swarm Optimization for Color Image Segmentation

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Abstract. *In this paper, a hybrid Otsu and improved Particle Swarm Optimization (PSO) algorithm is presented to deal with multilevel color image thresholding problem, named APSOW. In APSOW, the historical information represented by the local best solutions found so far by PSO population are permuted among the current population, using a randomized greedy process. APSOW also implements a weedout operator to prune the worst individuals from the population. The proposed APSOW is compared to other hybrid EAs and Otsu approaches from literature (include standard PSO model) through twelve benchmark color image problems, showing its potential and robustness.*

1. Introduction

Image segmentation is one of the most fundamental processes in image understanding and computer vision. Image segmentation methods have two basic properties: discontinuity and similarity [Ye et al. 2015]. Thresholding is an important technique for image segmentation based on the similarity of image components. One of the most popular image thresholding methods is Otsu's algorithm [Otsu 1979].

The original Otsu algorithm was proposed as an effective method to segment an input image in gray level in two classes (image binarization). Otsu algorithm is easily adapted as multi-level thresholding approach, but multi-level Otsu has found little use in practice, since its computational cost increases significantly if the number of selected thresholds is high.

Recently, many Evolutionary Algorithms (EAs) and Swarm Intelligence (SIs) methods have been adapted to the context of multi-level image thresholding [Suresh and Lal 2017, Elaziz et al. 2019, Wang and Tan 2019]. In [Zhou and Yang 2011], Genetic Algorithm (GA) [Holland 1992] and Otsu have been used to infrared image segmentation. In [Kumar et al. 2011], Otsu algorithm have been combined to Differential Evolution (DE) [Storn and Price 1997] algorithm. In [Rodríguez-Esparza et al. 2020], Harris Hawks Optimization (HHO) haven been adapted to multi-level thresholding, using the Minimum Cross-Entropy as the fitness function. Particle Swarm Optimization (PSO) [Kennedy and Eberhart 1995] and Group Search Otpimization (GSO) [He et al. 2009] have also been hybridized with Otsu algorithm in [Zhang and Zhou 2012] and [Ye et al. 2015, Pacifico et al. 2018], respectively.

In this work, we propose a hybrid improved PSO and Otsu algorithm, named APSOW, for color image segmentation problem based on multi-level image thresholding. The proposed APSOW performs a permutation on the historical information of the search (represented by the local best solutions found so far for each particle) in an attempt to speedup the convergence of the swarm, and also, to help slow particles to escape from local optima regions of the problem search space. APSOW also employs a weedout operator to prune a percentage of the weakest individuals from the population.

The paper is organized as follows. Otsu's algorithm, multi-level Otsu and PSO presented in Section 2. The proposed APSOW is presented in Section 3. The experimental results are discussed in Section 4. Finally, some conclusions and leads for future works are presented (Section 5).

2. Background

This section presents the background information concerning Otsu's algorithm and Multi-Level Otsu (Section 2.1), and also the main characteristics of standard PSO algorithm (Section 2.2). Brief descriptions for each model are presented as follows.

2.1. Multi-Level Otsu Method

The original Otsu method [Otsu 1979] aims to divide a given gray image I in two classes C_0 and C_1 by the threshold T. But in multilevel Otsu, we consider that D thresholds will be applied to partition I in $(D+1)$ classes $C_0, C_1, ..., C_D$. Assuming that I is represented in L gray levels $[0, 1, \ldots, L-1]$. Let N_p denote the total number of pixels in I, and n_i is the number of pixels at level i. The probability of gray level i is denoted by eq. (1).

$$p_i = n_i / N_p,\ p_i \geq 0,\ \sum_{i=0}^{L-1} p_i = 1 \tag{1}$$

The gray level probability distribution for class C_k is obtained by:

$$r_k = \begin{cases} \sum_{i=0}^{T_i} p_i, \text{ if } k=0 \\ \sum_{i=T_k}^{T_{k+1}} p_i, \text{ if } 1 \leq k \leq D-1 \\ \sum_{i=T_D}^{L-1} p_i, \text{ if } k=D \end{cases} \tag{2}$$

where T_i is i-th threshold, such that $T_1 < T_2 < ... < T_{D-1} < T_D$. The mean of class C_k is denoted by:

$$\mu_k = \begin{cases} (\sum_{i=0}^{T_i} i \cdot p_i) / \sum_{i=0}^{T_i} p_i, \text{ if } k=0 \\ (\sum_{i=T_k}^{T_{k+1}} i \cdot p_i) / \sum_{i=T_k}^{T_{k+1}} p_i, \text{ if } 1 \leq k \leq D-1 \\ (\sum_{i=T_D}^{L-1} i \cdot p_i) / \sum_{i=T_D}^{L-1} p_i, \text{ if } k=D \end{cases} \tag{3}$$

The Between-Class Variance (BCV) for multilevel Otsu is given by eq. (4) [Ye et al. 2015]. Multilevel Otsu algorithm finds the optimal set of thresholds by **maximizing** the BCV [Liu and Yu 2009].

$$f_I = \sum_{i=0}^{D} r_i \cdot (\mu_i - \mu_I)^2,\ \mu_I = \sum_{i=0}^{D} r_i \mu_i \tag{4}$$

2.2. Particle Swarm Optimization

Particle Swarm Optimization algorithm consists of a SI stochastic global search metaheuristic originated from the attempt to graphically simulate the social behavior of a flock of birds looking for resources [Kennedy and Eberhart 1995]. Later, looking for theoretical foundations, studies were realized concerning the way individuals in groups interact, exchanging information and reviewing personal concepts improving their adaptation to the environment [Kennedy et al. 2001]. In standard PSO, all individuals of the population follow the best point found so far by any individual of the population (global best or g_{best} strategy).

In PSO, a population (*swarm*) *G* of *S* individuals (*particles*) is kept. Each particle keeps track of its position, velocity and best position. As the algorithm iterates, the velocity $\mathbf{V}_i^t$ of each particle $\mathbf{X}_i^t$ is determined according to the two main exemplar points of the search: the individual local best position visited so far $\mathbf{Y}_i^t$ and the global best position visited so far $\hat{\mathbf{Y}}^t$ by the swarm. The eq. (5) and eq. (6) describe respectively, how the new velocity and the new position of a particle are determined.

$$v_{ij}^{t+1}=wv_{ij}^{t}+c_1r_1(y_{ij}^{t}-x_{ij}^{t})+c_2r_2(\hat{y}_{j}^{t}-x_{ij}^{t}) \tag{5}$$

$$x_{ij}^{t+1}=x_{ij}^{t}+v_{ij}^{t+1} \tag{6}$$

where $1 \leq i \leq S, 1 \leq j \leq n$, w is the scalar inertia weight (momentum term usually in the interval [0.4, 0.9]), the values r_1 and r_2 are random variables taken from an uniform distribution $U(0,1)$, and the values $0 < c_1, c_2 \leq 2$ are individual and global acceleration coefficients, respectively. The new local best position $\mathbf{Y}_i^{t+1}$ and the new global best position $\hat{\mathbf{Y}}^{t+1}$ are determined by eq. (7) and eq. (8), respectively:

$$\mathbf{Y}_i^{t+1}=\begin{cases}\mathbf{Y}_i^t, \text{ if } f(\mathbf{Y}_i^{t+1}) \text{ is better than } f(\mathbf{X}_i^t)\\ \mathbf{X}_i^{t+1}, \text{ otherwise}\end{cases} \tag{7}$$

$$\hat{\mathbf{Y}}^{t+1}=\arg best_{1\leq i\leq S}\ f(\mathbf{Y}_i^{t+1}) \tag{8}$$

3. Proposed Approach: APSOW

Let I be a color image in RGB color space. The image I is composed by three color channels, R, G and B. In APSOW, I is decomposed for each color channel, generating the images I_R, I_G and I_B in gray level. Each gray level image is treated independently for segmentation purposes [Parthasarathy and Chitra 2015, Pacifico et al. 2018]. The i-th particle $\mathbf{X}_i \in \Re^n$ in APSOW represents D dimensions (thresholds) for each RGB channel, where $n = 3 \times D$. Fig. 1 represents an APSOW particle for $D = 4$ (i.e., $\mathbf{X}_i \in \Re^12$), where the first four features represent the thresholds $T_{R1} < T_{R2} < T_{R3} < T_{R4}$ for I_R, the 5-th to 8-th features represent the thresholds $T_{G1} < T_{G2} < T_{G3} < T_{G4}$ for I_G and the last four characteristics represent the thresholds $T_{B1} < T_{B2} < T_{B3} < T_{B4}$ for I_B.

