Modeling Uncertainty in Crime Underreporting: An Indicator Kriging Approach

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Abstract. *This study confronts crime underreporting in São Paulo, a critical challenge to public policy in emerging nations. We introduce a suitable geostatistical methodology to estimate crime probability from official police reports. The framework employs Indicator Kriging, a non-parametric method that excels with incomplete and missing data. It transforms crime counts into binary indicators based on hotspots, enabling robust probability estimation even in areas with sparse or no records. This approach effectively overcomes critical data gaps, generating detailed and accurate spatial probability maps of known criminal patterns. The resulting model offers a powerful analytical tool for targeted public security, revealing latent crime dynamics.*

1. Introduction

The underreporting of crime, the phenomenon where the actual number of crimes committed is higher than the number officially recorded or reported to law enforcement, is a chronic challenge in major urban centers, creating a "dark figure of crime" that severely hinders the development of effective, evidence-based public policies [Jaitman and Anauati 2020, Buil-Gil et al. 2021]. In São Paulo, for instance, statistics indicate that more than half of all robbery victims do not file an official police report [Carvalho et al. 2021]. This pervasive data gap leads to incomplete and biased official records, which can misguide resource allocation and preventative strategies by presenting a flawed map of criminal activity. Addressing this missing data is therefore a critical first step toward more intelligent and equitable public security.

In response, a variety of analytical approaches have been explored. Predictive models, while powerful, rely heavily on the completeness of historical data and thus falter in areas where such records are sparse or absent [Gorr et al. 2003, Kounadi et al. 2020]. Common spatial analysis techniques such as Kernel Density Estimation (KDE) are effective for visualizing known crime hotspots, but they cannot estimate risk in unsampled locations where crimes go unreported, thus failing to account for the "dark figure" [Prathap 2022]. More advanced geostatistical methods have also been applied, including regression kriging with socioeconomic variables [Usman et al. 2021] and spatiotemporal cokriging with human mobility data [Yu et al. 2020]. However, these approaches often depend on external covariates and do not directly model the probability of occurrence that arises from the uncertainty of underreporting itself.

This study proposes a different path by employing Indicator Kriging (IK), a non-parametric geostatistical technique well-suited for incomplete datasets [Journel 1983]. Rather than predicting crime counts, our methodology transforms raw data into binary indicators—identifying locations as hotspots or not—to estimate the probability of crime occurrence across the entire geographical space. Because IK does not require assumptions about the underlying data distribution, it is ideal for handling the uncertainty inherent in underreported crime data. The result is a continuous and detailed probability surface that provides a more robust foundation for public policy, overcoming the limitations imposed by missing data in traditional methods.

Foundational criminological theories underscore the spatially constrained nature of crime and validate the adoption of geostatistical tools. Routine activity [Cohen and Felson 2010], [Osgood et al. 1996] and crime pattern theories [Brantingham and Brantingham 1993], [Cohen and Felson 2010] argue that offense opportunities cluster in micro-geographic "nodes, paths, and edges", where environmental features create predictable hotspots [Lisowska-Kierepka 2022], [Braga et al. 2019]. Moreover, the broken windows paradigm highlights how disorderly surroundings propagate further crime within their immediate vicinity [Windows 1982]. When coupled with the probabilistic interpolation offered by indicator kriging, these perspectives lay a solid theoretical foundation for applying this method to enrich AI-driven crime prediction frameworks, ultimately contributing to more effective, data-scarce public safety solutions.

2. Related Works

The challenge of analyzing crime patterns is fundamentally a problem of data quality. The vast number of incidents that are never officially reported creates significant gaps and biases in law enforcement datasets. This issue is particularly acute in Latin America [Jaitman and Anauati 2020], with studies in São Paulo confirming that a majority of certain crimes, such as robbery, go unreported [Carvalho et al. 2021]. This phenomenon not only complicates the measurement of crime [Buil-Gil et al. 2021] but also has a direct impact on data-driven policing, as analytical models trained on such incomplete data risk reinforcing existing biases and misdirecting public resources [Akpinar et al. 2021]. Understanding and mitigating the impact of this missing data is therefore a central concern in modern crime analysis.

