

# Optimizing Empirical Methods for Calculating the Bearing Capacity of Concrete Piles

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**Abstract.** *Designing concrete piles that are low-cost and safe requires reliable methods to predict their bearing capacity. Empirical design methods are a popular alternative, like Meyerhof's method (MH), which suits better temperate soils, and Décourt-Quaresma's (DQ), which is more suitable for tropical soils. Coefficients empirically calibrate these methods; nevertheless, they frequently become inaccurate for specific cases. This work aims to recalibrate these two empirical design methods using datasets containing static load tests, all obtained for tropical soil. The Lichtenberg Algorithm (LA) is applied to find optimal coefficients, considering three pile types for MH and two for DQ. The study tested three different objective functions. The new coefficients improved MH concerning  $R^2$ , RMSE, and MAE.  $R^2$  increased from 0.32 to 0.90 for one case of bored piles, the most notable improvement observed throughout the study. The same did not occur for DQ, although RMSE and MAE significantly decreased. The original calibration of the methods can explain this difference once this work uses data from tropical soil.*

**Resumo.** *Projetar estacas de concreto que sejam seguras e de baixo custo requer métodos confiáveis para prever sua capacidade de suporte. Os métodos de projeto empírico são uma alternativa popular, como o método de Meyerhof (MH), que se adapta melhor a solos temperados, e o de Décourt-Quaresma (DQ), que é mais adequado para solos tropicais. Os coeficientes calibram empiricamente esses métodos; no entanto, frequentemente tornam-se imprecisos para casos específicos. Este trabalho visa recalibrar esses dois métodos de projeto empíricos utilizando conjuntos de dados contendo testes de carga estática, todos obtidos para solos tropicais. O Algoritmo de Lichtenberg (LA) é aplicado para encontrar coeficientes ótimos, considerando três tipos de estacas para MH e dois para DQ. O estudo testou três funções objetivo diferentes. Os novos coeficientes melhoraram o MH em relação a  $R^2$ , RMSE e MAE.  $R^2$  aumentou de 0,32 para 0,90 para um caso de estacas escavadas, a melhoria mais notável observada ao longo do estudo. O mesmo não ocorreu para o DQ, embora o RMSE e o MAE tenham diminuído significativamente. A calibração original dos métodos pode explicar esta diferença, uma vez que este trabalho utiliza dados de solos tropicais.*

## 1. Introduction

The evolution of cities requires new constructions, which frequently use piles in their projects. Designers need reliable techniques to predict their bearing capacity to produce

low-cost projects without jeopardizing safety. Literature provides many techniques based on different approaches, and the most popular are the so-called empirical design methods [Robert 1997]. These methods typically use standard penetration test (SPT) or cone penetration test (CPT) data and apply formulas calibrated with coefficients that depend on soil type and pile type [Salgado 2008].

Meyerhof and Geoffrey [Meyerhof 1976] presented an empirical method that gives the pile bearing capacity from its geometry and SPT values, here called MH. The employed calibration coefficients were set mainly from the author's self-experience [Salgado 2008]. Despite being inaccurate for some pile types like bored and continuous flight auger (CFA) piles, MH's method is popular. Al-Atroush et al. [Al-Atroush et al. 2022] proposed modifying the MH formula to become more accurate for bored piles. Nonetheless, the new formula makes soil parameters expensive to measure compared to SPT.

In tropical countries, geotechnical engineers usually prefer empirical design methods from local researchers because they use tropical soil to calibrate their formulas. For example, the formula proposed by Décourt and Quaresma [Décourt and Quaresma 1978] (denoted as DQ here) used SPT values from Brazilian soil. It evolved to more sophisticated calibration depending on soil and pile types [Décourt et al. 1996].

Many authors have recently applied modern computational techniques to obtain more accurate and reliable empirical design methods. One strand includes the use of Machine Learning (ML) techniques. Some authors use soil parameters measured in the laboratory as inputs [Moayedi and Hayati 2019], and others use parameters obtained from in-situ experiments. Shahin [Shahin 2010] applies an artificial neural network algorithm to CPT data. Alkroosh et al. [Alkroosh et al. 2015] also use CPT data to predict the bearing capacity using least squares support vector machines. Some studies apply ML to SPT data, like [Pham et al. 2020], which applies artificial neural networks to a comprehensive dataset, and [Pham et al. 2022], which uses SPT associated with three ML techniques.

