# A Study on the Effects of Reputation-based Decision on the Dynamics of Public Goods Game with Punishment, Signaling and Gossiping Mechanisms

Mariana R. Mendoza<sup>1</sup>, Ana L. C. Bazzan<sup>1</sup>

<sup>1</sup>Instituto de Informática – Universidade Federal do Rio Grande do Sul Caixa Postal 15.064 – 91.501-970 – Porto Alegre – RS – Brazil

{mrmendoza,bazzan}@inf.ufrgs.br

Abstract. In public goods game individuals contribute in favor of a common benefit. However this attracts free-riders, who profit the benefits generated by the group regardless their contribution decision. Although one would expect a cooperation collapse within a rational society, what is observed, in fact, is the emergence of cooperation. In the present work we not only address mechanisms such as punishment, signaling, and gossiping, but also add reputation-based decision-making to the process. We show that the dynamics of public goods game changes with the inclusion of the latter mechanism. Specifically, a decrease in the average wealth and contribution is observed because agents tend to be more selective. This holds a similarity with real-world situations: Human beings do act based on reputation about their partners.

## 1. Introduction

In the last years the evolution of cooperation has been a widely studied topic in manifold disciplines, such as biology, economy and computer science, just to mention some. What intrigues researchers of this field is how cooperation is maintained in such a competitive world. Through both theoretical and experimental investigation, researchers aim to answer questions like why do people cooperate when they could just profit from the benefit generated by others. Assuming rational decision making, one would expect an uncooperative behavior from individuals. Yet, what we observe is the emergence of cooperation. In [Nowak 2006], many mechanisms involved with the evolution of cooperation are discussed. The author explains that competition takes place between all organisms, no matters their complexity, due to natural selection's principles, but that cooperation is needed in the construction of new levels of organization.

One well established paradigm for discussing altruistic behavior is the public goods game. In this game, individuals incur a cost to create a benefit for a group. Notwithstanding individuals' decision about contributing, they enjoy the benefits generated by the group. We can observe cooperation emergence in free-software development, blood donation, wikipedia, e-bay reviews about sellers, etc. This is an issue if, again, we suppose individuals act rationally. Free-riders would be attracted by the benefits, proliferate and, at some moment, cooperation would collapse. However, human societies have somehow managed to solve this and great interest and effort has been dedicated to the study of public goods game dilemma. Several mechanisms have been proposed in order to explain this phenomenon, like sanction, reputation and signaling. See [Henrich 2006, Nowak 2006] for an overview. Punishment mechanism has been experimentally studied by [Gürek et al. 2006] with real subjects playing the public goods game. Players should select among two institutions to participate, one of which involving sanction. Authors show that individuals do prefer a sanctioning institution over a sanctioning free one, demonstrating the importance of punishment to the stabilization of cooperation. This mechanism was also addressed in [Bazzan and Dahmen 2010] to investigate evolution of cooperation among synthetic persons in a public goods game. Authors propose the creation of a more detailed scenario, where agents can punish, are exposed to bribery and can spread gossips about their acquaintances' contribution profiles.

The contribution profile is inspired by the so called green-beard model, proposed by Hamilton [Hamilton 1964] and named by Dawkins [Dawkins 1976]. In the latter, author suggests that if a gene arises that not only gives individuals a very distinguishable physical trait but also a tendency to be altruistic towards other individuals who carry the same trait, mutual altruism between these individuals could evolve. In the public goods game context, the green-beard effect is a metaphor to denote individuals who contribute when they see other green beards. [Bazzan and Dahmen 2010] modified the green-beard principle to include two kinds of beards: the blue and the red ones. These colors come from the seminal work of Nowak and May [Nowak and May 1992] on the spatial Iterated Prisoner's Dilemma, where blue is used to depict cooperation while red means defection.