The initial population for APSOW is randomly determined, respecting the restriction $T_1 < T_2 < \cdots < T_D$ for each RGB channel. After random initialization, the fitness

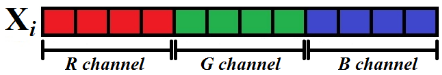

Figura 1. Particle representation for four thresholds per RGB channel.

function $f_I(\mathbf{X}_i^0)$ is computed for each particle $\mathbf{X}_i^0$ as:

$$f_{I_{RGB}}(\mathbf{X}_i) = f_{I_R}(\mathbf{X}_i) + f_{I_G}(\mathbf{X}_i) + f_{I_B}(\mathbf{X}_i) \tag{9}$$

After initialization, the generational process begins. At the beginning of each t-th generation in APSOW, the local best solutions for each image (I_R, I_G and I_B) are permuted among the particles using a greedy operator, in an attempt to speedup the convergence of the swarm and help weak particles to escape from local optma areas of the problem search space. Firstly, a random permutation is performed according to eq. (10):

$$\mathbf{TY}^t = permuting(\mathbf{Y}^t) \tag{10}$$

After that, each individual $\mathbf{TY}_i^t$ that represents a better solution than its corresponding particle $\mathbf{Y}_i^t$ will be used to replace $\mathbf{Y}_i^t$ in current local best population $\mathbf{Y}^t$ (eq. (11)).

$$\mathbf{Y}_i^t = \begin{cases} \mathbf{Y}_i^t, \text{ if } f(\mathbf{Y}_i^{t+1}) \text{ is better than } f(\mathbf{TY}_i^t) \\ \mathbf{TY}_i^t, \text{ otherwise} \end{cases} \tag{11}$$

Once the $\mathbf{Y}^t$ is defined, PSO operators are executed as in standard PSO. In each generation, after the evaluation of the new swarm G^{t+1}, a selection operator to prune the weakest particles from the current swarm. A percentage of the weakest particles for each image I_R, I_G and I_B will be selected to be pruned by random reinitialization according to the partial fitness function $f_R(\mathbf{X}_i^t)$, $f_G(\mathbf{X}_i^t)$ and $f_B(\mathbf{X}_i^t)$. Since each RGB channel is evaluated independently, one particle $\mathbf{X}_i^t$ may be considered weak for some RGB channel but not weak considering the other channels. In this situation, only the characteristics from $\mathbf{X}_i^t$ corresponding to the weak channels are pruned. This operator aims to promote a better escaping from local minima points. The algorithm for APSOW is presented in Algorithm 1.

4. Experimental Results

In this section, the proposed APSOW is tested by means of twelve benchmark color images using four other EAs from literature as comparison methods: GA, DE, PSO and GSO. The selected benchmark color images are presented inf Fig. 2. The parameters for each algorithm are presented in Table 1, according to [Ye et al. 2015, Pacifico et al. 2018].

All algorithms have been run in MATLAB 7.6 environment, and all tests have been executed in a computer with an i5-5250U CPU and 8 GB RAM. In the experiments, the dimension of thresholding value is set from 2 to 5 for each RGB channel [Ye et al. 2015]. For each test, thirty independent executions have been run for each color image. All evolutionary algorithms have started with the same initial population in each test. The initial population has been acquired by random sampling, where, for each RGB channel, all thresholds have been randomly determined and sorted, so we have $T_1 < T_2 < \cdots < T_D$.

Algorithm 1 APSOW

$t \leftarrow 0$
Initialization: For each particle $\mathbf{X}_i{}^{(0)}$, initialize randomly each threshold $T_i (i = 1, 2, \ldots, D)$ such that $T_1 < T_2 < \cdots < T_D$ for each RGB channel.
Calculate the initial fitness function for each particle $\mathbf{X}_i^{(0)}$ using eq. (9).
while (termination conditions are not met) **do**
  For each image I_R, I_G and I_B, execute the random permutation operator, according to eq. (10) and eq. (11).
  for each particle $\mathbf{X}_i^t$ **do**
    Update its velocity $\mathbf{V}_i^{t+1}$ and position $\mathbf{X}_i^{t+1}$ according to eq. (5) and eq. (6).
    Calculate its fitness $f(\mathbf{X}_i^{t+1})$.
    Update its local best $\mathbf{Y}_i^{t+1}$ position according to eq. (7).
  end for
  Obtain the current *gBest* $\hat{\mathbf{Y}}^{t+1}$ according to eq. (8).
  Calculate the new fitness value for each particle using eq. (9).
  Sort all particles from the current swarm according to their values for $f_{I_R}(\mathbf{X}_i^t)$.
  Reinitialize randomly the features corresponding to D thresholds for I_R for a percentage of the weakest members $\mathbf{X}_i^t$ according to $f_{I_R}(\mathbf{X}_i^t)$, such that $T_{R1} < T_{R2} < \ldots < T_{R(D-1)} < T_{RD}$.
  Repeat the sorting and reinitialization processes according to $f_{I_G}(\mathbf{X}_i^t)$ and $f_{I_B}(\mathbf{X}_i^t)$ for images I_G and I_B, respectively.
  $t := t + 1$.
end while
Return the best particle $\hat{\mathbf{Y}}^{t_{max}}$.

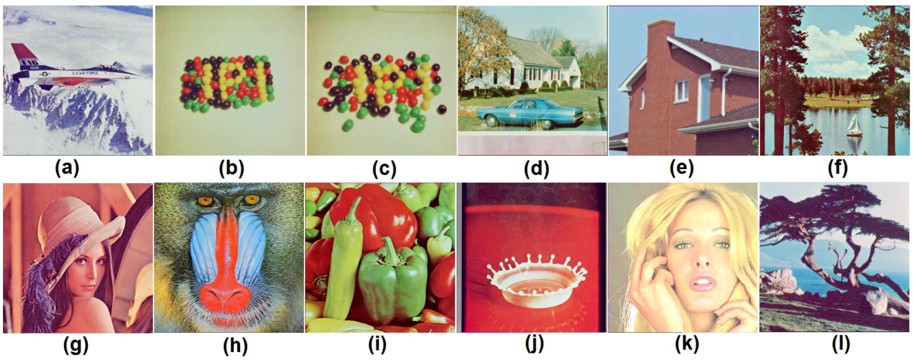

Figura 2. Benchmark color images: (a) Airplane, (b) Gellybeans1, (c) Geallybeans2, (d) House1, (e) House2, (f) Lake, (g) Lena, (h) Mandrill, (i) Peppers, (j) Splash, (k) Tiffany and (l) Tree.

Tabela 1. Parameters for each algorithm.

Algorithm	Parameter	Value
All EAs	S t_{max}	20 100
GA	C_r and M_r	0.8 and 0.1
DE	C_r and F	0.3 and 0.5
PSO-based Approaches	c_1 and c_2 w	2.0 0.9 to 0.4
GSO	l_max and θ_{max}	5 and 5
APSOW	*Weedout Rate*	20%

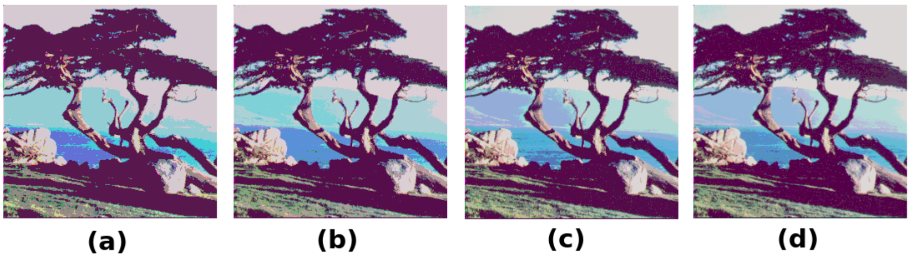

Figura 3. APSOW sample results for Tree image: (a) 2-D, (b) 3-D, (c) 4-D, (d) 5-D.

The evaluation criterion includes an overall rank system employed through the application of Friedman hypothesis test [Friedman 1937], adopting Nemenyi test as the *post-hoc* test, in relation to the fitness function (eq. (9)) [Demšar 2006] and its components to each RGB channel (images I_R, I_G and I_B). The experimental results are shown from Table 2 to Table 5. Some sample results for the proposed APSOW are shown in Fig. 3.