Another approach combines ML with optimization techniques, leading to better models. Nevertheless, those models are complex, and engineers prefer classical empirical design methods. Kardani et al. [Kardani et al. 2020], for example, use particle swarm optimization to find the best hyperparameters for the ML algorithms, using soil parameters as inputs. Ardalan et al. [Ardalan et al. 2009] use a similar approach, using CPT parameters as inputs to obtain the shaft capacity of the pile, and Kordjazi et al. [Kordjazi et al. 2014] employ a broader database and get the total bearing capacity. Pham and Tran [Pham and Tran 2022] associate random forest with two optimization algorithms.

One lack observed in the literature is that none of the previous works try to improve the empirical design methods without significant conceptual changes in their original formula, which is the aim of this work. Finding the optimized parameters of these methods for all load tests can be formulated as an optimization problem. Metaheuristics have been the most used optimization algorithms for solving complex engineering problems [Yang 2014].

Here, we use an efficient metaheuristic inspired by lightning named the Lichtenberg Algorithm (LA) to solve our problem. LA is based on trajectory and population,

was recently proposed [Pereira et al. 2021] and applied successfully to many engineering problems [Francisco et al. 2021, Pereira et al. 2022a, Brendon Francisco et al. 2022, de Souza et al. 2022]. This study applies this algorithm for the first time in a geotechnical problem.

The advantage of this approach is that it maintains all attractive aspects of the original methods, only changing which coefficients to substitute. Two methods were selected: MH [Meyerhof 1976] and DQ [Décourt et al. 1996], frequently used in temperate countries and popular in tropical countries like Brazil. The objective is to recalibrate these methods using 168 load tests performed in precast concrete piles, 70 in bored piles, and 95 in continuous flight auger (CFA) piles collected from soil samples in Brazil. Examples include testing five objective functions for different pile types and improving both empirical design methods. The optimization demonstrated helpful in adjusting the parameters of MH, with significant gains. There were also gains for DQ, although not as substantial as those observed for MH.

This paper is structured as follows: Section 2 presents the Lichtenberg Algorithm, the metaheuristic chosen for the optimization. Section 3 presents the MH and DQ empirical design methods. Section 4 presents the methodology of the experiments, whose results are presented and discussed in Section 5. Section 6 concludes this work.

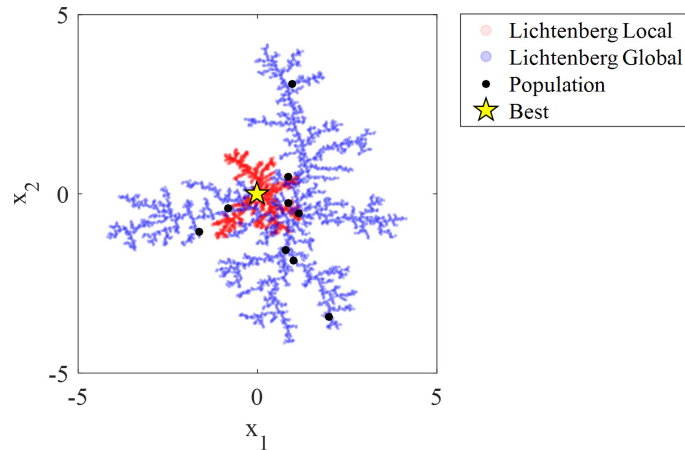
## **2. Lichtenberg Algorithm (LA)**

A new meta-heuristic inspired by the physical phenomena of lightning storms and Lichtenberg figures (LF) was recently created [Pereira et al. 2021] and tested against traditional and recent meta-heuristics using famous and complex test function groups. As a result, the LA proved to be an effective metaheuristic. Moreover, it surpassed other traditional and recent algorithms.

The algorithm is hybrid because it uses an LF thrown in the search space from its center at each iteration, being, at this point, the best one of all iterations. It uses limited LF points as a population for evaluation in the objective function. The algorithm creates the LF as follows: it builds a binary matrix (0 and 1) like a map, and in the center, a particle, represented by the number one, is fixed. It builds the cluster by the unitary values of the matrix, and the empty spaces have zero values. Each unitary matrix element is a particle of the cluster. The program starts defining the population of particles, and the creation radius defines the construction space of the figure. The lines and columns of the matrix correspond to twice the creation radius.

