# Parameter Estimation of a Wave Energy Converter Model using Physics Informed Neural Networks

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***Abstract.*** *Wave Energy Converters (WECs) play a vital role in the pursuit of sustainable marine energy, offering a promising solution to harness the vast potential of ocean waves to generate energy. However, the accurate identification of physical parameters governing WEC dynamics is a major challenge, especially when direct measurements of the system are limited or noisy. This paper addresses the inverse problem of estimating unknown parameters in a physical model for a WEC system using Physics-Informed Neural Networks (PINNs). The proposed approach incorporates the governing physical equations as soft constraints, giving rise to a physical loss function that is combined with a data loss function and optimized during training. By generating synthetic data that simulates the dynamic response of a WEC, we employ PINNs to infer three critical physical parameters of the model The approach demonstrates high accuracy, particularly when sufficient data is available for training, and remains robust under varying levels of noise. These findings highlight the suitability of PINNs for inverse problems in wave energy systems, supporting their use in system identification and control.*

## 1. Introduction

Wave Energy Converters (WECs) are innovative systems designed to extract energy from ocean waves, representing a promising source of sustainable and renewable power [Falnes 2007, Bhinder et al. 2020]. Accurate modeling of WEC dynamics is essential for their optimal design, control, and deployment [Falcão 2010]. These dynamics are governed by complex integro-differential equations involving parameters such as radiation damping, added mass, and hydrodynamic stiffness, many of which are difficult to measure directly in real-world conditions. In this scenario, the inverse problem consists of estimating these model parameters (that represent physical quantities) from limited or noisy observational data from the system.

Traditionally, inverse problems have been addressed using numerical optimization methods coupled with physics-based solvers [Kaipio and Somersalo 2006, Tarantola 2005]. While effective, these approaches are computationally expensive and often sensitive to noise and initial conditions. In recent years, advances in machine learning have introduced Physics-Informed Neural Networks (PINNs) [Raissi et al. 2019], a novel class of function approximators that incorporate prior physical knowledge into their training process by embedding the governing system equations directly into the loss function used to train the neural network. This allows PINNs to leverage both observational data and constraints imposed by the physical system, resulting in improved generalization and data efficiency [Karniadakis et al. 2021].

PINNs have shown remarkable success in solving forward (i.e., prediction of system state) and inverse problems across various domains, including fluid dynamics [Raissi et al. 2020], solid mechanics [Zhang et al. 2020], and geophysics [Song et al. 2021]. Their ability to encode physical laws enables stable training even in the presence of noise, making them especially suited for real-world engineering applications where high-fidelity measurements are scarce.

In this work, we investigate the application of PINNs to the inverse problem of parameter estimation in the context of WEC dynamics. In particular, we consider a specific physical model for a WEC recently proposed and deployed. This model has the following three physical parameters: radiation damping coefficient ($R_r$), added mass ($M_r$), and elasticity coefficient ($K_e$). Our goal is to estimate this parameters using PINNs under limited and noisy observational data from the system dynamics. Using synthetic data from the simulation of a system where these physical parameters are known, we demonstrate that PINNs can accurately and robustly infer them. This study not only validates the use of PINNs for marine energy systems but also highlights their potential for broader applications in physics-based modeling and control.

The remainder of this paper is organized as follows. Section 2 presents the background on WEC and the related work on using PINNs for inverse problems. Section 3 presents the specific WEC model considered in this work and details of the proposed approach, including experimental design. The results are presented and discussed in Section 4 indicating the potential of the proposed methodology. Last, Section 5 concludes the paper.

## 2. Background and Related Work

This section outlines the theoretical and application-specific background on Wave Energy Converters and Physics-Informed Neural Networks. We first review the mathematical formulation of damped harmonic oscillators, which serve as simplified representations for a variety of physical systems, including marine energy converters. Next, we examine the core principles behind Wave Energy Converters (WECs), with a focus on the Hyperbaric WEC developed at COPPE/UFRJ, as illustrated in Figure 1. We then discuss recent advances in solving inverse problems using Physics-Informed Neural Networks (PINNs), particularly in the context of renewable energy applications.

### 2.1. Damped Harmonic Oscillators and System Identification

The damped harmonic oscillator is a canonical model in physics and engineering, used to describe the dynamic response of systems subject to restoring and dissipative forces. Its equation of motion is given by:

$$m\frac{d^2x}{dt^2} + \mu\frac{dx}{dt} + kx = 0, \tag{1}$$

where $x(t)$ is the position of the object at time $t$, $m$ is the mass, $\mu$ is the damping coefficient, and $k$ is the stiffness constant. In the underdamped regime ($\mu^2 < 4mk$), the system exhibits decaying oscillatory behavior that converges to some position. This formulation underlies a variety of engineering systems, including suspension systems, electrical RLC circuits, and floating bodies in ocean environments.