The overall evaluation performed through the application of the Friedman tests are presented in Table 6. The overall evaluation shows that the proposed APSOW is able to find the best performances according to the ranking system for $D = 2, 3$ and 5 (except for I_G when $D = 3$, where it has achieved the second best value), with statistical significance in relation to GA and GSO, and it has outperformed PSO and DE with statistical significance in most of the cases. For $D = 4$, APSOW has been able to find the best ranks for I_B and considering the whole image I, reaching the second best performances in relation to I_R and I_B. The overall evaluation showed the potential and robustness of the proposed APSOW to tackle image segmentation problems.

The evaluation considering the average execution times (see Table 2 to Table 5) shows that the execution time of APSOW is slightly higher than standard PSO, as expected, but its average execution times are completely compatible with the execution times of any evolutionary and swarm intelligence algorithms, making APSOW an interesting option to deal with image thresholding applications.

5. Conclusions

In this work, we presented a new hybrid improved PSO and Otsu algorithm for multi-level color image thresholding, called APSOW. APSOW employs a permutation to randomize the historical information stored in local best population to speedup the swarm convergence, an a weedout operator to prune a percentage of the weakest particles from the swarm, promoting a better escaping mechanism from local minima points.

Experimental evaluation has been conducted using twelve benchmark color images and by the comparison of proposed APSOW in relation to four EAs from the literature: GA, DE, standard PSO and GSO. The experimental results have been evaluated by hypothesis tests of type Friedman test in relation to the Between-Class Variance index (the adopted fitness function for all evaluated EAs), considering the original color image (in RGB) and each one its color channels individually. According to Friedman/Nemenyi tests, the proposed APSOW is able to outperform the comparison approaches in most of the evaluated scenarios, showing the potential of the proposed method for image segmentation tasks.

Tabela 2. Experimental results for 2-D: Mean $\pm$ standard deviation.

Image	Algorithm	f_{I_R}	f_{I_G}	f_{I_B}	f_I	Time (s)
Airplane	GA	1763.1 $\pm$ 19.83	2403.3 $\pm$ 20.11	906.35 $\pm$ 23.95	5072.7 $\pm$ 30.00	2.3797 $\pm$ 0.0160
	DE	1787.9 $\pm$ 0.0053	2436.4 $\pm$ 0.0006	930.78 $\pm$ 0.0032	5155.1 $\pm$ 0.0061	2.3459 $\pm$ 0.0157
	PSO	1787.9 $\pm$ 0.0087	2436.4 $\pm$ 0.0004	930.78 $\pm$ 0.0	5155.1 $\pm$ 0.0087	2.3319 $\pm$ 0.0249
	GSO	1787.9 $\pm$ 0.2361	2436.3 $\pm$ 0.0959	925.51 $\pm$ 27.23	5149.7 $\pm$ 27.24	2.4438 $\pm$ 0.0216
	APSOW	1787.9 $\pm$ 0.0052	2436.4 $\pm$ 0.0002	9307.8 $\pm$ 0.0	5155.1 $\pm$ 0.0052	2.4796 $\pm$ 0.0668
Gellybeans1	GA	1034.8 $\pm$ 15.50	1883.8 $\pm$ 16.00	1641.0 $\pm$ 11.13	4559.6 $\pm$ 13.02	0.8600 $\pm$ 0.0242
	DE	1052.5 $\pm$ 0.0027	1904.2 $\pm$ 0.0075	1653.9 $\pm$ 0.0056	4610.5 $\pm$ 0.0117	0.8256 $\pm$ 0.0242
	PSO	1050.8 $\pm$ 9.197	1904.2 $\pm$ 0.0008	1653.9 $\pm$ 0.0064	4608.8 $\pm$ 9.197	0.8172 $\pm$ 0.0242
	GSO	1047.4 $\pm$ 15.37	1904.2 $\pm$ 0.0	1648.1 $\pm$ 31.54	4599.7 $\pm$ 34.22	0.9354 $\pm$ 0.0262
	APSOW	1052.5 $\pm$ 0.0065	1904.2 $\pm$ 0.0011	1653.9 $\pm$ 0.0052	4610.5 $\pm$ 0.0105	0.8452 $\pm$ 0.0085
Gellybeans2	GA	1374.2 $\pm$ 22.79	2374.5 $\pm$ 22.33	1940.5 $\pm$ 20.47	5689.2 $\pm$ 28.30	0.8573 $\pm$ 0.0171
	DE	1398.3 $\pm$ 0.0035	2401.1 $\pm$ 0.0	1969.7 $\pm$ 0.0	5769.1 $\pm$ 0.0035	0.8261 $\pm$ 0.0241
	PSO	1395.4 $\pm$ 16.20	2401.1 $\pm$ 0.0103	1969.7 $\pm$ 0.0	5766.1 $\pm$ 16.20	0.8182 $\pm$ 0.0232
	GSO	1394.1 $\pm$ 16.52	2401.1 $\pm$ 0.1291	1969.6 $\pm$ 0.2913	5764.8 $\pm$ 16.89	0.9395 $\pm$ 0.0291
	APSOW	1398.3 $\pm$ 0.0035	2401.1 $\pm$ 0.0103	1969.7 $\pm$ 0.0069	5769.1 $\pm$ 0.0123	0.8461 $\pm$ 0.0069
House1	GA	2173.4 $\pm$ 45.18	1971.5 $\pm$ 39.97	3369.8 $\pm$ 36.38	7514.7 $\pm$ 64.99	2.3902 $\pm$ 0.0662
	DE	2223.4 $\pm$ 0.0	2023.9 $\pm$ 0.0090	3413.0 $\pm$ 0.0	7660.2 $\pm$ 0.0090	2.3464 $\pm$ 0.0137
	PSO	2223.4 $\pm$ 0.0097	2023.9 $\pm$ 0.0090	3413.0 $\pm$ 0.0162	7660.2 $\pm$ 0.0197	2.3360 $\pm$ 0.0135