Particle releasing is random across the matrix, and if they reach the cluster with just a central particle in the beginning, they have an  $S$  probability of fixing.  $S$  is called the stickiness coefficient and controls the density of the cluster. The particle walks are plotted randomly in radial directions, all at the same plane. The final position is the closest matrix element, and the algorithm adds the new particle only if a neighbor matrix element already contains a particle. If the particle walk reaches a radius more extensive than the creation radius, the algorithm deletes it, and another random walk starts. The procedure repeats until all particles defined in the beginning are inserted in the cluster or until it reaches its construction limit.

All cluster particles belong to the same plane, and the algorithm can plot an LF with any size, slope, or starting point. The first position and size of the figure coincide



**Figure 1. Basic LA's search strategy [Pereira et al. 2022b]**

with the search space. Each iteration can plot the figure with different sizes and rotations, selected randomly. The procedure uses measures to improve the algorithm's exploration and exploitation capabilities and prevent a flawed reading of the search space.

Another optimizer parameter is the refinement, which ranges from 0 to 1. Figure 1 illustrates how this parameter works, presenting the primary LF in blue and the LF created by the refinement in red. New LF sizes range from zero to the size of the main one, depending on the value of the refinement. For example, only the primary LF acts on the optimizer if refinement is zero.

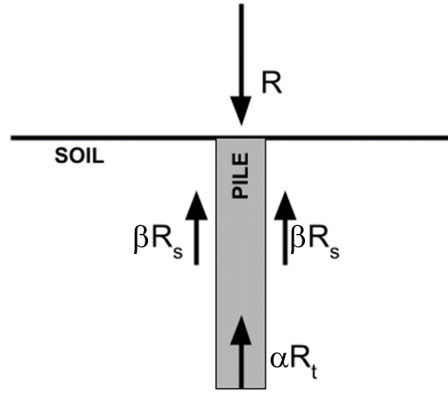
The computation of the objective function does not use all LF particles, just the population points defined at the beginning of the algorithm and represented by black dots in Figure 1. The population usually is 10 times the number of design variables of the problem. The procedure chooses the LF particles throughout the LF structure, modified at each iteration. It always places them in the search space, so LA is a hybrid algorithm that merges population and trajectory strategies. This hybrid approach gives the algorithm a better capacity for exploitation and exploration.

Another parameter is the switching factor, which changes the LF in the optimizer input data. this discrete parameter can be 0, 1 or 2. When it is 0, the algorithm uses the same LF throughout all iterations. If the discrete parameter is 1, the algorithm generates a new figure at each iteration. The optimizer uses a previously saved LF for the switching factor equal to 2 and generates no figure. Changing this parameter impacts computational cost once generating an LF is costly.

This algorithm can be used to construct 2D or 3D LFs corresponding to two and three decision variables, respectively. It is also possible to use the algorithm for more than three decision variables, which is a projection of these figures. The final parameter set is the number of iterations, which is defined initially and usually equal to 100.

### 3. Empirical design methods

This section presents the empirical design methods for calculating bearing capacity. Both decompose the pile bearing capacity  $R$  in tip resistance  $R_t$  and shaft resistance  $R_s$ , as presented in Figure 2. These components are magnified by coefficients  $\alpha$  and  $\beta$ , respectively:



**Figure 2. Components of pile bearing capacity.**

$$R = \alpha R_t + \beta R_s \quad (1)$$

The procedure uses SPT values and pile geometry to estimate  $R_t$  and  $R_s$ . For the method MH [Meyerhof 1976]:

$$R_t = A_t L \frac{N_t}{D} \leq 400 N_t \quad (2)$$

$$R_s = N_s U L \quad (3)$$

where  $A_t$  is the area of the pile base,  $L$  is pile length,  $D$  is pile diameter,  $N_t$  is the mean SPT within  $8D$  above the pile tip and  $3D$  below,  $N_s$  is the mean SPT value along the pile shaft and  $U$  is the mean pile perimeter along the shaft. Notice that the expression for  $N_t$  is discontinuous due to the inequality. Meyerhof and Geoffrey [Meyerhof 1976] recommend employing  $\alpha = 40$  and  $\beta = 2$  for driven piles in Equation 1. The recommendation for bored and low displacement piles (CFA) is  $\alpha = 12$  and  $\beta = 1$ .