In the present paper we are particularly interested in agent-based simulation of public goods game to investigate some open questions about the effect of different population sizes and punishment costs over the dynamics of the public goods game, raised in [Bazzan and Dahmen 2010]. Also, we extend the model proposed by the authors by including a reputation-based decision mechanism to bring gossip phenomenon closer to reality. In the work developed by [Bazzan and Dahmen 2010], agents believe in any rumors received about other agents' profile, regardless who the sender is. In our model, all received information is filtered according to the reputation assigned to other agents, such that only rumors sent by trustful source will be accepted.

Reputation refers to a collection of opinions that agents have about coexisting individuals in the society based in past experiences. When this observation is measured and quantified, the resulting rating may represent a valuable information to assist in decision making. In [Brandt et al. 2003], for instance, neighbors' reputation was used for deciding about whether to contribute with the common pool in spatial public goods game. Although green-beard effect somehow represents a reputation about an individual's behavior, in our work reputation has nothing to do with contribution decisions, instead, it is related to gossip spreading. The main motivation of aggregating a reputation mechanism to the model proposed in [Bazzan and Dahmen 2010] is that in real world humans do keep a register of their past experiences in their memories so that they can be later used to help them decide about what actions to take.

This paper is organized as follows. In Section 2 a brief review about the public goods game dilemma will be made. In the sequence, its modeling and dynamics according to [Bazzan and Dahmen 2010] will be explained and further details about the reputation mechanism will be given. Section 4 presents the settings and results of simulations run. Finally, in Section 5 we expose our concluding remarks.

## 2. Public Goods Game

In its original formulation, this game deals with public spending on libraries, community roads, etc. Individuals are offered to invest their money in a common pool, knowing that the overall invested amount is multiplied by an interest rate and equally divided by all participants, regardless their contributions. Therefore, the group as a whole does better when all individuals decide to contribute with the public pool. However, each individual faces the temptation to defect and to free-ride on the other individuals' contribution.

In theory, the Nash equilibrium in this game is simply zero contributions by all. Nevertheless, the Nash equilibrium is rarely seen in experiments; people do tend to add something into the pool. From a theoretical point of view, the reasons for this outcome are not fully understood but probably involve issues related to signalling, punishment and reputation. The two latter have been proven by [Brandt et al. 2003] as useful mechanisms to improve readiness of cooperation between agents. In [Gintis et al. 2001], the evolution of cooperation is explained as a natural consequence of the cooperation as an honest signal of the member's quality as a mate. Yet, the effects of sharing such information between agents have not been studied by the authors. Signalling mechanisms have been later explored by [Bazzan and Dahmen 2010]. Other former works have shown, for instance, that altruistic behavior prevails in the context of strong reciprocity [Gintis 2000] or between local interacting agents adopting an imitation mechanism [Bergstrom 2002].

#### 3. Modeling

#### 3.1. General Model

The basic behavior of our formulation of the public goods game is defined by a group of connected individuals or agents, usually disposed in a  $n \times n$  grid. Each individual must decide at each time step whether or not to contribute a number of tokens to the common pool. If contributing, s/he must also decide about the value  $q_i(t) \in [q_{min}, q_{max}]$  to be contributed. Overall contributions are multiplied by an interest rate r > 0 and are then equally divided between all participating agents, increasing their wealth. Therefore, the average contribution and the return per agent are given by Eqs. 1 and 2.

$$\bar{q}(t) = \frac{1}{n} \sum_{i=1}^{n} q_i(t)$$
 (1)

$$R_i(t) = \bar{q}(t) \times r - q_i(t) \tag{2}$$

An accumulated wealth W is computed at each time step as shown in Eq 3. Also, each agent has an average wealth, which is calculated over time, according to Eq 4, where  $q_{max}$  is the number of tokens given to each agent at each time step.

$$W_i(t+1) = W_i(t) + R_i(t) + q_{max}$$
 (3)

$$\bar{w}_i = \frac{W_i(t)}{\sum t} \tag{4}$$

In what follows we explain the model proposed in [Bazzan and Dahmen 2010] and already mentioned in Section 1. The dynamics of public goods game, repeated for several time steps, is summarized in three main steps:

- 1. select value  $q_i(t) \in [q_{min}, q_{max}]$  to contribute;
- 2. randomly draw a free-rider from known agents list and decide whether and how much to punish; punished agents may bribe;
- 3. propagate gossip about other agents.