	GSO	2223.4 $\pm$ 0.0239	2023.9 $\pm$ 0.0	3413.0 $\pm$ 0.0182	7660.2 $\pm$ 0.0311	2.4438 $\pm$ 0.0166
	APSOW	2223.4 $\pm$ 0.0297	2023.9 $\pm$ 0.0	3413.0 $\pm$ 0.0182	7660.2 $\pm$ 0.0324	2.4641 $\pm$ 0.0164
House2	GA	816.19 $\pm$ 19.10	2945.8 $\pm$ 20.61	3754.6 $\pm$ 15.21	7516.5 $\pm$ 24.68	0.8656 $\pm$ 0.0340
	DE	843.20 $\pm$ 4.442	2965.4 $\pm$ 0.0902	3769.7 $\pm$ 0.5307	7578.3 $\pm$ 4.479	0.8177 $\pm$ 0.0189
	PSO	834.47 $\pm$ 8.845	2965.4 $\pm$ 0.0045	3769.7 $\pm$ 0.5375	7569.5 $\pm$ 8.968	0.8255 $\pm$ 0.0416
	GSO	836.22 $\pm$ 8.905	2965.4 $\pm$ 0.0	3769.7 $\pm$ 0.5385	7571.3 $\pm$ 8.955	0.9491 $\pm$ 0.0576
	APSOW	844.41 $\pm$ 0.0114	2965.4 $\pm$ 0.0089	3769.5 $\pm$ 0.5108	7579.4 $\pm$ 0.5103	0.8579 $\pm$ 0.0137
Lake	GA	1548.6 $\pm$ 30.11	5595.0 $\pm$ 34.09	5599.2 $\pm$ 36.65	12742.7 $\pm$ 48.61	2.3840 $\pm$ 0.0424
	DE	1589.9 $\pm$ 0.0042	5636.6 $\pm$ 0.0	5640.5 $\pm$ 0.0104	12867.0 $\pm$ 0.0132	2.3542 $\pm$ 0.0729
	PSO	1589.9 $\pm$ 0.0	5636.6 $\pm$ 0.0083	5640.5 $\pm$ 0.0062	12867.0 $\pm$ 0.0102	2.3355 $\pm$ 0.0177
	GSO	1589.9 $\pm$ 0.0042	5636.6 $\pm$ 0.0	5640.5 $\pm$ 0.0	12867.0 $\pm$ 0.0042	2.4881 $\pm$ 0.0952
	APSOW	1589.9 $\pm$ 0.0042	5636.6 $\pm$ 0.0115	5640.5 $\pm$ 0.0062	12867.0 $\pm$ 0.0132	2.4858 $\pm$ 0.0238
Lena	GA	2140.5 $\pm$ 35.25	2328.9 $\pm$ 44.02	956.20 $\pm$ 53.37	5425.6 $\pm$ 67.83	2.3729 $\pm$ 0.0122
	DE	2183.7 $\pm$ 0.0	2393.6 $\pm$ 0.0	1004.9 $\pm$ 0.0	5582.3 $\pm$ 0.0	2.3683 $\pm$ 0.0322
	PSO	2183.7 $\pm$ 0.0163	2393.6 $\pm$ 0.0079	1004.9 $\pm$ 0.0	5582.3 $\pm$ 0.0178	2.3349 $\pm$ 0.0497
	GSO	2183.7 $\pm$ 0.0276	2393.6 $\pm$ 0.0	1004.9 $\pm$ 0.0	5582.3 $\pm$ 0.0276	2.4688 $\pm$ 0.0591
	APSOW	2183.7 $\pm$ 0.0117	2393.6 $\pm$ 0.0	1004.9 $\pm$ 0.0150	5582.3 $\pm$ 0.0187	2.4646 $\pm$ 0.0147
Mandril	GA	2584.3 $\pm$ 39.10	1946.4 $\pm$ 40.56	3295.5 $\pm$ 60.86	7826.1 $\pm$ 66.89	2.3880 $\pm$ 0.0275
	DE	2642.2 $\pm$ 0.0112	2004.7 $\pm$ 0.0138	3360.0 $\pm$ 0.0064	8006.9 $\pm$ 0.0169	2.3463 $\pm$ 0.0130
	PSO	2642.2 $\pm$ 0.0081	2004.7 $\pm$ 0.0138	3360.0 $\pm$ 0.0150	8006.9 $\pm$ 0.0242	2.3496 $\pm$ 0.0242
	GSO	2642.2 $\pm$ 0.1318	2004.7 $\pm$ 0.0641	3360.0 $\pm$ 0.0246	8006.9 $\pm$ 0.2169	2.5016 $\pm$ 0.0891
	APSOW	2642.2 $\pm$ 0.0081	2004.7 $\pm$ 0.0147	3360.0 $\pm$ 0.0150	8006.9 $\pm$ 0.0189	2.4791 $\pm$ 0.0143
Peppers	GA	1710.7 $\pm$ 33.16	5146.3 $\pm$ 43.45	1611.6 $\pm$ 35.25	8468.5 $\pm$ 40.41	2.4016 $\pm$ 0.0799
	DE	1759.2 $\pm$ 0.0	5203.9 $\pm$ 0.0	1656.2 $\pm$ 0.0	8619.3 $\pm$ 0.0	2.3407 $\pm$ 0.0118
	PSO	1759.2 $\pm$ 0.0173	5203.9 $\pm$ 0.0110	1656.2 $\pm$ 0.0117	8619.3 $\pm$ 0.0229	2.3422 $\pm$ 0.0284
	GSO	1759.2 $\pm$ 0.0185	5203.9 $\pm$ 0.0228	1656.2 $\pm$ 0.0	8619.3 $\pm$ 0.0281	2.4646 $\pm$ 0.0287
	APSOW	1759.2 $\pm$ 0.0149	5203.9 $\pm$ 0.0184	1656.2 $\pm$ 0.0	8619.3 $\pm$ 0.0229	2.4802 $\pm$ 0.0429
Splash	GA	3244.5 $\pm$ 19.45	3170.7 $\pm$ 23.00	2005.9 $\pm$ 30.94	8421.1 $\pm$ 33.84	2.3777 $\pm$ 0.0229
	DE	3268.2 $\pm$ 0.0502	3206.0 $\pm$ 6.797	2039.4 $\pm$ 0.0263	8513.5 $\pm$ 6.789	2.3338 $\pm$ 0.0096
	PSO	3268.2 $\pm$ 0.0	3183.9 $\pm$ 17.07	2039.4 $\pm$ 0.0	8491.5 $\pm$ 17.07	2.3220 $\pm$ 0.0171
	GSO	3263.5 $\pm$ 25.72	3185.1 $\pm$ 17.45	2039.4 $\pm$ 0.0	8488.0 $\pm$ 33.06	2.4802 $\pm$ 0.0466
	APSOW	3268.2 $\pm$ 0.0029	3207.6 $\pm$ 0.0	2039.4 $\pm$ 0.0115	8515.2 $\pm$ 0.0117	2.4875 $\pm$ 0.0656
Tiffany	GA	846.46 $\pm$ 8.954	865.38 $\pm$ 9.889	464.44 $\pm$ 12.85	2176.3 $\pm$ 13.31	2.3918 $\pm$ 0.0536
	DE	861.34 $\pm$ 2.512	883.93 $\pm$ 3.505	481.46 $\pm$ 0.5415	2226.7 $\pm$ 3.787	2.3500 $\pm$ 0.0568
	PSO	854.43 $\pm$ 6.917	880.53 $\pm$ 6.572	469.47 $\pm$ 22.68	2204.4 $\pm$ 22.92	2.3167 $\pm$ 0.0092
	GSO	851.01 $\pm$ 8.626	867.10 $\pm$ 16.32	464.19 $\pm$ 25.27	2182.3 $\pm$ 32.88	2.4614 $\pm$ 0.0419
	APSOW	861.12 $\pm$ 3.321	885.80 $\pm$ 0.0422	481.76 $\pm$ 0.0107	2228.7 $\pm$ 3.315	2.5302 $\pm$ 0.1415
Tree	GA	2983.5 $\pm$ 36.72	5494.9 $\pm$ 32.69	3763.3 $\pm$ 38.78	12241.7 $\pm$ 58.70	0.8807 $\pm$ 0.0276
	DE	3031.4 $\pm$ 0.0	5530.6 $\pm$ 0.0	3806.7 $\pm$ 0.0	12368.7 $\pm$ 0.0	0.8189 $\pm$ 0.0182
	PSO	3031.4 $\pm$ 0.0	5530.6 $\pm$ 0.0	3806.7 $\pm$ 0.0122	12368.7 $\pm$ 0.0122	0.8067 $\pm$ 0.0088
	GSO	3031.4 $\pm$ 0.0	5530.6 $\pm$ 0.0	3806.7 $\pm$ 0.0167	12368.7 $\pm$ 0.0167	0.9344 $\pm$ 0.0150
	APSOW	3031.4 $\pm$ 0.0	5530.6 $\pm$ 0.0	3806.7 $\pm$ 0.0147	12368.7 $\pm$ 0.0147	0.8583 $\pm$ 0.0099