Décourt and Quaresma [Décourt et al. 1996] proposed the second empirical design method considered in this work. It defines  $R_t$  and  $R_s$  as:

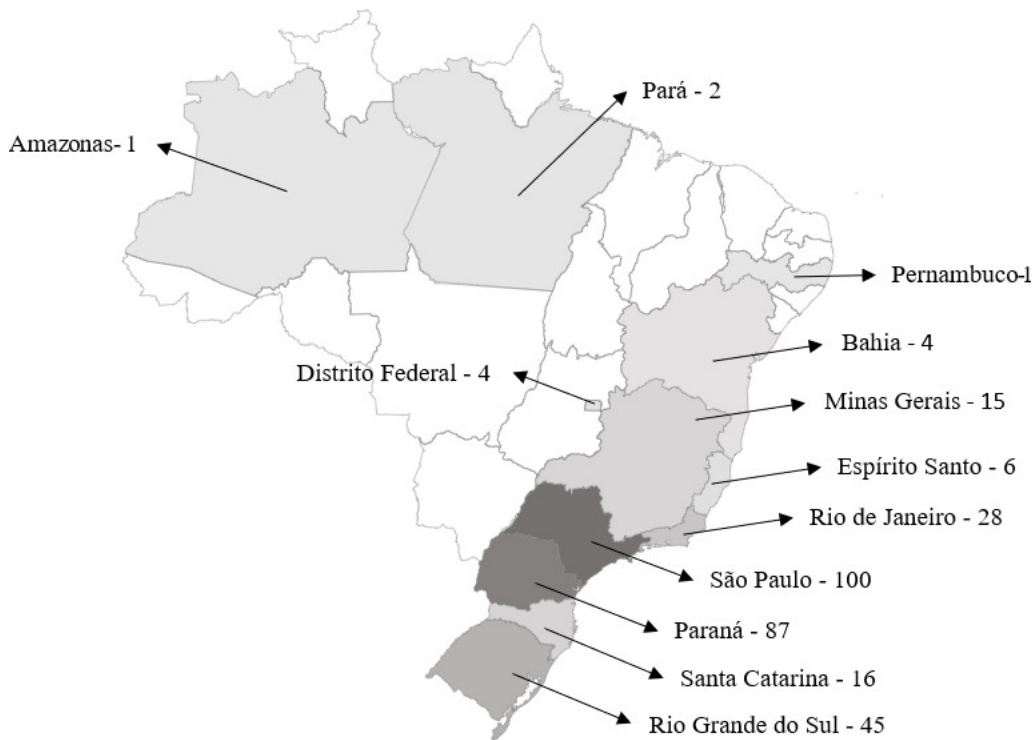
$$R_t = K N_t^* A_t \quad (4)$$

$$R_s = 10 \left( \frac{N_s^*}{3} + 1 \right) U L \quad (5)$$

where  $K$  is a factor that depends on soil type,  $N_t^*$  is the mean SPT considering values at the pile tip, the one above and the one below, and  $N_s^*$  is the mean SPT value along the shaft (does not include values used for the tip). DQ [Décourt et al. 1996] recommends  $\alpha = 0.3$  and  $\beta = 1.0$  for CFA piles and  $\alpha = \beta = 1.0$  for precast concrete piles in Equation 1. It is also possible to use this method for bored piles. Nevertheless, once  $\alpha$  and  $\beta$  depend on soil type, this study does not include this case.

#### 4. Methodology

The first step of the study was constructing a dataset for each case to be optimized. The information required to build a dataset includes pile dimensions, SPT values of the soil,



**Figure 3. Number of piles in Brazilian states.**

and the measured pile-bearing capacity via static load test. This information was gathered from the literature using the works of [Lobo 2005], [Santos Jr. 1988], and [Vianna 2000]. Authors produced all information from these references according to Brazilian standards, reducing differences between measurements obtained by different authors. This work gathers pile geometry, soil SPT values, and pile bearing capacity for 168 precast piles, 95 CFA piles, and 70 bored piles. Figure 3 presents the Brazilian states covered and the number of piles in each.

One dataset for each pile type is available for MH, including the inputs of Equations 2 and 3 and the measured pile bearing capacity ( $Q_u$ ). For DQ, this study produced similar datasets for precast piles and CFA piles. They include the inputs of Equations 4 and 5 and the measured pile bearing capacity.

The procedure divides each dataset into training and testing at a 75-25% rate. First, it selects the subsets at random. Then, the LA finds the optimized coefficients using the training set and obtains the performance metrics  $R^2$ , RMSE, and MAE for the test set. This procedure is needed to avoid overfitting. Each random selection of the training and test sets corresponds to 10 algorithm executions, and it also repeats random selection 10 times to obtain results less biased by a specific data partition. The final  $\alpha$  and  $\beta$  are the mean values considering the 100 executions of the LA.