Agents are created with labels which indicate her/his contribution's profile: freeriders (FC) are in red, while high-contributors' (HC) identification is blue. The proportion of red beards in the environment is defined by parameter  $p_R$ . The idea of the green beard effect in this scenario is that individuals' profiles are visible for others agents and, most important, they influence their decision about contribution. Also, each agent keep for her/himself a list of observed blue beards (high-contributors,  $B_i$ ) and red beards (freeriders,  $R_i$ ). Free-riders never contribute to the common pool, i.e., they contribute  $q_{min} =$ 0. High-contributors decide how much to contribute depending on their knowledge about others players' profiles. They may contribute with any value  $q_i(t) \in [q_{min_{HC}}, q_{max}]$ , defined according to the following rules (where for instance,  $|B_i|$  is the cardinality of set  $B_i$ ):

- $q_i = q_{min_{HC}}$ , if  $|B_i| < |R_i|$ ;
- $q_i = q_{max}$ , if  $|B_i| > |R_i|$ ;
- $q_i = random(q_{min_{HC}}, q_{max})$ , if  $|B_i| = |R_i|$ .

Return and wealth are then computed. After that, agents decide whether and how much they will punish. Any member of the red beards' list may be punished and the choice is random. Punishment mechanism occurs after all agents have contributed and it depends on the average wealth of a given individual and on the willingness to punish ( $\omega$ ). Agents will punish only if their average wealth ( $\bar{w}_i$ ) is higher than the cost of punishing ( $c_{me}$ ) multiplied by the factor  $\omega$ . Punished agents may have their balance decreased by  $c_{you}$  while those who had applied the punishment may lose  $c_{me}$ . However, before punishment costs are applied, the punished agent may try to bribe her/his punisher as long as they are in close neighborhood, i.e. immediate neighbors.

Bribery is only accepted if its value is higher than the internal threshold of the agent who is practising punishment. The internal threshold is a private information computed as follows: agent *i* is initialized with a factor  $\beta_i$  whose value is drawn from a normal curve with mean 2 and deviation 1. This factor is multiplied by the agent's average wealth, determining the minimum bribe accepted by this agent. Also, agents are created with a factor  $\lambda_i$ , which is multiplied by its average wealth in order to specify the maximum bribe paid by this agent when s/he wishes to tempt bribe. The wealth of players *i* and *j*, involved in punishment and bribery processes, are then updated according to the following rules. If bribery did not happened or was not accepted, accumulated wealths of *i* and *j* are updated to  $W_i = W_i - c_{me}$  and  $W_j = W_j - c_{you}$  respectively. However, if bribery was accepted, player *j* is transferred from red beards' list to blue beards' list of player *i* and accumulated wealths are updated such that  $W_i = W_i + \beta_j \overline{w}_j$  and  $W_j = W_j - \lambda_i \overline{w}_i$ .

In the last stage of the algorithm described in [Bazzan and Dahmen 2010], agents may spread rumors about their blue and red beards' lists with probability  $p_g$ . When this condition is verified, agent *i* sends a message to each one of its *k* neighbors containing this information. When an agent receives a message, s/he filters out all rumors involving the close neighbors. This avoids that this agent is led to believe in erroneous information about her/his own neighbors. This is a reasonable assumption since the existing relationship allows them to check directly the real beard color of its k-th neighbors. At the end of each round, agents with negative average wealth are eliminated. The number of steps for which this dynamics will be repeated is given by  $t_{max}$ .

## 3.2. Reputation Mechanism

As previously mentioned, in this work we extend the model described in the previous section, aggregating a reputation mechanism between agents. Reputation refers to a collection of opinions that agents have about coexisting individuals in the society. When this observation is measured and quantified, the resulting rating may represent a valuable information to assist in decision making. In [Mui et al. 2003], a study about reputation across several subjects was discussed and authors concluded that reputation is a multiple parts notion and a context-dependent quantity. In electronic commerce environment, for instance, reputation has been applied as a modeling framework of sellers' reliability aiming to encourage transactions. In social contexts, such as the iterated prisoners' dilemma game, reputation was already used to explain cooperation between selfish individuals: agents decide whether to cooperate or defect based on the opponent's reputation, which is inferred from the ratio of cooperation over defection.