Parameter identification in such systems involves estimating the quantities $m$, $\mu$, and $k$ from observations of the object position over time (i.e., a time series). Classical system identification techniques include frequency-domain analysis and time-domain optimization, but these often require dense, noise-free data and are sensitive to initialization and overfitting [Kaipio and Somersalo 2006].

### 2.2. Wave Energy Converters

WECs are renewable energy devices that harvest ocean wave energy and convert it into electricity. They are subject to highly dynamic and stochastic marine environments, necessitating accurate dynamical models for simulation, control, and optimization [Falcão 2010]. Many WECs, including point absorbers and oscillating water columns (OWCs), can be modeled as second-order dynamical systems akin to damped harmonic oscillators. The general governing equation takes the form:

$$(M + M_r)\ddot{x}(t) + R_r\dot{x}(t) + K_e x(t) = F_{\text{exc}}(t), \tag{2}$$

where $M$ is the system's movable component mass, $M_r$ is the added water mass, $R_r$ is the radiation damping, $K_e$ is the hydrostatic stiffness, and $F_{\text{exc}}$ is the excitation force from waves [Falnes 2007].

The Hyperbaric WEC consists of a floating body coupled to an enclosed air chamber, producing complex dynamic interactions between mechanical and fluid subsystems [Shadman et al. 2019]. Estimating the hydrodynamic coefficients of such systems is essential for design and operational performance, yet challenging due to environmental variability and limited sensing infrastructure.

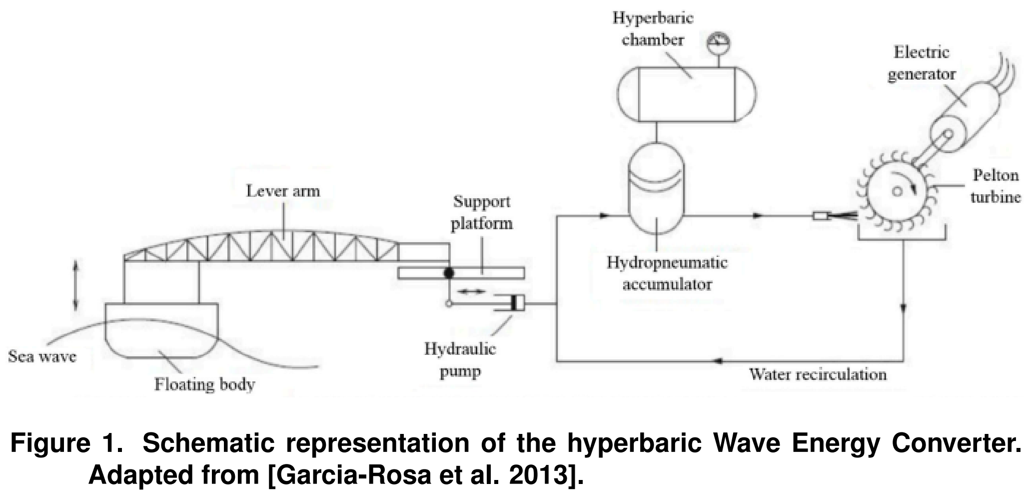


**Figure 1. Schematic representation of the hyperbaric Wave Energy Converter. Adapted from [Garcia-Rosa et al. 2013].**

### 2.3. Inverse Problems and PINNs

Inverse problems involve estimating unknown parameters or source terms from observed outputs, often under constraints imposed by known physical models [Tarantola 2005]. In marine systems, these may include hydrodynamic coefficients or structural parameters that are difficult to measure directly. Classical methods for inverse problems rely on iterative optimization and regularization, but may struggle with ill-posedness and sensitivity to noise.

PINNs have emerged as a robust alternative by embedding the governing equations into the neural network's loss function [Raissi et al. 2019]. This formulation ensures physical consistency while simultaneously estimating unknown quantities from sparse data. Recent studies have demonstrated PINNs capacity for inferring hidden parameters in fluid flow [Raissi et al. 2020], elasticity [Zhang et al. 2020], and biomedical systems [Kissas et al. 2020]. Their ability to integrate physics as a soft constraint makes them particularly attractive for solving inverse problems in partially observed systems.