The procedure repeats twice for each case to refine the solution search. In the first one, considering that the original method recommends coefficients  $\alpha_0$  and  $\beta_0$ , ranges are  $[0, 10\alpha_0]$  and  $[0, 10\beta_0]$ . Notice that  $\alpha$  and  $\beta$  must be positive; otherwise, the pile reaction at the tip or shaft would pull the pile into the soil instead of resisting the load applied at the top (see Figure 2 and Equation 1). In the second round, the procedure defines smaller

ranges considering values obtained for  $\alpha$  and  $\beta$  in the first round.

This study performed preliminary tests with five objective functions. Three of them use metrics employed to evaluate the accuracy of the optimized method: the coefficient of determination with a minus sign  $f_{R^2}$ , the root-mean-square deviation  $f_{RMSE}$ , and the mean absolute error  $f_{MAE}$ . The other two are least squares fitting  $f_{LE}$  and the Kullback–Leibler divergence  $f_{KLD}$ . The best results were observed for  $f_{LE}$ ,  $f_{RMSE}$  and  $f_{MAE}$ , which are defined next:

$$f_{LE} = \sum_{i=1}^N (x_i - \hat{x}_i)^2 \quad (6)$$

$$f_{RMSE} = \sqrt{\frac{\sum_{i=1}^N (x_i - \hat{x}_i)^2}{N}} \quad (7)$$

$$f_{MAE} = \frac{\sum_{i=1}^N (x_i - \hat{x}_i)}{N} \quad (8)$$

where  $N$  is the number of observations,  $x_i$  is an observation,  $\bar{x}$  is the mean value of all observations,  $\hat{x}_i$  is an estimated value,  $y_i$  and  $\hat{y}_i$  are the normalized values of  $x_i$  and  $\hat{x}_i$ , respectively.

## 5. Results and discussion

This section summarizes and discusses the main results, considering the methodology presented in Section 4. Section 5.1 presents a study of MH considering precast, CFA, and bored concrete piles. Section 5.2 includes results obtained for DQ, considering precast and CFA concrete piles. Bored piles are not included in DQ as discussed in Section 3. As presented in Section 4, the initial search space for the optimal solution ranges from zero to ten times the coefficients of the original methods. This procedure leads to the following ranges:

- MH:
  - Precast  $\Rightarrow \alpha \in [0, 400]$  and  $\beta \in [0, 20]$
  - CFA  $\Rightarrow \alpha \in [0, 120]$  and  $\beta \in [0, 10]$
  - Bored  $\Rightarrow \alpha \in [0, 120]$  and  $\beta \in [0, 10]$
- DQ:
  - Precast  $\Rightarrow \alpha \in [0, 10]$  and  $\beta \in [0, 10]$
  - CFA  $\Rightarrow \alpha \in [0, 3]$  and  $\beta \in [0, 10]$

The refined space ranges are different for each case and objective function, as presented in the following Sections.

### 5.1. Meyerhof's method (MH)

Table 1 presents the LA and MH results obtained for precast concrete piles. Below each objective function, one can find the values found for  $\alpha$  and  $\beta$  and the resulting values of  $R^2$ , RMSE and MAE, considering the test dataset. The best results per metric are highlighted in boldface.

Comparing the results of the optimized (LA) and original (MH) coefficients, one can observe that  $R^2$ , RMSE, and MAE results are better for the optimized coefficients

**Table 1. Results for MH and LA in precast concrete piles.**

Func	LA			MH		
	$f_{LE}$	$f_{RMSE}$	$f_{MAE}$	$f_{LE}$	$f_{RMSE}$	$f_{MAE}$
$\alpha$	5.8	6.5	6.1	40	40	40
$\beta$	4.9	5.8	5.5	2	2	2
$R^2$	<b>0.76</b>	<b>0.76</b>	0.75	0.70	0.70	0.67
RMSE	686	691	<b>644</b>	2755	2553	2489
MAE	432	436	<b>412</b>	1911	1777	1710