The main goal of including a reputation mechanism in the public goods game's dynamics described in Section 3.1 is to make the model even more detailed and allow agents to decide whether or not they will believe in the gossip spread by their neighbors based on the reputation associated to them. As rumors may include erroneous information about red beards who have previously bribed other agents to lie about their contribution profile, the fact that an agent will only believe in information originating from a trustful source may influence in the overall cooperation and wealth. By adding a reputation verification to the gossip mechanism, all information coming from high-reputation neighbors will be accepted, while those sent by low-reputation neighbors will be rejected. This notion of low and high reputation is variable between agents, providing more dynamism to the environment.

At the beginning of the simulation, each agent sets the reputation of her/his neighbors to 0. Simultaneously, a factor  $\alpha_i$  is sampled for each agent from a normal distribution with mean 1 and standard deviation 1. This factor represents the agents' demanding profile in respect to their neighbors' reputation, i.e. the minimum reputation required to accept any propagated rumors. Consequently, on average individuals will demand a minimum reputation equal to 1. Also, two extreme situations will happen, characterizing agents with antagonistic profiles: some agents will be very tolerant with their neighbors' ( $\alpha_i \leq 0$ ), while others will be very rigorous about their neighbors' reputation ( $\alpha_i \geq 2$ ). Therefore, the variation in agents' demanding profiles is associated to the  $\alpha_i$  factor.

When rumors are received, the agent verifies if the reputation associated to the neighbor who sent them is greater than her/his internal threshold. If not, this agent will ignore all the information received. However, if the neighbor has a higher reputation, the agent will not only believe in the rumors, but also check if they do not conflict with her/his own knowledge, i.e., if agents labelled as free-riders are in fact in her/his red beards' list and if the high-contributors spread by the neighbors are in fact in her/his blue beards' list. This process is done only for gossip about agents who are already member of any of the agent's lists; unknown players will be directly added in the corresponding list according to the received information.

The reputation associated to a gossiper neighbor is a dynamic value, which is updated according to the information previously sent by her/him. This is done such that neighbors who usually propagate correct information will have higher reputation and therefore, higher chances to spread her/his gossips. If any sent gossip goes against the receiver's knowledge about other agents' contribution profile, the responsible neighbor will have her/his respective reputation decreased by a constant rate  $\delta_{dec}$ . Otherwise, the reputation associated to this neighbor will be increased by  $\delta_{inc}$ , as well as the probability of acceptance of her/his gossips in subsequent time steps.

## **3.3. Tools**

The public goods game was modeled in Netlogo<sup>1</sup>, a commonly used tool for agent-base simulations. Each agent has two types of attributes: local and global. Global attributes are shared by all agents and remain constant over time, while local attributes may vary between agents and during the simulation. Agents' attributes are summarized in Table 1, where G denotes global attributes and L stands for local attributes. The list of scenario's attributes and their possible values according to model's definitions are shown in Table 2. The environment is set up at the beginning of each simulation according to the data retrieved from those variables.