Tabela 3. Experimental results for 3-D: Mean $\pm$ standard deviation.

Image	Algorithm	f_{I_R}	f_{I_G}	f_{I_B}	f_I	Time (s)
Airplane	GA	1827.9 $\pm$ 16.39	2508.7 $\pm$ 18.40	948.54 $\pm$ 17.31	5285.1 $\pm$ 19.02	2.4828 $\pm$ 0.0227
	DE	1855.6 $\pm$ 0.6661	2533.4 $\pm$ 0.7497	970.29 $\pm$ 0.7321	5359.3 $\pm$ 1.261	2.4433 $\pm$ 0.0138
	PSO	1856.3 $\pm$ 0.3349	2534.0 $\pm$ 0.1129	969.51 $\pm$ 7.401	5359.8 $\pm$ 7.379	2.4375 $\pm$ 0.0270
	GSO	1853.4 $\pm$ 12.40	2533.8 $\pm$ 0.6388	960.97 $\pm$ 14.83	5348.2 $\pm$ 18.32	2.5954 $\pm$ 0.0217
	APSOW	1856.3 $\pm$ 0.0861	2534.0 $\pm$ 0.0986	971.07 $\pm$ 0.0789	5361.4 $\pm$ 0.1805	2.5731 $\pm$ 0.0189
Gellybeans1	GA	1054.4 $\pm$ 10.45	1937.9 $\pm$ 9.369	1672.8 $\pm$ 10.55	4665.0 $\pm$ 13.65	0.9459 $\pm$ 0.0469
	DE	1075.4 $\pm$ 0.6677	1952.9 $\pm$ 0.4598	1689.9 $\pm$ 0.3213	4718.2 $\pm$ 0.9855	0.8948 $\pm$ 0.0313
	PSO	1074.3 $\pm$ 5.947	1953.1 $\pm$ 0.0593	1684.8 $\pm$ 10.81	4712.2 $\pm$ 12.01	0.8938 $\pm$ 0.0270
	GSO	1074.5 $\pm$ 4.250	1952.5 $\pm$ 1.235	1682.5 $\pm$ 11.80	4709.5 $\pm$ 11.89	1.0865 $\pm$ 0.0320
	APSOW	1075.9 $\pm$ 0.0611	1953.1 $\pm$ 0.0569	1689.2 $\pm$ 2.191	4718.2 $\pm$ 2.193	0.9141 $\pm$ 0.0127
Gellybeans2	GA	1405.4 $\pm$ 19.33	2440.0 $\pm$ 15.30	1991.4 $\pm$ 16.68	5836.8 $\pm$ 14.61	0.9349 $\pm$ 0.0243
	DE	1431.5 $\pm$ 0.3986	2465.2 $\pm$ 0.4082	2015.7 $\pm$ 1.318	5912.4 $\pm$ 1.467	0.8912 $\pm$ 0.0211
	PSO	1428.5 $\pm$ 10.22	2465.5 $\pm$ 0.0842	2010.0 $\pm$ 16.14	5904.0 $\pm$ 17.84	0.8897 $\pm$ 0.0232
	GSO	1430.5 $\pm$ 6.090	2465.3 $\pm$ 0.4169	2011.1 $\pm$ 11.53	5907.0 $\pm$ 12.92	1.0802 $\pm$ 0.0309
	APSOW	1431.8 $\pm$ 0.0520	2465.5 $\pm$ 0.0609	2016.5 $\pm$ 0.0639	5913.8 $\pm$ 0.1147	0.9240 $\pm$ 0.0223
House1	GA	2308.9 $\pm$ 45.35	2123.5 $\pm$ 42.61	3520.3 $\pm$ 43.94	7952.6 $\pm$ 45.50	2.4917 $\pm$ 0.0467
	DE	2369.6 $\pm$ 0.3048	2171.9 $\pm$ 0.4432	3575.6 $\pm$ 0.1726	8117.1 $\pm$ 0.5222	2.4298 $\pm$ 0.0140
	PSO	2369.8 $\pm$ 0.0964	2172.1 $\pm$ 0.0457	3575.7 $\pm$ 0.0465	8117.7 $\pm$ 0.1218	2.4355 $\pm$ 0.0494
	GSO	2369.8 $\pm$ 0.2255	2172.0 $\pm$ 0.2266	3570.3 $\pm$ 29.72	8112.1 $\pm$ 29.69	2.6109 $\pm$ 0.0804
	APSOW	2369.8 $\pm$ 0.0923	2172.0 $\pm$ 0.1270	3575.7 $\pm$ 0.0554	8117.6 $\pm$ 0.1888	2.5600 $\pm$ 0.0259
House2	GA	886.40 $\pm$ 24.08	2991.7 $\pm$ 19.09	3822.4 $\pm$ 20.61	7700.5 $\pm$ 23.05	0.9391 $\pm$ 0.0194
	DE	918.86 $\pm$ 0.0405	3023.5 $\pm$ 0.0277	3855.1 $\pm$ 0.0544	7797.5 $\pm$ 0.0901	0.8939 $\pm$ 0.0219
	PSO	918.86 $\pm$ 0.0151	3023.5 $\pm$ 0.0191	3849.4 $\pm$ 21.54	7791.8 $\pm$ 21.53	0.8943 $\pm$ 0.0294
	GSO	909.37 $\pm$ 24.11	3015.2 $\pm$ 11.82	3852.2 $\pm$ 15.53	7776.8 $\pm$ 31.75	1.0766 $\pm$ 0.0226
	APSOW	918.86 $\pm$ 0.0207	3023.5 $\pm$ 0.0508	3855.1 $\pm$ 0.0379	7797.4 $\pm$ 0.0702	0.9198 $\pm$ 0.0114
Lake	GA	1651.2 $\pm$ 25.36	5736.9 $\pm$ 23.73	5703.3 $\pm$ 30.49	13091.4 $\pm$ 32.10	2.4880 $\pm$ 0.0628
	DE	1693.7 $\pm$ 0.4003	5776.3 $\pm$ 0.4174	5739.9 $\pm$ 0.2470	13209.9 $\pm$ 0.7330	2.4391 $\pm$ 0.0186
	PSO	1687.1 $\pm$ 26.42	5776.7 $\pm$ 0.0543	5740.3 $\pm$ 0.0755	13204.0 $\pm$ 26.41	2.4276 $\pm$ 0.0088
	GSO	1690.5 $\pm$ 19.01	5775.8 $\pm$ 1.909	5739.4 $\pm$ 1.576	13205.8 $\pm$ 18.95	2.6068 $\pm$ 0.0314