Current computational approaches to crime analysis have struggled to overcome this core limitation. Standard forecasting methods are highly dependent on complete historical time-series data, limiting their utility in contexts with significant data gaps [Gorr et al. 2003, Kounadi et al. 2020]. Spatial analysis techniques like Kernel Density Estimation (KDE) are widely used for visualizing crime "hotspots" but are inherently unable to estimate risk in areas with no reported incidents, effectively ignoring the spatial dimensions of the underreporting problem [Prathap 2022]. Other, more advanced geostatistical methods have incorporated external variables to improve predictions; for example, regression kriging has been used with socioeconomic indicators [Usman et al. 2021], and spatiotemporal cokriging has been combined with human mobility data [Yu et al. 2020]. While these methods add valuable context, they do not directly address the primary challenge of estimating occurrence probability from the binary presence or absence of data points stemming from underreporting.

The existing literature thus reveals a clear gap: a need for a robust, non-parametric method capable of estimating spatial risk directly from incomplete crime data without relying on external covariates. This study addresses this gap by applying Indicator Kriging (IK), a geostatistical technique first formalized by Journel [Journel 1983]. Unlike other methods, IK is specifically designed to estimate the probability of a variable exceeding a certain threshold by transforming data points into binary indicators. While IK has been successfully applied in other fields to model spatial probabilities, such as disease mapping or environmental contamination [Guimarães et al. 2012, Piccini et al. 2012], its application to the criminological problem of underreporting remains underexplored. Our work demonstrates that this approach is uniquely suited to model crime probability, providing a more reliable surface of risk even in the face of severe data sparsity.

3. Methodology

To better understand and predict where offenses are most likely to occur, this study draws on the São Paulo State Public Security Secretariat's open-access database of cell phone theft and robbery reports from January 2024 to December 2024 [1]. All raw incident records in the dataset were first cleansed by deduplicating entries ensuring a one-to-one correspondence between police bulletins and occurrences, and any records lacking valid geographic coordinates were subsequently removed. The cleaned dataset was then spatially constrained by intersecting the remaining georeferenced events with the official Brazilian Institute of Geography and Statistics (IBGE) municipal boundary mesh of São Paulo city. Finally, these filtered occurrences were aggregated by summation within each cell of a uniform 300 m × 300 m grid.

To identify hotspots of criminal activity, we employ a simple Poisson model following the pioneering work of Sherman et al. [Sherman et al. 1989]. The global rate parameter is estimated as

$$\hat{\lambda} = \frac{1}{N} \sum_{i=1}^{N} X_i,$$

where X_i denotes the observed count in cell i and N is the total number of cells in the study area. For each cell, the one-tailed p-value associated with observing at least X_i events is computed via the complementary cumulative distribution function:

$$p_i = 1 - \sum_{k=0}^{X_i - 1} \frac{\hat{\lambda}^k e^{-\hat{\lambda}}}{k!}.$$

By setting a significance threshold $\alpha = 0.05$, an indicator variable is defined as

$$I(\mathbf{u_i}) = \begin{cases} 1, & \text{if } p_i < \alpha, \\ 0, & \text{otherwise}, \end{cases}$$

thereby classifying cell $I(\mathbf{u_i})$ as a hotspot when $I(\mathbf{u_i}) = 1$ and as not a hotspot when $I(\mathbf{u_i}) = 0$. This step produces the binary indicator values 0 and 1 required for subsequent Indicator Kriging.

[1] `https://www.ssp.sp.gov.br/estatistica/consultas`

The Indicator kriging (IK) [Journel 1983] is applied to the binary indicator field $I(\mathbf{u_i})$ to estimate the probability of event occurrence at unsampled locations. The empirical indicator semivariogram is computed as

$$\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{|\mathbf{u_i}-\mathbf{u_j}|=h} \big[I(\mathbf{u_i}) - I(\mathbf{u_j})\big]^2,$$

where $N(h)$ denotes the number of data pairs separated by lag distance h. A theoretical model $\gamma(h)$ is then fitted to $\hat{\gamma}(h)$. For any unsampled location $\mathbf{u_0}$, the IK estimator is

$$\hat{I}(\mathbf{u_0}) = \sum_{i=1}^{n} \lambda_i \, I(\mathbf{u_i}),$$

where the weights $\{\lambda_i\}$ are obtained by solving the ordinary kriging system:

$$\sum_{j=1}^{n} \lambda_j \, \gamma(\mathbf{u_i} - \mathbf{u_j}) + \mu = \gamma(\mathbf{u_i} - \mathbf{u_0}), \quad \sum_{j=1}^{n} \lambda_j = 1.$$

The resulting $\hat{I}(\mathbf{u_0})$ values range between 0 and 1 and represent the estimated local probability of hotspot occurrence.