| Attribute      | Description  | Туре |  |  |  |
|----------------|--|------|--|--|--|
| $c_{me}$       | cost to punish                                     | G    |  |  |  |
| $c_{you}$      | cost if punished                                   | G    |  |  |  |
| ω              | willingness to punish                              | G    |  |  |  |
| $w_0$          | initial wealth                                     |      |  |  |  |
| $\delta_{dec}$ | decrease rate of reputation                        |      |  |  |  |
| $\delta_{inc}$ | increase rate of reputation                        | G    |  |  |  |
| $q_{min}$      | minimum contribution for free-riders               | G    |  |  |  |
| $q_{min_{HC}}$ | minimum contribution for HC                        | G    |  |  |  |
| $q_{max}$      | maximum contribution                               | G    |  |  |  |
| $T_i$          | tag of agent <i>i</i>                              | L    |  |  |  |
| $N_i$          | neighbors' list of <i>i</i>                        | L    |  |  |  |
| $B_i$          | blue beards' list of <i>i</i>                      | L    |  |  |  |
| $R_i$          | red beards' list of <i>i</i>                       | L    |  |  |  |
| $q_i$          | contribution of <i>i</i>                           | L    |  |  |  |
| $W_i$          | accumulated wealth of <i>i</i>                     | L    |  |  |  |
| $R_i$          | return received by <i>i</i>                        | L    |  |  |  |
| $\bar{w}_i$    | average wealth of <i>i</i> over time               | L    |  |  |  |
| $\beta_i$      | percentage of $\bar{w}_i$ offered as bribe         | L    |  |  |  |
| $\lambda_i$    | factor to compute minimum accepted bribe           | L    |  |  |  |
| $\alpha_i$     | minimum reputation required by <i>i</i>            | L    |  |  |  |
| $\gamma_i$     | reputation associated by <i>i</i> to its neighbors | L    |  |  |  |

Table 1. Agents' attributes.

# 4. Simulations and Results

## 4.1. Settings

Different tests were made with the model created in Netlogo. The experiments' goal is to investigate the impact of the population size, the punishment costs and the in-

<sup>&</sup>lt;sup>1</sup>http://ccl.northwestern.edu/NetLogo/

clusion of a reputation mechanism over the basic dynamic exposed in Section 3.1. The main model's parameters were configured exactly as the variables' values used in [Bazzan and Dahmen 2010], so that results comparison is possible: r = 1.2,  $c_{me} = 5$ ,  $c_{you} = 20$ ,  $\omega = 3$ ,  $q_{min} = 0$ ,  $q_{min_{HC}} = 10$ ,  $q_{max} = 20$  and  $p_R = \{25, 50, 75\}$ . The scenario is originally composed by n = 225 agents disposed in a  $15 \times 15$  grid. For experiments involving the test of different population sizes or punishment costs, we varied parameters n and the rate  $c_{me}/c_{you}$ , respectively, while the tests of the reputation mechanism were performed maintaining the original variables' values. Results' analysis is made based on the average contribution (Eq. 1) and the average accumulated wealth (Eq. 4) collected at the last time step over 30 runs of the same simulation setting. We remark that contribution is an instantaneous score.

| Attribute       | Description                      | Possible values            |  |  |  |  |  |  |
|-----------------|----------------------------------|----------------------------|--|--|--|--|--|--|
| n               | population size                  | $\{21, 49, 81, 121, 225\}$ |  |  |  |  |  |  |
| $p_R$           | percentage of free-riders        | $\{25, 50, 75\}$           |  |  |  |  |  |  |
| r               | interest rate                    | [1.0:0.1:1.5]              |  |  |  |  |  |  |
| $p_g$           | probability of gossip            | [10:10:100]                |  |  |  |  |  |  |
| $t_{max}$       | number of game steps             | [50:50:300]                |  |  |  |  |  |  |
| $n_s$           | number of simulations to perform | [1:1:30]                   |  |  |  |  |  |  |
| p?              | activate/deactivate punishment   | $\{true, false\}$          |  |  |  |  |  |  |
| $\overline{g}?$ | activate/deactivate gossip       | $\{true, false\}$          |  |  |  |  |  |  |
| $\overline{r?}$ | activate/deactivate reputation   | $\{true, false\}$          |  |  |  |  |  |  |

Table 2. Model's attributes and their possible values.

#### 4.2. Experiments

The first experiments derive from [Bazzan and Dahmen 2010] and consist of a variation of the free-riders' percentage, combining it with the activation of punishment and bribery mechanisms and with different gossip probabilities. The purpose of these experiments was to test the model and verify if the observed behavior matches with the one described at [Bazzan and Dahmen 2010]<sup>2</sup>. Results are shown in Table 3. As expected, the greater the percentage of free-riders in the environment, the lower the average wealth and contribution of players, since the decision of whether and how much to contribute is directly related to the number of red beards an agent knows. Besides, the activation of punishment, and hence bribery, and gossip mechanisms seems to increase the average contribution due to more elimination of free-riders.