	APSOW	1694.0 $\pm$ 0.1277	5776.6 $\pm$ 0.1894	5740.2 $\pm$ 0.1274	13210.7 $\pm$ 0.3213	2.5684 $\pm$ 0.0357
Lena	GA	2227.4 $\pm$ 28.53	2549.6 $\pm$ 24.12	1014.4 $\pm$ 28.45	5791.4 $\pm$ 34.59	2.4818 $\pm$ 0.0207
	DE	2272.3 $\pm$ 0.2780	2597.8 $\pm$ 0.1679	1069.9 $\pm$ 0.2474	5940.0 $\pm$ 0.4014	2.4417 $\pm$ 0.0711
	PSO	2263.5 $\pm$ 27.08	2591.1 $\pm$ 37.30	1061.4 $\pm$ 22.53	5916.1 $\pm$ 47.13	2.4318 $\pm$ 0.0662
	GSO	2262.9 $\pm$ 26.86	2597.9 $\pm$ 0.1255	1056.0 $\pm$ 24.83	5916.8 $\pm$ 32.96	2.6109 $\pm$ 0.0182
	APSOW	2272.4 $\pm$ 0.2046	2597.9 $\pm$ 0.0755	1070.0 $\pm$ 0.1244	5940.4 $\pm$ 0.2826	2.5536 $\pm$ 0.0246
Mandril	GA	2766.8 $\pm$ 41.40	2055.0 $\pm$ 40.72	3501.8 $\pm$ 43.47	8323.6 $\pm$ 33.76	2.5089 $\pm$ 0.0284
	DE	2830.3 $\pm$ 0.3190	2113.2 $\pm$ 0.4475	3555.6 $\pm$ 0.2109	8499.2 $\pm$ 0.6765	2.4532 $\pm$ 0.0188
	PSO	2830.6 $\pm$ 0.0482	2110.0 $\pm$ 19.89	3555.8 $\pm$ 0.1699	8496.4 $\pm$ 19.87	2.4370 $\pm$ 0.0111
	GSO	2830.6 $\pm$ 0.1471	2113.4 $\pm$ 0.7137	3555.8 $\pm$ 0.1919	8499.8 $\pm$ 0.8534	2.6402 $\pm$ 0.0530
	APSOW	2830.5 $\pm$ 0.2417	2113.5 $\pm$ 0.2280	3555.7 $\pm$ 0.1694	8499.7 $\pm$ 0.4823	2.5771 $\pm$ 0.0311
Peppers	GA	1822.5 $\pm$ 42.87	5324.6 $\pm$ 34.07	1725.3 $\pm$ 31.76	8872.3 $\pm$ 32.62	2.4787 $\pm$ 0.0265
	DE	1884.5 $\pm$ 1.134	5381.1 $\pm$ 3.245	1770.7 $\pm$ 11.49	9036.4 $\pm$ 11.28	2.4604 $\pm$ 0.0545
	PSO	1881.2 $\pm$ 23.04	5382.5 $\pm$ 0.0664	1773.0 $\pm$ 10.55	9036.8 $\pm$ 24.60	2.4235 $\pm$ 0.0222
	GSO	1885.4 $\pm$ 0.2749	5378.8 $\pm$ 11.07	1754.5 $\pm$ 23.91	9018.7 $\pm$ 25.08	2.6114 $\pm$ 0.0219
	APSOW	1885.4 $\pm$ 0.0985	5382.5 $\pm$ 0.1394	1777.1 $\pm$ 0.0833	9045.0 $\pm$ 0.2260	2.5714 $\pm$ 0.0323
Splash	GA	3366.1 $\pm$ 39.74	3423.3 $\pm$ 27.74	2044.0 $\pm$ 26.87	8833.3 $\pm$ 35.94	2.4928 $\pm$ 0.0382
	DE	3413.9 $\pm$ 0.1396	3453.0 $\pm$ 0.1008	2081.0 $\pm$ 1.193	8948.0 $\pm$ 1.079	2.4474 $\pm$ 0.0352
	PSO	3414.0 $\pm$ 0.0131	3453.1 $\pm$ 0.0189	2077.8 $\pm$ 10.50	8944.9 $\pm$ 10.50	2.4198 $\pm$ 0.0100
	GSO	3412.0 $\pm$ 11.14	3447.5 $\pm$ 30.68	2080.4 $\pm$ 2.394	8939.9 $\pm$ 32.48	2.6229 $\pm$ 0.0478
	APSOW	3414.0 $\pm$ 0.0849	3453.1 $\pm$ 0.0420	2080.8 $\pm$ 1.353	8947.9 $\pm$ 1.324	2.5709 $\pm$ 0.0498
Tiffany	GA	883.08 $\pm$ 12.42	971.56 $\pm$ 11.59	496.72 $\pm$ 11.70	2351.4 $\pm$ 13.45	2.4938 $\pm$ 0.0282
	DE	905.57 $\pm$ 2.936	987.90 $\pm$ 0.3377	521.74 $\pm$ 0.5117	2415.2 $\pm$ 2.944	2.4230 $\pm$ 0.0101
	PSO	892.51 $\pm$ 12.96	984.96 $\pm$ 7.164	511.11 $\pm$ 9.761	2388.6 $\pm$ 16.49	2.4173 $\pm$ 0.0158
	GSO	8778.0 $\pm$ 18.88	974.35 $\pm$ 28.03	507.28 $\pm$ 17.47	2359.4 $\pm$ 35.13	2.6031 $\pm$ 0.0106
	APSOW	905.71 $\pm$ 2.956	987.98 $\pm$ 0.2496	522.05 $\pm$ 0.0977	2415.7 $\pm$ 2.952	2.5605 $\pm$ 0.0217
Tree	GA	3075.8 $\pm$ 27.92	5632.8 $\pm$ 25.59	3838.0 $\pm$ 23.37	12546.7 $\pm$ 27.71	0.9698 $\pm$ 0.0303
	DE	3109.9 $\pm$ 0.7436	5662.7 $\pm$ 0.5664	3871.9 $\pm$ 0.4114	12644.5 $\pm$ 1.212	0.8876 $\pm$ 0.0105
	PSO	3108.8 $\pm$ 9.552	5663.2 $\pm$ 0.0976	3865.3 $\pm$ 15.87	12637.3 $\pm$ 21.62	0.8802 $\pm$ 0.0104
	GSO	3103.7 $\pm$ 15.28	5663.2 $\pm$ 0.1161	3866.6 $\pm$ 14.41	12633.5 $\pm$ 22.94	1.0703 $\pm$ 0.0090
	APSOW	3110.6 $\pm$ 0.0610	5663.1 $\pm$ 0.1387	3872.3 $\pm$ 0.1467	12646.0 $\pm$ 0.2657	0.9318 $\pm$ 0.0149