**Table 2. Results for MH and LA in CFA piles.**

Func	LA			MH		
	$f_{MQ}$	$f_{RMSE}$	$f_{MAE}$	$f_{MQ}$	$f_{RMSE}$	$f_{MAE}$
$\alpha$	0.0	1.8	0.2	12	12	12
$\beta$	5.8	5.4	6.1	1	1	1
$R^2$	0.68	<b>0.69</b>	0.60	0.45	0.40	0.30
RMSE	616	<b>574</b>	658	767	782	877
MAE	476	<b>449</b>	501	598	616	681

when compared to the original method in all cases.  $R^2$  was higher and RMSE and MAE were lower for the optimized coefficients than the original values produced by standard MH without any optimization. These results provide evidence that changing MH coefficients for this case is advisable. The mean values of  $\alpha$  and  $\beta$  considering  $f_{LE}$ ,  $f_{RMSE}$  and  $f_{MAE}$  are 6.1 and 5.4, respectively. These are the coefficients recommended here.

Notice that  $\alpha$  represents the magnitude of the pile tip reaction, while  $\beta$  represents the magnitude of the pile shaft reaction. Decreasing  $\alpha$  from 40 to 6.1 implies reducing the pile tip's contribution at pile bearing capacity while increasing  $\beta$  from 2 to 5.4 means augmenting the participation of the shaft. One can conclude that this change in coefficients implies a better understanding of how the foundation works as a mechanism.

Table 2 presents the main LA results for CFA piles, and accuracy is reasonable despite the lower values compared to Table 1. The best results per metric are highlighted in boldface. One can observe that concerning  $R^2$ , the accuracy with optimized coefficients surpasses that obtained with the original coefficients. Considering RMSE and MAE, the optimized coefficients perform better in all cases. This case shows a more notable improvement than in the previous pile type.  $f_{RMSE}$  was the optimization function with the best results.

For CFA piles, the mean values of the optimized coefficients are  $\alpha = 0.8$  and  $\beta = 5.7$ . Thus, the original MH method overestimates the pile tip's contribution ( $\alpha = 12$ ) and underestimates the pile shaft's contribution ( $\beta = 1$ ) for CFA piles.

Table 3 presents the MH and LA main results for bored piles. The best results per metric are highlighted in boldface. This case achieved the higher  $R^2$  values, all close to 0.9. The same was not observed for the original coefficients of MH as presented in Table 3, with values around 0.33. The difference observed in RMSE and MAE is also very prominent, more than double the original coefficients. For this case, the mean values for  $\alpha$  and  $\beta$  are 1.4 and 3.7, respectively.



**Table 3. Results for bored piles.**

Func	LA			MH		
	$f_{MQ}$	$f_{RMSE}$	$f_{MAE}$	$f_{MQ}$	$f_{RMSE}$	$f_{MAE}$
$\alpha$	3.1	0.0	1.1	12	12	12
$\beta$	3.3	3.9	3.8	1	1	1
$R^2$	0.88	<b>0.90</b>	0.88	0.33	0.32	0.32
RMSE	2643	<b>1965</b>	2310	5874	5132	5248
MAE	1476	<b>1124</b>	1316	3148	2560	2700

The original coefficients for bored piles are  $\alpha = 12$  and  $\beta = 1.0$ . Therefore, this third case reinforces the previous conclusion that the original coefficients used in MH overestimate the pile tip's contribution and underestimate the pile shaft's contribution.

## 5.2. Décourt-Quaresma's method (DQ)

Table 4 contains the LA main results with optimized coefficients for precast piles, and also presents results obtained with the original DQ coefficients. The best results per metric are highlighted in boldface. We observe no significant change in  $R^2$ . Nonetheless, RMSE and MAE can be considered better for optimized coefficients. The objective function  $f_{MAE}$  has yielded the best overall results. In this case, the mean values for the coefficients are  $\alpha = 1.1$  and  $\beta = 1.5$ , which are close to the original values. That is evidence that this case has no significant margin of improvement.

**Table 4. DQ and LA results for precast concrete piles.**

Func	LA			DQ		
	$f_{MQ}$	$f_{RMSE}$	$f_{MAE}$	$f_{MQ}$	$f_{RMSE}$	$f_{MAE}$
$\alpha$	1.3	0.9	1.2	1.1	1.1	1.1
$\beta$	1.2	1.8	1.4	1.5	1.5	1.5
$R^2$	0.73	0.77	<b>0.79</b>	0.73	0.78	<b>0.79</b>
RMSE	652	643	<b>559</b>	737	748	658
MAE	387	428	<b>354</b>	433	459	415

Interestingly, both coefficients increased in this case, which means that the overall force sum was augmented when coefficients were optimized. That is evidence that DQ intentionally underestimates the overall bearing capacity of piles to increase safety.