In the sequence, we repeated the experiments of [Bazzan and Dahmen 2010] for different population sizes, aiming to examine if the system dynamics is affected by this parameter. We tested  $n = \{49, 81, 121\}$  for a scenario with and without punishment mechanism and compared it to the results published by the authors for a scenario with 225 agents. The simulations' results are shown in Figure 1. It is possible to observe that the mean for both average contribution and average accumulated wealth haven't suffered significant variation, meaning that this parameter seems not to interfere in the basic dynamics of the public goods game. The most visible changes have happened when punishment mechanism is activated (right column), specially for a scenario with 50% of free-riders.

<sup>&</sup>lt;sup>2</sup>The original implementation was not made in Netlogo, but using SeSAm.

|       |       | $<\bar{w}_i>$ |       | $\langle q_i \rangle$ |       |      |
|-------|-------|---------------|-------|-----------------------|-------|------|
| $p_R$ | p?    | $p_g$         | avg.  | std.                  | avg.  | std. |
| 25%   | false | 0             | 22.97 | 0.16                  | 13.86 | 0.86 |
|       | true  | 0             | 21.97 | 0.84                  | 18.86 | 0.46 |
|       | true  | 20            | 24.04 | 0.04                  | 19.98 | 0.03 |
|       | true  | 50            | 24.02 | 0.03                  | 20.00 | 0.00 |
|       | true  | 70            | 24.02 | 0.03                  | 20.00 | 0.00 |
| 50%   | false | 0             | 21.62 | 0.16                  | 7.13  | 0.85 |
|       | true  | 0             | 17.30 | 0.94                  | 13.50 | 1.67 |
|       | true  | 20            | 23.42 | 0.26                  | 19.93 | 0.09 |
|       | true  | 50            | 23.62 | 0.13                  | 19.99 | 0.00 |
|       | true  | 70            | 23.66 | 0.10                  | 20.00 | 0.00 |
| 75%   | false | 0             | 20.77 | 0.10                  | 2.89  | 0.50 |
|       | true  | 0             | 16.34 | 0.49                  | 6.59  | 0.76 |
|       | true  | 20            | 13.70 | 0.56                  | 8.08  | 2.25 |
|       | true  | 50            | 14.20 | 0.54                  | 7.82  | 2.91 |
|       | true  | 70            | 14.13 | 0.45                  | 8.76  | 2.75 |

Table 3. Average wealth and average contribution in the last time step over 30 simulation runs obtained with our model for scenarios proposed in [Bazzan and Dahmen 2010].

Later, we ran simulations using distinct values of punishment costs, varying the rate between  $c_{me}/c_{you}$ . We tested ratios 1:1, 2:1, 3:1 e 5:1. Ratio 4:1 is the original one, used in [Bazzan and Dahmen 2010], thus it was not included in this set of simulations. The results, depicted in Figure 2, show that the greater the difference between  $c_{me}$  and  $c_{you}$ , the higher the contribution level and, therefore, the average wealth of agents. Thus, this parameter has a direct effect over the game's dynamics. This behavior is probably due to the fact that a higher punishment cost causes more free-riders to be eliminated, which affects the decision of other agents about how much to contribute. At this point it is worth to remind that the decision about whether and how much to punish is directly related to the number of free-riders known by each agent. The effect over the mean average wealth was more remarkable for ratio 5:1, in which  $c_{me} = 5$  and  $c_{you} = 25$ . In this case, a significant raise in individuals' wealth can be observed for all percentages of free-riders. Other proportions have resulted in similar marks for scenarios with 50% and 70% of free-riders, and a more expressive variation is observed when the rate of free-riders is 25%.