Tabela 4. Experimental results for 4-D: Mean $\pm$ standard deviation.

Image	Algorithm	f_{I_R}	f_{I_G}	f_{I_B}	f_I	Time (s)
Airplane	GA	1871.5 ± 18.66	255.45 ± 17.10	963.03 ± 16.65	5389.1 ± 16.73	2.5365 ± 0.0166
	DE	1900.8 ± 0.7848	2586.6 ± 0.7555	993.90 ± 0.5915	5481.3 ± 1.356	2.5302 ± 0.0263
	PSO	1898.7 ± 11.56	2587.3 ± 0.3159	989.85 ± 9.183	5475.9 ± 16.11	2.5318 ± 0.0498
	GSO	1896.6 ± 14.39	2585.6 ± 6.747	979.21 ± 15.64	5461.4 ± 20.90	2.6557 ± 0.0245
	APSOW	1901.7 ± 0.2286	2587.3 ± 0.3411	994.67 ± 0.2273	5483.7 ± 0.5306	2.6516 ± 0.0352
Gellybeans1	GA	1071.5 ± 8.708	1957.0 ± 11.28	1694.2 ± 10.43	4722.7 ± 9.509	0.9959 ± 0.0539
	DE	1089.4 ± 0.6389	1974.4 ± 0.8169	1718.0 ± 1.982	4781.9 ± 3.067	0.9589 ± 0.0347
	PSO	1088.0 ± 4.833	1974.0 ± 4.541	1707.7 ± 17.27	4769.7 ± 16.29	0.9662 ± 0.0422
	GSO	1082.9 ± 8.614	1971.0 ± 7.501	1707.7 ± 15.16	4761.7 ± 22.17	1.1298 ± 0.0329
	APSOW	1089.8 ± 0.1630	1975.1 ± 0.1775	1718.8 ± 0.1565	4783.7 ± 0.3207	0.9839 ± 0.0086
Gellybeans2	GA	1425.7 ± 12.99	2478.8 ± 13.96	2017.5 ± 13.70	5921.9 ± 16.49	0.9839 ± 0.0266
	DE	1448.4 ± 0.6094	2496.3 ± 0.4967	2045.5 ± 0.4752	5990.2 ± 0.9319	0.9656 ± 0.0305
	PSO	1446.2 ± 6.556	2494.7 ± 7.948	2034.1 ± 14.91	5974.9 ± 20.87	0.9563 ± 0.0252
	GSO	1442.7 ± 7.625	2492.8 ± 9.460	2030.5 ± 18.59	5966.0 ± 23.23	1.1261 ± 0.0355
	APSOW	1448.8 ± 0.8718	2496.6 ± 0.3538	2045.9 ± 0.4971	5991.3 ± 1.382	0.9876 ± 0.0104
House1	GA	2386.6 ± 28.13	2197.9 ± 25.78	3593.3 ± 22.10	8177.8 ± 34.24	2.5531 ± 0.0527
	DE	2437.1 ± 0.9650	2239.4 ± 1.455	3631.9 ± 2.163	8308.5 ± 2.497	2.5204 ± 0.0190
	PSO	2436.2 ± 12.56	2241.0 ± 0.3367	3629.4 ± 14.69	8306.5 ± 18.62	2.5094 ± 0.0113
	GSO	2438.2 ± 0.8670	2239.9 ± 2.625	3630.3 ± 10.66	8308.5 ± 10.95	2.6678 ± 0.0338
	APSOW	2438.3 ± 0.4987	2240.8 ± 0.4893	3633.8 ± 1.548	8312.9 ± 1.634	2.6416 ± 0.0209
House2	GA	919.25 ± 13.07	3027.5 ± 12.75	3855.7 ± 12.03	7802.5 ± 14.86	0.9793 ± 0.0181
	DE	944.90 ± 2.030	3050.7 ± 2.705	3878.1 ± 1.691	7873.8 ± 3.790	0.9677 ± 0.0328
	PSO	943.23 ± 7.124	3050.7 ± 5.679	3875.8 ± 9.497	7869.7 ± 11.53	0.9604 ± 0.0201
	GSO	9421.6 ± 6.878	3050.0 ± 6.484	3871.1 ± 21.74	7863.3 ± 23.27	1.1296 ± 0.0379
	APSOW	946.24 ± 1.802	3052.7 ± 2.275	3880.1 ± 0.8102	7879.0 ± 3.184	0.9875 ± 0.0096
Lake	GA	1703.5 ± 20.82	5823.2 ± 20.14	5748.2 ± 23.15	13275.0 ± 21.76	2.5479 ± 0.0191
	DE	1741.1 ± 1.420	5866.6 ± 1.050	5795.3 ± 1.454	13403.0 ± 2.507	2.5370 ± 0.0325
	PSO	1739.1 ± 12.26	5867.9 ± 0.2356	5794.6 ± 10.27	13401.6 ± 15.57	2.5094 ± 0.0089
	GSO	1735.9 ± 12.56	5866.4 ± 42.06	5789.8 ± 17.31	13392.1 ± 19.36	2.6734 ± 0.0488
	APSOW	1742.2 ± 0.3352	5867.7 ± 0.3839	5796.3 ± 0.5681	13406.1 ± 0.9190	2.6428 ± 0.0258
Lena	GA	2276.4 ± 21.49	2619.5 ± 23.90	1053.1 ± 26.14	5948.9 ± 32.29	2.5500 ± 0.0282
	DE	2321.0 ± 1.181	2673.8 ± 2.382	1095.4 ± 1.223	6090.1 ± 3.691	2.5308 ± 0.0693
	PSO	2312.1 ± 20.23	2665.1 ± 26.81	1090.0 ± 18.49	6067.1 ± 38.18	2.5115 ± 0.0226
	GSO	2307.5 ± 20.77	2673.5 ± 3.752	1081.3 ± 24.15	6062.3 ± 27.25	2.6735 ± 0.0534
	APSOW	2321.9 ± 0.5466	2675.2 ± 0.4719	1096.3 ± 0.8450	6093.4 ± 1.555	2.6395 ± 0.0259
Mandril	GA	2863.8 ± 36.65	2122.2 ± 27.08	3587.6 ± 34.85	8573.6 ± 35.04	2.5719 ± 0.0257
	DE	2922.8 ± 1.159	2167.8 ± 1.267	3644.7 ± 0.9718	8735.4 ± 1.853	2.5407 ± 0.0261
	PSO	2924.0 ± 0.2345	2160.0 ± 21.13	3639.4 ± 22.74	8723.5 ± 28.99	2.5406 ± 0.0270
	GSO	2923.7 ± 0.6716	2165.6 ± 10.78	3645.2 ± 0.8095	8734.5 ± 10.88	2.7010 ± 0.0826
	APSOW	2923.4 ± 0.9006	2169.4 ± 0.3442	3645.4 ± 0.2333	8738.2 ± 1.080	2.6593 ± 0.0248
Peppers	GA	1889.4 ± 28.18	5427.5 ± 2.997	1791.5 ± 31.11	9108.4 ± 32.10	2.5495 ± 0.0193
	DE	1935.0 ± 1033.7	5478.2 ± 1.240	1847.1 ± 1.705	9260.2 ± 2.699	2.5646 ± 0.0867
	PSO	1934.5 ± 9.283	5479.6 ± 0.1274	1836.0 ± 33.09	9250.2 ± 33.72	2.5260 ± 0.0283
	GSO	1933.2 ± 4.137	5478.9 ± 1.162	1836.6 ± 24.33	9248.7 ± 24.56	2.6594 ± 0.0255
	APSOW	1935.9 ± 0.5011	5479.2 ± 0.4551	1848.6 ± 0.2473	9263.7 ± 0.8627	2.7281 ± 0.1791
Splash	GA	3452.3 ± 29.73	3480.6 ± 22.92	2074.7 ± 19.29	9007.5 ± 26.43	2.5672 ± 0.0623
	DE	3487.1 ± 0.8918	3520.3 ± 6.159	2116.7 ± 1.767	9124.1 ± 6.798	2.5370 ± 0.0385
	PSO	3487.9 ± 0.5831	3514.5 ± 19.68	2117.0 ± 4.905	9119.4 ± 20.86	2.5255 ± 0.0724
	GSO	3488.0 ± 0.2391	3503.2 ± 19.60	2112.6 ± 10.46	9103.8 ± 23.65	2.6818 ± 0.0755
	APSOW	3487.9 ± 0.1822	3522.2 ± 0.2994	2117.8 ± 0.1569	9128.0 ± 0.4447	2.6510 ± 0.0710
Tiffany	GA	907.09 ± 7.918	1014.3 ± 9.706	525.00 ± 10.83	2446.4 ± 10.72	2.5480 ± 0.0508
	DE	922.62 ± 0.5295	1029.3 ± 0.4187	543.85 ± 2.059	2495.8 ± 1.923	2.5224 ± 0.0320
	PSO	921.16 ± 9.121	1029.2 ± 0.7900	539.79 ± 10.34	2490.1 ± 12.99	2.5026 ± 0.0118
	GSO	909.96 ± 20.28	1028.8 ± 1.717	534.06 ± 14.81	2472.8 ± 24.36	2.6600 ± 0.0374
	APSOW	922.83 ± 0.1525	1029.4 ± 0.4738	542.93 ± 2.532	2495.1 ± 2.383	2.6386 ± 0.0430
Tree	GA	3126.3 ± 18.29	5694.7 ± 16.02	3872.6 ± 18.67	12693.7 ± 22.95	0.9927 ± 0.0220
	DE	3157.7 ± 2.174	5722.8 ± 2.157	3908.9 ± 2.325	12789.4 ± 3.839	0.9641 ± 0.0211
	PSO	3153.8 ± 10.42	5724.8 ± 1.073	3905.5 ± 9.540	12784.0 ± 14.83	0.9490 ± 0.0099
	GSO	3148.3 ± 11.92	5724.0 ± 1.450	3902.0 ± 8.431	12774.4 ± 13.21	1.1156 ± 0.0179
	APSOW	3158.9 ± 4.339	5724.7 ± 0.7861	3909.8 ± 3.073	12793.4 ± 5.359	0.9969 ± 0.0126