To calibrate the theoretical variogram model parameters, a Bayesian optimization procedure [Močkus 1975], [Jones et al. 1998] was employed to minimize the Indicative Goodness of Fit (IGF) criterion [Pannatier 1996]. The IGF quantifies the normalized squared deviation between the empirical and modeled semivariogram values:

$$\mathrm{IGF}(\theta) = \frac{\sum_{k=1}^{K} \big[\hat{\gamma}(h_k) - \gamma(h_k;\theta)\big]^2}{\sum_{k=1}^{K} \hat{\gamma}(h_k)^2},$$

where $\hat{\gamma}(h_k)$ is the empirical semivariogram at lag h_k, $\gamma(h_k;\theta)$ is the theoretical model value, and K is the number of lag classes.

A Gaussian process surrogate model was fitted to approximate $\mathrm{IGF}(\theta)$ over the parameter domain. At each iteration t, a new candidate parameter vector $\theta^{(t)}$ was selected by maximizing an acquisition function (e.g., Expected Improvement), and the true IGF was evaluated at $\theta^{(t)}$. This procedure was repeated until convergence [2], yielding the optimal parameters

$$\hat{\theta} = \arg\min_{\theta} \, \mathrm{IGF}(\theta).$$

To measure and understand the efficacy of the method's application, first we need to calculate the global spatial autocorrelation statistic I [Moran 1950]to test whether the spatial distribution of crime counts across the $300\,\mathrm{m} \times 300\,\mathrm{m}$ grid departs from randomness. The statistic is defined as follows:

$$I \;=\; \frac{n}{W} \, \frac{\displaystyle\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\,(x_i - \bar{x})(x_j - \bar{x})}{\displaystyle\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

[2]In this study, 800 rounds were performed

where n is the number of grid cells, x_i is the crime count in cell i, $\bar{x}$ is the overall mean, w_{ij} is the spatial weight between cells i and j, and $W = \sum_i \sum_j w_{ij}$ is the sum of all spatial weights.

4. Results and Discussion

A statistically significant positive value of I, indicated by its standardized z-score and associated p-value, demonstrates spatial clustering and therefore satisfies the fundamental prerequisite for applying Indicator Kriging, which assumes an underlying spatial dependence structure. The estimated I statistic and its inferential measures are reported in Table 1.

Table 1. Global Moran's I for the 300 m $\times$ 300 m crime grid.

Statistic	Value
Moran's I	0.5752
z-score	141.3510
p-value	< 0.001

The Global Moran's I statistic for the crime grid yielded a value of 0.5752, a standardized z-score of 141.35, and a p-value below 0.001 (Table 1), indicating significant positive spatial autocorrelation in crime incidence. This result satisfies the prerequisite of spatial dependence for applying Indicator Kriging. Consequently, the spatial structure of crime counts was characterized through the empirical semivariogram demonstrated in Fig. 1 was calibrated via Bayesian optimization of the Indicative Goodness of Fit (IGF) criterion and estimated using Matheron's method-of-moments [Matheron 1963].