The last step performed in this study consists of the inclusion of the reputation mechanism, described in Section 3, by which agents will consider the rumors received from their neighbors depending on the reputation assigned to them. Reputations are dynamically modified based on the veracity of information received. When an agent receives rumors from her/his neighbor that goes against her/his own knowledge, this neighbor will have the reputation decreased by factor  $\delta_{dec}$ . Otherwise, if the information matches or if it refers to a new knowledge, the neighbor's reputation will be increased by factor  $\delta_{inc}$ . The motivation is that individuals will become more selective about the rumors spread by their neighbors, ignoring the erroneous information usually sent by those with low reputation.

First, we tested the case where  $\delta_{dec} = \delta_{inc} = 0.2$  and observed how the inclusion of a reputation mechanism affects the basic dynamics of the public goods game. Figure 3 shows a comparison between the average wealth and contribution for a scenario with



Figure 1. Average contribution and accumulated wealth of simulations with different population size for scenarios with and without punishment mechanism. The graphs' legends are shown in the bottom left subfigure.



Figure 2. Average contribution and wealth for different ratios  $c_{me}/c_{uou}$ .

and without this mechanism. The graphs' legends are shown in the bottom left subfigure. To help in results' interpretation, we also plot the average number of agents eliminated. As one can notice, for most of used settings, the activation of this mechanism caused a reduction in the average wealth and contribution. The lower average contribution is closely related to the reduction in the amount of eliminated individuals (Figure 3-c), who generally have a free-rider profile.

A larger concentration of free-riders in the scenario causes contribution levels to decrease. This fact alone justifies the small variation in wealth and contribution values in the scenario with 25% of free-riders, in which the elimination of non-contributing agents has little impact on the dynamics. However, an unexpected small increase in the average wealth may be observed when the percentage of free-riders is equal to 75%. We suspect that the reason of this behavior is related to the higher wealth concentration between free-riders. Since the only cost they have is the punishment cost, when not punished their accumulated wealth increases substantially. When more free-riders agents remain in the scenario, which is the case as players elimination rate decreased, total accumulated wealth is higher and, consequently, also the average wealth.



Figure 3. (a) average wealth, (b) average contribution and (c) average number of eliminated players between scenarios with and without reputation mechanism. The graphs' legends are shown in the left subfigure.



Figure 4. Average contribution and wealth for group of players with lowest and highest demand about their neighbors' reputation.

To better understand the consequences of including a reputation mechanism in a scenario like the public goods game, we run some simulations following more closely agents who have antagonistic profiles. We selected the group of agents who require low reputation from their neighbors, i.e. agents who have  $-1.0 \le \alpha_i \le 0$ , whom we call tolerant agents, and the group of most demanding players, i.e. those who have  $2.0 \le \alpha_i \le 3.0$ , to whom we refer to as rigorous agents. For these, we plot the average contribution and wealth in Figure 4. In this figure one can observe that the difference is modest but yet, the group of rigorous individuals tend to contribute less as they rarely accept information about briberies propagated by their neighbors. Therefore, their red beards' list is more extensive than their blue beards' list. Thus, according to the relationships in Eqs. 2 and 3, their average accumulated wealth is higher on average.

Two other variants were tested: i) the case where  $\delta_{dec} = 0.2$  and  $\delta_{inc} = 0.3$ ; and ii)  $\delta_{dec} = 0.3$  and  $\delta_{inc} = 0.2$ . These values simulate the cases in which agents attach distinct relevance levels to mistakes and hits contained in received rumors. Results are depicted respectively in Figures 5-a and 5-b. No significant variation in the behavior of curves for average accumulated wealth and average contribution plotted in both figures is observed. Therefore, one concludes that the fact that an individual attaches more importance to the mistakes ( $\delta_{dec} > \delta_{inc}$ ) or hits ( $\delta_{inc} > \delta_{dec}$ ) made by her/his neighbors seems not influence the results qualitatively. In quantitative terms, the changes are minor and occur mainly in

scenarios with 50% of free-riders.



Figure 5. Average contribution and wealth for agents with lowest and highest demand about their neighbors' reputation when a)  $\delta_{dec} < \delta_{inc}$  and b)  $\delta_{dec} > \delta_{inc}$ .