Tabela 5. Experimental results for 5-D: Mean ± standard deviation.

Image	Algorithm	f_{I_R}	f_{I_G}	f_{I_B}	f_I	Time (s)
Airplane	GA	1898.4 ± 14.29	2586.2 ± 13.95	977.68 ± 14.14	5462.4 ± 15.64	2.6348 ± 0.0336
	DE	1921.3 ± 1.349	2616.3 ± 2.035	1001.7 ± 3.091	5539.4 ± 4.710	2.6083 ± 0.0238
	PSO	1920.6 ± 6.426	2614.5 ± 10.24	1000.4 ± 7.258	5535.6 ± 13.42	2.6068 ± 0.0184
	GSO	1919.2 ± 8.976	2611.8 ± 11.01	994.30 ± 11.65	5525.4 ± 16.82	2.7636 ± 0.0195
	APSOW	1922.5 ± 0.7313	2618.1 ± 0.7350	1003.9 ± 3.481	5544.5 ± 3.675	2.7536 ± 0.0518
Gellybeans1	GA	1082.9 ± 7.351	1971.7 ± 10.17	1711.0 ± 10.54	4765.7 ± 10.69	1.0515 ± 0.0250
	DE	1094.8 ± 1.765	1986.9 ± 0.5476	1729.6 ± 0.7525	4811.4 ± 1.890	1.0349 ± 0.0267
	PSO	1092.0 ± 5.317	1985.7 ± 4.230	1721.7 ± 11.02	4799.5 ± 13.23	1.0261 ± 0.0236
	GSO	1089.6 ± 5.827	1983.0 ± 6.324	1716.6 ± 15.12	4789.3 ± 15.69	1.2010 ± 0.0273
	APSOW	1095.4 ± 0.5348	1986.7 ± 2.284	1727.5 ± 4.987	4809.8 ± 5.048	1.0506 ± 0.0079
Gellybeans2	GA	1441.4 ± 8.232	2495.2 ± 11.62	2038.8 ± 11.94	5975.6 ± 16.28	1.0489 ± 0.0195
	DE	1456.8 ± 2.182	2514.8 ± 1.106	2061.5 ± 4.977	6033.2 ± 6.744	1.0287 ± 0.0221
	PSO	1455.4 ± 4.696	2512.7 ± 7.402	2054.2 ± 8.927	6022.4 ± 12.91	1.0224 ± 0.0200
	GSO	1451.3 ± 7.218	2511.9 ± 7.239	2051.4 ± 12.47	6014.6 ± 13.20	1.1969 ± 0.0285
	APSOW	1458.4 ± 0.7110	2515.4 ± 3.563	2059.7 ± 7.227	6033.6 ± 9.179	1.0625 ± 0.0352
House1	GA	2427.7 ± 21.99	2235.3 ± 21.87	3634.4 ± 20.64	8297.5 ± 25.37	2.6224 ± 0.0247
	DE	2467.7 ± 1.733	2285.6 ± 1.876	3676.0 ± 2.019	8429.4 ± 3.274	2.6073 ± 0.0219
	PSO	2465.3 ± 10.79	2281.4 ± 14.91	3676.8 ± 7.688	8423.6 ± 17.55	2.616 ± 0.0649
	GSO	2463.9 ± 9.188	2280.4 ± 12.62	3668.4 ± 17.11	8412.9 ± 21.46	2.7776 ± 0.0494
	APSOW	2469.4 ± 0.6580	2287.0 ± 0.6757	3677.8 ± 0.8035	8434.2 ± 1.449	2.7333 ± 0.0191
House2	GA	938.02 ± 11.45	3050.5 ± 14.99	3872.1 ± 13.17	7860.7 ± 13.53	1.0615 ± 0.0331
	DE	966.72 ± 3.789	3076.9 ± 3.162	3893.5 ± 1.717	7937.2 ± 6.481	1.0781 ± 0.0811
	PSO	960.59 ± 12.64	3078.2 ± 21.12	3890.6 ± 8.959	7929.4 ± 15.41	1.0651 ± 0.0726
	GSO	957.45 ± 11.99	3077.6 ± 2.164	3885.7 ± 10.69	7920.8 ± 16.86	1.1974 ± 0.0467
	APSOW	968.41 ± 0.8029	3078.9 ± 1.505	3894.6 ± 2.723	7941.9 ± 3.029	1.0542 ± 0.0091
Lake	GA	1724.9 ± 23.78	5880.0 ± 20.81	5787.7 ± 19.18	13392.7 ± 20.43	2.6240 ± 0.0115
	DE	1768.5 ± 1.605	5917.1 ± 1.429	5828.0 ± 1.693	13513.7 ± 2.730	2.8870 ± 0.1690
	PSO	1766.8 ± 8.546	5913.4 ± 15.58	5823.5 ± 1.370	13503.8 ± 21.58	2.8792 ± 0.1447
	GSO	1760.6 ± 12.77	5916.3 ± 4.290	5822.4 ± 11.46	13499.4 ± 17.14	2.7734 ± 0.0556
	APSOW	1769.8 ± 0.8341	5918.4 ± 0.6596	5830.0 ± 0.8718	13518.3 ± 1.588	2.7521 ± 0.0933
Lena	GA	2308.7 ± 22.49	2672.6 ± 20.11	1076.8 ± 18.21	6058.3 ± 19.53	2.6308 ± 0.0244
	DE	2336.0 ± 6.397	2707.1 ± 1.958	1113.9 ± 2.264	6157.1 ± 6.803	2.7755 ± 0.0869
	PSO	2333.5 ± 9.541	2703.4 ± 12.83	1107.8 ± 9.628	6144.8 ± 18.53	2.7531 ± 0.0880
	GSO	2320.4 ± 20.14	2704.7 ± 9.857	1100.1 ± 14.24	6125.3 ± 26.45	2.7776 ± 0.0419
	APSOW	2340.1 ± 3.977	2708.5 ± 0.6935	1114.2 ± 4.086	6162.9 ± 6.271	2.7322 ± 0.0608
Mandril	GA	2920.4 ± 31.67	2156.4 ± 19.95	3640.0 ± 22.86	8716.9 ± 29.53	2.6557 ± 0.0388
	DE	2967.6 ± 2.762	2199.8 ± 1.980	3690.7 ± 1.822	8858.2 ± 3.931	2.7917 ± 0.0673
	PSO	2970.7 ± 0.8978	2196.8 ± 11.38	3685.1 ± 18.08	8852.7 ± 19.48	2.7942 ± 0.0348
	GSO	2969.6 ± 2.888	2195.4 ± 12.39	3692.5 ± 1.869	8857.7 ± 13.50	2.8016 ± 0.0684
	APSOW	2970.5 ± 1.146	2201.7 ± 0.9516	3692.6 ± 0.9582	8865.0 ± 1.946	2.7625 ± 0.0465
Peppers	GA	1924.5 ± 20.50	5485.3 ± 18.68	1834.9 ± 18.03	9244.7 ± 23.55	2.6458 ± 0.0497
	DE	1963.9 ± 1.940	5523.2 ± 1.897	1877.5 ± 2.268	9364.7 ± 4.274	2.6062 ± 0.0112
	PSO	1961.9 ± 10.25	5518.9 ± 14.74	1869.8 ± 14.76	9350.8 ± 24.21	2.6328 ± 0.1370
	GSO	1957.9 ± 15.28	5523.8 ± 2.094	1872.6 ± 10.93	9354.4 ± 19.77	2.7906 ± 0.0711
	APSOW	1965.8 ± 0.6985	5524.0 ± 1.121	1880.3 ± 0.6652	9370.2 ± 1.775	2.7307 ± 0.0199
Splash	GA	3480.3 ± 15.39	3523.7 ± 14.23	2100.6 ± 17.24	9104.6 ± 19.32	2.6740 ± 0.1000
	DE	3507.3 ± 2.704	3547.7 ± 2.135	2129.5 ± 2.568	9184.6 ± 3.391	2.6146 ± 0.0531
	PSO	3507.9 ± 5.747	3548.8 ± 5.480	2129.0 ± 6.296	9185.8 ± 10.00	2.6031 ± 0.0709
	GSO	3504.8 ± 7.828	3545.9 ± 8.745	2129.7 ± 2.605	9180.6 ± 12.88	2.7635 ± 0.0306
	APSOW	3510.3 ± 0.9714	3550.6 ± 2.150	2132.0 ± 3.272	9193.0 ± 5.064	2.7157 ± 0.0087
Tiffany	GA	919.23 ± 9.068	1047.6 ± 8.907	540.53 ± 11.27	2507.3 ± 9.670	2.6302 ± 0.0230
	DE	933.71 ± 3.807	1063.9 ± 2.278	563.06 ± 0.7680	2560.7 ± 4.205	2.6047 ± 0.0137
	PSO	930.59 ± 8.803	1061.7 ± 1.089	562.80 ± 4.246	2555.1 ± 17.41	2.5895 ± 0.0123
	GSO	923.90 ± 13.02	1056.2 ± 13.09	553.29 ± 9.277	2533.4 ± 18.52	2.7682 ± 0.0477
	APSOW	934.70 ± 4.336	1065.2 ± 0.2536	563.50 ± 0.4289	2563.4 ± 4.467	2.718 ± 0.0081
Tree	GA	3160.0 ± 12.45	5727.5 ± 16.93	3899.5 ± 17.78	12787.0 ± 16.92	1.0849 ± 0.0298
	DE	3181.7 ± 1.335	5765.0 ± 1.834	3929.4 ± 1.728	12876.2 ± 2.722	1.0360 ± 0.0357
	PSO	3181.2 ± 5.859	5764.0 ± 10.59	3926.3 ± 9.951	12871.5 ± 16.07	1.0292 ± 0.0240
	GSO	3182.9 ± 1.448	5761.0 ± 12.54	3930.0 ± 4.199	12874.1 ± 12.84	1.2016 ± 0.0179
	APSOW	3181.9 ± 4.258	5766.5 ± 0.6905	3931.7 ± 1.166	12880.3 ± 4.494	1.0599 ± 0.0083

Tabela 6. Overall Evaluation: Average Ranks for the Friedman Test, with a critical distance $CD = 1.7608$.

2-D				
Algorithm	f_{I_R}	f_{I_G}	f_{I_B}	f_I
GA	18.4958	17.7833	17.0750	17.5083
DE	91.2903	88.7222	88.3625	**95.2403**
PSO	88.1458	88.9806	**91.3972**	86.1708
GSO	84.8153	86.0931	89.3347	82.5542
APSOW	**94.7528**	**95.9208**	**91.3306**	**96.0264**
3-D				
Algorithm	f_{I_R}	f_{I_G}	f_{I_B}	f_I
GA	19.5750	17.3472	20.5778	17.2694
DE	72.4042	67.6625	74.1125	77.5958
PSO	**100.4986**	**107.9736**	97.9556	101.7583
GSO	84.4417	91.3319	83.6264	75.1444
APSOW	**100.5806**	93.1847	**101.2278**	**105.7319**
4-D				
Algorithm	f_{I_R}	f_{I_G}	f_{I_B}	f_I
GA	19.3917	18.2069	22.2167	16.6333
DE	71.1847	67.4778	76.7292	80.8486
PSO	**105.4986**	**106.5014**	97.0500	98.5611
GSO	81.6278	85.5736	78.7264	68.6472
APSOW	99.7972	99.7403	**102.7778**	**112.8097**
5-D				
Algorithm	f_{I_R}	f_{I_G}	f_{I_B}	f_I
GA	21.5556	19.4778	21.8556	16.4778
DE	77.1069	74.1931	80.7833	87.3750
PSO	95.8583	**100.4625**	93.1431	88.2361
GSO	76.1528	83.0236	75.9778	67.6722
APSOW	**106.8264**	**100.3431**	**105.7403**	**117.7389**

As future works, we intend to adapt some PSO variants to the context of image segmentation by multi-level thresholding. We also intend to hybridize PSO with other EAs from the literature to perform image segmentation.

Acknowledgment

The authors would like to thank FACEPE, CNPq and CAPES (Brazilian Research Agencies) for their financial support.

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