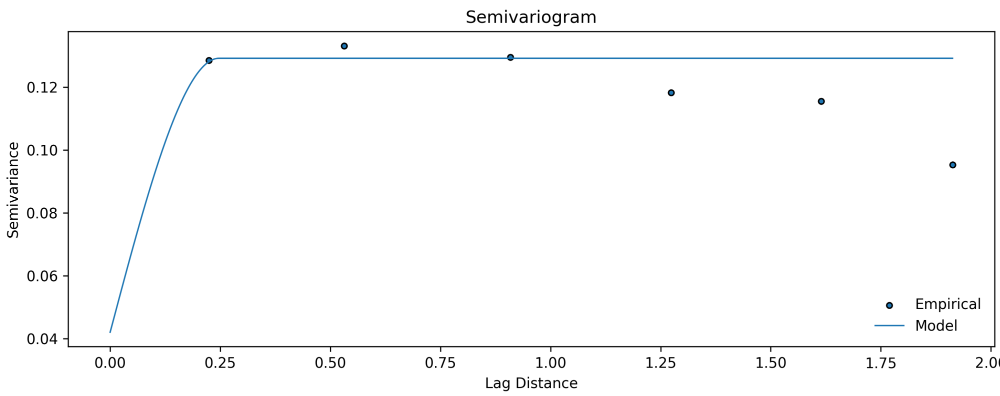

Figure 1. Empirical semivariogram.

As shown in Fig. 1, the empirical semivariogram points depict how semivariance increases with lag distance, reflecting stronger spatial similarity at short separations and weaker similarity as cells become more distant. The fitted spherical model shows a nonzero intercept at the origin (the nugget effect) indicating microscale variability or measurement error that is unaccounted for at infinitesimal lags. As lag increases, the model curve rises until it reaches a constant value (the sill) beyond which additional separation does not increase semivariance. The distance at which the curve flattens(the range) therefore defines the limit of spatial autocorrelation in the crime counts. Practically, the semivariogram quantifies the spatial dependence structure and provides the functional form needed by Indicator Kriging to compute distance-based weights: closer cells with lower semivariance receive higher influence in the estimation, thereby minimizing prediction variance under the stationarity assumption. By making the nugget, sill, and range directly visible, it is possible to delve into the variogram parameters separately (see Table 2).

Table 2. Calibrated variogram parameters for Indicator Kriging

Parameter	Value
Partial sill	0.08721
Range	0.24714
Nugget	0.04199
Anisotropy scaling	4.55112
Anisotropy angle (°)	94.48043

The variogram calibration reveals several key insights into the spatial dynamics of theft in São Paulo. First, the total sill (partial sill + nugget ≈ 0.1292) indicates that roughly 67.5% of the observed variability in the binary theft indicator is explained by structured spatial dependence, while the remaining 32.5% is attributable to micro-scale variation or reporting noise. The partial sill of 0.08721 thus quantifies the strength of crime clustering beyond random chance, showing that theft occurrences are not uniformly distributed but concentrated in localized pockets.

Second, the range of 0.24714 identifies the effective correlation distance: theft indicators remain positively correlated across distances up to approximately one quarter of the study extent (i.e., adjacent and near-adjacent grid cells), implying block-level clustering likely driven by street-network connectivity and opportunity structures.

Third, the nugget effect of 0.04199 captures inherent randomness at scales below the grid resolution—reflecting underreporting, irregular patrol patterns, or truly isolated incidents.

Finally, the pronounced anisotropy (scaling ≈ 4.55 at an angle of $94.48°$) demonstrates that spatial dependence decays much more slowly along a principal axis aligned nearly north–south—consistent with major arterial corridors and transit lines—than in the perpendicular direction. These parameter values together depict a theft process that is both strongly clustered at street-block scales and directionally biased by urban infrastructure, underscoring the need for anisotropic, data-driven policing strategies. Therefore, this fitted semivariogram supplies the spatial continuity model required for Indicator Kriging, and the analysis now turns to the visual interpretation of the resulting probability maps (See Fig 2 and Fig 3).

To better see the results, the interpolated surface is examined through a perceptually uniform *viridis* color gradient that progresses from deep violet ($probability \approx 0$), through emerald–green ($probability \approx 0.5$), to luminous yellow ($probability \approx 1$), thereby preserving ordinal meaning while maximising visual contrast. This rendering reveals a pronounced high-risk epicentre occupying the municipal core: contiguous yellow–green cells coalesce into a compact hotspot whose geometry mirrors the short-range continuity encoded in the fitted semivariogram. Risk decays outward, yet does so anisotropically, three greenish lobes extend west, southwest, and northeast, suggesting directional persistence in criminal opportunity flows. Peripheral districts appear largely quiescent in violet hues, but scattered teal patches attest that danger is not uniformly absent; rather, isolated cells concentrate moderate probability where local socio-spatial conditions outweigh distance decay. In sum, the map shows crime risk to be highly concentrated, decaying with distance from the centre while resurging sporadically where local conditions override the overall trend, thereby illustrating the capacity of Indicator Kriging to honour both global structure and local particularities on the area studied.