#### 5. Conclusion

In this work we have investigated the effects of parameters such as population size and punishment costs on the dynamics of public goods game using an agent-based simulation. Also, we proposed a more detailed scenario for gossip spreading based in the concept of reputation. Our main concluding remarks may be summarized as follows. The basic dynamics, in which the more red beards are seen the lesser the contribution and wealth of agents, is insensible to variations in the size of population. We tested values  $n = \{49, 81, 121, 225\}$  and no significant change in average wealth and contribution was observed. Regarding the punishment costs, we modified the ratio 4:1 applied in [Bazzan and Dahmen 2010], where  $c_{me} = 5$  and  $c_{you} = 20$ , using ratios 1:1, 2:1, 3:1 e 5:1. We observed that the greater the difference between  $c_{me}$  and  $c_{you}$ , the higher the contribution level and, therefore, the average wealth of agents. This effect is related to the fact that a higher punishment cost causes more free-riders to be eliminated, which affects the decision of other agents about how much to contribute.

The reputation mechanism caused a decrease in the average wealth and contribution. The lower average contribution is closely related to the reduction in the amount of eliminated agents, who generally have a free-rider profile. Also, we observed that the rigorous agents rarely accept information about briberies and, therefore, they tend to contribute less because their red beards' list is more extensive than their blue beards' list. The relevance of such simulations is the studying more realistic behaviors. In fact, our results hold a similarity with real-world situations: Human beings do select in which partners to believe, according to a reputation degree assigned to them. This reputation reflects somehow the trustful level of agents based on past experiences. However it causes a loss of utility (here contribution level). For instance, someone may believe in a partner even after this partner has shared wrong information a couple of times. However, at some point, a bad reputation will be attached to the sender and no more gossip spread by her/him will be accepted in the future.

Once a bad reputation is earned, agents will no longer have the opportunity to

spread their gossips to those neighbors whose threshold is higher than their own reputation. This is surely not representative of real life. Therefore, it would be interesting to refine the model so that it becomes even more close to reality. The first direction in which it could be modified is to allow agents to decide, with a given probability, if they wish to give a second chance for neighbors with low reputation. This may be referred as the "forgiveness probability". Also, we judge interesting to test a different and nonlinear strategy of reputation update, which is more characteristic of the real world.

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# References

- Bazzan, A. L. C. and Dahmen, S. R. (2010). Bribe and punishment: Effects of signaling, gossiping, and bribery in public goods games. *Advances in Complex Systems*, 13(6):755–771.
- Bergstrom, T. (2002). Evolution of social behavior: Individual and group selection. *Journal of Economic Perspectives*, 16(2):67–78.
- Brandt, H., Hauert, C., and Sigmund, K. (2003). Punishment and reputation in spatial public goods games. *Proc Biol Sci*, 270(1519):1099–1104.
- Dawkins, R. (1976). The Selfish Gene. Oxford University Press, New York. 224 pp.
- Gintis, H. (2000). Strong reciprocity and human sociality. *Journal of Theoretical Biology*, 206(2):169–179.
- Gintis, H., Smith, E. A. L. D. E. N., and Bowles, S. (2001). Costly signaling and cooperation. *Journal of Theoretical Biology*, 213(1):103–119.
- Gürek, O., Irlenbusch, B., and Rockenbach, B. (2006). The competitive advantage of sanctioning institutions. *Science*, 312(5770):108–111.
- Hamilton, W. D. (1964). The genetic evolution of social behavior ii. *J Theoretical Biology*, 7(1):17–52.
- Henrich, J. (2006). Cooperation, punishment, and the evolution of human institutions. *Science*, 312(5770):60–61.
- Mui, L., Halberstadt, A., and Mohtashemi, M. (2003). Evaluating reputation in multiagents systems. In *Proc. of the AAMAS*, number 2631 in LNAI, pages 123–137.
- Nowak, M. A. (2006). Five rules for the evolution of cooperation. *Science*, 314(5805):1560–1563.
- Nowak, M. A. and May, R. M. (1992). Evolutionary games and spatial chaos. *Nature*, 359:826–829.