Table 5 presents results for LA optimized and original DQ coefficients, respectively. The best results per metric are highlighted in boldface. One can observe that conclusions are similar to the ones of the previous case, with no significant changes for  $R^2$  but a relevant improvement for RMSE and MAE. The mean values obtained for this case are  $\alpha = 0.4$  and  $\beta = 1.3$ .

Again, both  $\alpha$  and  $\beta$  increased for the optimized coefficients. This increase reinforces the conclusion that DQ underestimates the overall bearing capacity of piles for better safety.

**Table 5. Results for CFA piles.**

Func	LA			DQ		
	$f_{MQ}$	$f_{RMSE}$	$f_{MAE}$	$f_{MQ}$	$f_{RMSE}$	$f_{MAE}$
$\alpha$	0.3	0.4	0.4	0.3	0.3	0.3
$\beta$	1.4	1.3	1.3	1	1	1
$R^2$	0.66	0.64	<b>0.67</b>	<b>0.67</b>	0.64	<b>0.67</b>
RMSE	618	<b>566</b>	624	778	772	871
MAE	491	<b>425</b>	479	613	601	694

## 6. Conclusion

Empirical design methods are valuable tools for predicting the bearing capacity of piles. Nevertheless, classical approaches like Meyerhof's method (MH) give an over-generic equation that tends to be ineffective for specific cases. This work used LA to calibrate MH for three types of concrete piles: precast, CFA, and bored. We used datasets of SPT and static load tests performed in Brazil, as well as three different objective functions. Considering the mean value obtained with these three objective functions, we recommend substituting the original values of  $\alpha = 40$  and  $\beta = 2$  in the MH equation by 6.1 and 5.4 for precast piles. We recommend replacing the original coefficients  $\alpha = 12$  and  $\beta = 1$  by 0.8 and 5.7 for CFA piles and 1.4 and 3.7 for bored piles. Changing the original values is especially recommended for CFA and bored piles, for which MH is inaccurate. The origin of MH can explain these results because its author originally proposed it for countries with temperate soil.

Using the same methodology, this study also optimized the coefficients of another empirical bearing capacity method, Décourt-Quaresma's (DQ). Although we observed some improvement, the final coefficients differed from those in the original method developed for Brazilian soil. For precast concrete piles, the original values are  $\alpha = 1.0$  and  $\beta = 1.0$ , and our study leads to 1.1 and 1.5. For CFA piles, the original values are  $\alpha = 0.3$  and  $\beta = 1.0$ , and our study leads to 0.4 and 1.3.

This methodology's main weakness is that the dataset includes only Brazilian soil. So, the coefficients tend to lose accuracy if used for piles placed in other countries. One possibility to overcome this problem in future research is to include more empirical design methods in the study and try to deduce a formula that would be adequate for a wider variety of soil types. Then, one can calibrate this formula using optimization techniques with a richer dataset. The study can also include other metaheuristics for optimizing the coefficients' values.

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## References

Al-Atroush, M. E., Hefny, A. M., and Sorour, T. M. (2022). Modified meyerhof approach for forecasting reliable ultimate capacity of the large diameter bored piles. *Scientific*

*Reports*, 12.