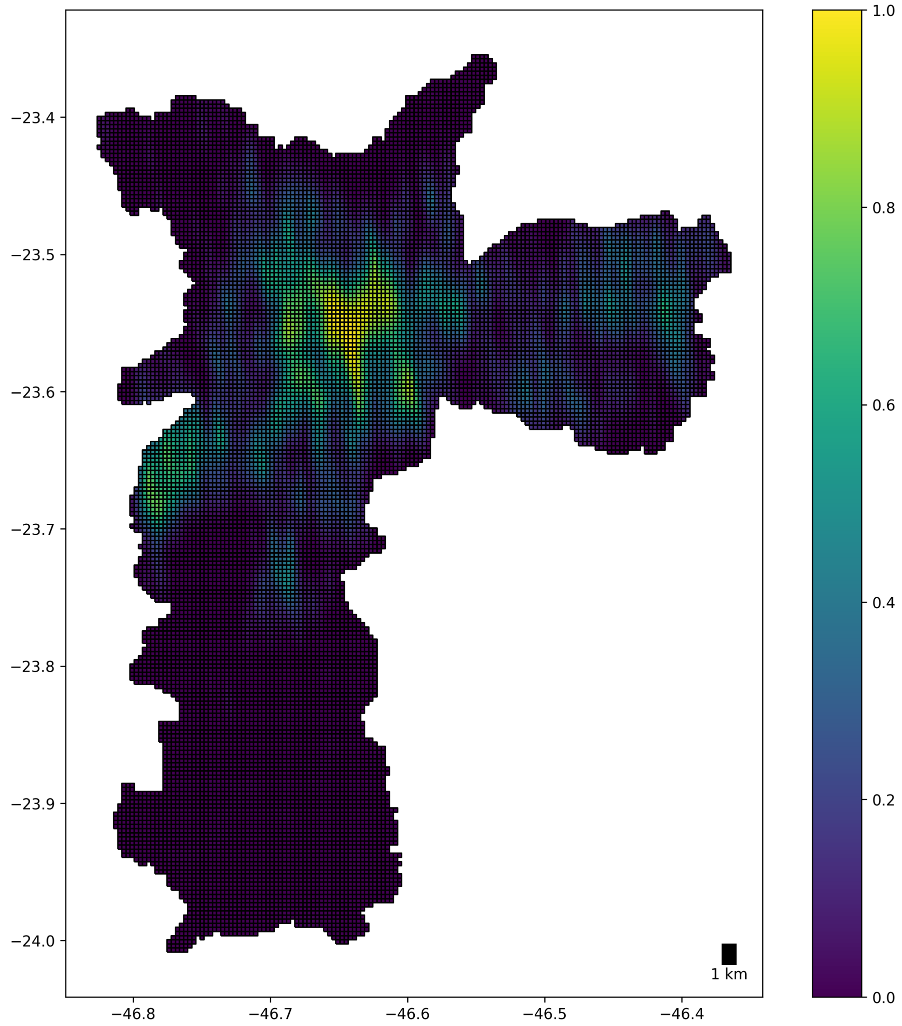

Figure 2. Probability map generated by Indicator Kriging for São Paulo (2024).

Figure 3 illustrates, on the street-block scale, a example of how the indicator Kriging reproduces the spatial mechanisms that the criminological theory ascribes to highly accessible transport nodes [Brantingham and Brantingham 1993] . The colour gradient (white → yellow → red) portrays increasing predicted probability within each 300 m cell, revealing a sharply elongated red core that aligns with the subway tracks and station concourse. Flanking avenues act as natural escape corridors, so the kriged surface sensibly grades into orange and yellow bands along those thoroughfares. Although presented here as a single exemplar, the same pattern recurs wherever metro stations, arterial roads or other high-flow facilities create convergences of offenders, targets and egress routes; This confirms that the Indicator Kriging model demonstrates a deeper analytical capability than simple interpolation. By aligning high-probability zones with features like transit nodes and escape corridors, the model is effectively encoding the spatial signature of criminological concepts such as 'nodes' and 'paths'. Consequently, the framework provides not just a predictive surface, but a valuable explanatory insight into the environmental factors that sustain crime concentration.

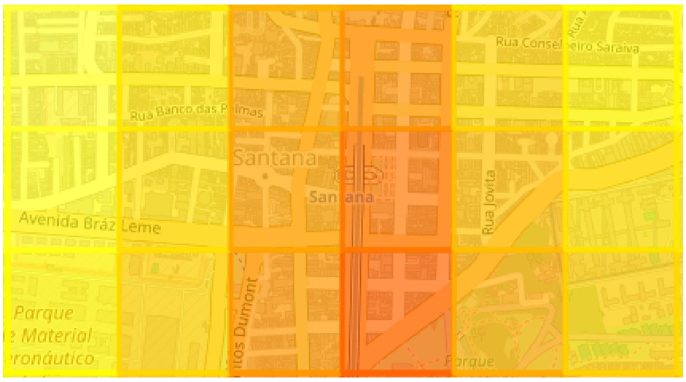

Figure 3. A São Paulo's subway station visualized with Folium's Heatmap (2024)

The significant positive spatial autocorrelation, confirmed by the Moran's I statistic of 0.5752, is a cornerstone of this analysis. This metric indicates that crime incidents in São Paulo are not randomly distributed; rather, they exhibit a strong tendency to cluster spatially, meaning high-crime areas are frequently adjacent to other high-crime areas. The statistical significance of this pattern is validated by the extremely high z-score (141.35) and the near-zero p-value (¡ 0.001), which demonstrate that this clustering is not a product of random chance. This clustering phenomenon is explained by foundational criminological frameworks, such as routine activity and crime pattern theories, which argue that criminal opportunities converge at specific "nodes, paths, and edges" within the urban environment, such as transit hubs or commercial streets that facilitate offender access to targets. Therefore, establishing this spatial dependence is a critical prerequisite; methods like Indicator Kriging are fundamentally based on spatial autocorrelation to interpolate values, and their application would be invalid on spatially random data. The real benefit of this result is twofold: first, for understanding the phenomenon, it makes the latent structure of crime risk visible even in areas with sparse reports. Second, for practical application, it provides public agencies with a continuous probability map, enabling a shift from reactive policing based on incomplete reports to proactive resource allocation—such as patrol prioritization or community outreach—in areas with the highest latent risk, not just the highest report counts.

5. Conclusion

This study successfully demonstrated the viability and effectiveness of Indicator Kriging (IK) as a geostatistical solution to the persistent problem of crime underreporting in urban analysis. By applying this non-parametric method to official crime data from São Paulo, we generated a continuous and robust probability surface that overcomes the critical limitations of data sparsity. The resulting model faithfully reproduced known spatial patterns of crime, such as the concentration of risk along major transit corridors, validating its ability to capture the underlying structure of criminal dynamics even when reported incidents are scarce. The primary methodological contribution lies in providing a framework that directly models the uncertainty of incomplete data, offering a significant advancement over traditional methods that rely on complete records or complex covariates.

The practical implications of this research for public policy are substantial. The probability maps produced by our model offer a powerful tool for public security agencies to move beyond reactive strategies based on flawed data. Instead, they can now identify areas with high latent risk and high uncertainty, allowing for a more proactive, equitable, and efficient allocation of resources, from patrol deployment to community-based prevention initiatives. Furthermore, the computational efficiency of the methodology, which aggregates individual data points into a grid, ensures that this approach is scalable and implementable even for public agencies with limited computational resources, making it a particularly relevant solution for emerging countries.

While this study underscores the power of a purely spatial approach, it also opens promising avenues for future research. The IK-derived probability surface can serve as a foundational layer in more complex predictive models, effectively "correcting" for underreporting at the input stage to improve their accuracy and fairness. Future work could also focus on integrating alternative data sources, such as victimization surveys or social media reports, to further refine the model and deepen the understanding of the "dark figure of crime". Ultimately, this research affirms the immense potential of applying rigorous geostatistical analytics to solve pressing public policy challenges, turning incomplete data into actionable intelligence.

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