- Alkroosh, I. S., Bahadori, M., Nikraz, H., and Bahadori, A. (2015). Regressive approach for predicting bearing capacity of bored piles from cone penetration test data. *Journal of Rock Mechanics and Geotechnical Engineering*, 7(5):584–592.
- Ardalan, H., Eslami, A., and Nariman-Zadeh, N. (2009). Piles shaft capacity from cpt and cptu data by polynomial neural networks and genetic algorithms. *Computers and Geotechnics*, 36(4):616–625.
- Brendon Francisco, M., Luiz Junho Pereira, J., Augusto Vilas Boas Vasconcelos, G., Simões da Cunha Jr, S., and Ferreira Gomes, G. (2022). Multiobjective design optimization of double arrowhead auxetic model using lichtenberg algorithm based on metamodelling. *Structures*, 45:1199–1211.
- de Souza, T. A. Z., Pereira, J. L. J., Francisco, M. B., Sotomonte, C. A. R., Ma, B. J., Gomes, G. F., and Coronado, C. J. R. (2022). Multi-objective optimization for methane, glycerol, and ethanol steam reforming using lichtenberg algorithm. *International Journal of Green Energy*, pages 1–18.
- Décourt, L., Albiero, J., and Cintra, J. (1996). Análise e projeto de fundações profundas. *Fundações: teoria e prática*, 2:265–327.
- Décourt, L. and Quaresma, A. R. (1978). Capacidade de carga de estacas a partir de valores de spt. In *Congresso Brasileiro de Mecânica dos Solos e Engenharia de Fundações*, volume 6, pages 45–53.
- Francisco, M. B., Junqueira, D. M., Oliver, G. A., Pereira, J. L. J., da Cunha Jr Jr, S. S., and Gomes, G. F. (2021). Design optimizations of carbon fibre reinforced polymer isogrid lower limb prosthesis using particle swarm optimization and lichtenberg algorithm. *Engineering Optimization*, 53(11):1922–1945.
- Kardani, N., Zhou, A., Nazem, M., and Shen, S.-L. (2020). Estimation of bearing capacity of piles in cohesionless soil using optimised machine learning approaches. *Geotechnical and Geological Engineering*, 38(2):2271–2291.
- Kordjazi, A., Pooya Nejad, F., and Jaksa, M. (2014). Prediction of ultimate axial load-carrying capacity of piles using a support vector machine based on cpt data. *Computers and Geotechnics*, 55:91–102.
- Lobo, B. O. (2005). Método de previsão de capacidade de carga de estacas: aplicação dos conceitos de energia do ensaio spt. Master's thesis, Universidade Federal do Rio Grande do Sul.
- Meyerhof, G. G. (1976). Bearing capacity and settlements of pile foundations. *Journal of the Geotechnical Engineering Division*, 102(3):197–228.
- Moayed, H. and Hayati, S. (2019). Artificial intelligence design charts for predicting friction capacity of driven pile in clay. *Neural Computing and Applications*, 31(11):7429–7445.
- Pereira, J. L. J., Brendon, F. M., Ribeiro, R. F., Cunha, S. S., and Gomes, G. F. (2022a). Deep multiobjective design optimization of cfrp isogrid tubes using lichtenberg algorithm. *Soft Computing*, pages 7195–7209.

- Pereira, J. L. J., Francisco, M. B., de Oliveira, L. A., Chaves, J. A. S., Jr, S. S. C., and Gomes, G. F. (2022b). Multi-objective sensor placement optimization of helicopter rotor blade based on feature selection. *Mechanical Systems and Signal Processing*, 180:109466.
- Pereira, J. L. J., Francisco, M. B., Diniz, C. A., Oliver, G. A., Cunha, S. S., and Gomes, G. F. (2021). Lichtenberg algorithm: A novel hybrid physics-based meta-heuristic for global optimization. *Expert Systems with Applications*, 170:114522.
- Pham, B. T., Nguyen, D. D., Bui, Q.-A. T., Nguyen, M. D., Vu, T. T., and Prakash, I. (2022). Estimation of load-bearing capacity of bored piles using machine learning models. *Science of the Earth*, 44(4).
- Pham, T. A., Ly, H.-B., Tran, V. Q., Giap, L. V., Vu, H.-L. T., and Duong, H.-A. T. (2020). Prediction of pile axial bearing capacity using artificial neural network and random forest. *Applied Sciences*, 10(5).
- Pham, T. A. and Tran, V. Q. (2022). Developing random forest hybridization models for estimating the axial bearing capacity of pile. *PLoS ONE*, 17(3).
- Robert, Y. (1997). A few comments on pile design. *Canadian Geotechnical Journal*, 34(4):560–567.
- Salgado, R. (2008). *The Engineering of Foundations*. McGraw-Hill.
- Santos Jr., O. (1988). Previsão do comportamento carga-recalque de estacas pré-moldadas de concreto. Master's thesis, Universidade de São Paulo, São Carlos, Brasil.
- Shahin, M. A. (2010). Intelligent computing for modeling axial capacity of pile foundations. *Canadian Geotechnical Journal*, 47(2):230–243.
- Vianna, A. P. F. (2000). *Análise de provas de carga estática em estacas pré-moldadas cravadas na cidade de Curitiba e região metropolitana*. PhD thesis, Universidade de São Paulo.
- Yang, X.-S. (2014). *Nature-Inspired Optimization Algorithms*. Elsevier.