### 2.4. Inverse Problems in Wave Energy Converters

Physics-Informed Neural Networks (PINNs) have been shown to accurately estimate damping and stiffness parameters in Oscillating Wave Surge Converters (OWSCs), even under limited data availability [Ayyad et al. 2025], motivated by the complexity of hydrodynamic modeling and the scarcity of high-resolution sensor data. Similarly, PINNs have been employed to recover control-relevant parameters in Oscillating Water Column systems, demonstrating their utility for operational optimization [Ni et al. 2024].

Despite these advances, current works are limited in generalizability across WEC architectures and typically assume simplified excitation models. Moreover, few studies systematically evaluate the robustness of PINNs under varying noise levels or different temporal sampling regimes, which this work seeks to address.

## 3. Methodology

This section describes the proposed methodology for solving the inverse problem of parameter estimation in Wave Energy Converters (WECs) using Physics-Informed Neural Networks (PINNs). We begin by presenting the mathematical model of the WEC system under consideration, followed by a formalization of the inverse problem within the PINN framework. Finally, we detail the simulation setup, neural network architecture, and training strategy.

### 3.1. A Wave Energy Converter Model

We adopt the model proposed in [Machado et al. 2015], which characterizes the WEC dynamics as a forced damped harmonic oscillator:

$$(M + M_r)\,\ddot{y}(t) + R_r\,\dot{y}(t) + K_e\,y(t) = F_e(t) + F_p(t), \tag{3}$$

where $M$ is the buoy's mass, $M_r$ the added mass, $R_r$ the radiation damping, and $K_e$ the hydrostatic stiffness. The right-hand side comprises the wave excitation force $F_e(t)$ and the actuation force $F_p(t)$ from the hyperbaric piston.

The excitation force follows a frequency-dependent sinusoidal model [Garcia-Rosa and et al. 2009]:

$$F_e(t) = H\left(\frac{\rho g^3 R_r}{2\omega^3}\right)^{1/2}\cos(\omega t), \tag{4}$$

where $H$ is the wave height, $\rho$ the water density, $g$ gravitational acceleration, and $\omega$ the wave frequency.

In order to determine the piston's force $F_p$, we will make the simplifying assumption that a control system is acting to keep the pressure of the hydraulic pump's piston optimal. Under this assumption, the piston force is modeled via a simple control law [Garcia-Rosa and et al. 2009]:

$$F_p(t) = \begin{cases} 0, & \dot{y}(t) \geq 0 \\ p_c, & \dot{y}(t) < 0 \end{cases}, \tag{5}$$

with optimal pressure defined as:

$$p_{c_{\text{opt}}} = \frac{H\pi}{4A_p} \left( \frac{\rho g^3 R_r}{2\omega^3} \right)^{1/2}, \tag{6}$$

where $A_p$ is the piston cross-sectional area. The model output is the buoy's vertical displacement $y(t)$, serving as input-output training data for the PINN.

Last, the residual equation used in the PINN loss is given by:

$$R(y, \dot{y}, \ddot{y}; M, M_r, R_r, K_e) = (M + M_r)\,\ddot{y} + R_r\,\dot{y} + K_e\,y - F_e(t) - F_p(t). \tag{7}$$

### 3.2. Inverse Problem Formulation with PINNs

The inverse problem consists of estimating the parameters $\boldsymbol{\theta} = \{M, M_r, R_r, K_e\}$ such that the neural network's prediction of the buoy motion satisfies both the observed displacement data and the physical law encoded in Eq. 7.

The PINN is trained by minimizing the composite loss:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{data}} + w_{\text{physics}} \cdot \mathcal{L}_{\text{physics}}, \tag{8}$$

where

$$\mathcal{L}_{\text{data}} = \text{MSE}(y, \hat{y}), \quad \mathcal{L}_{\text{physics}} = \text{MSE}(R(t)),$$

MSE denotes the Mean Squared Error function and $R(t)$ is the residual defined in Eq. 7. Note that this loss function combines observational error with respect to the available data $\mathcal{L}_{\text{data}}$ and the residual error of the model $\mathcal{L}_{\text{physics}}$. The weighting coefficient $w_{\text{physics}}$ is treated as a hyperparameter that balances the contribution of the physics-based constraint during training. This allows the network to infer both the dynamic trajectory and the hidden parameters $\boldsymbol{\theta}$ in a physics-constrained manner.

### 3.3. Experimental Design and Simulation Setup

To evaluate the effectiveness and robustness of the PINN approach, we define two experimental scenarios. In each scenario, different subsets of parameters are treated as unknowns. For every configuration, 10 independent training runs are performed. The *absolute relative error* (ARE) between true and inferred parameters is computed, and the median ARE is reported to account for initialization variance.

The training data is generated via numerical integration of Eq. 3 using the backward differentiation formula (BDF) solver in `SciPy` [SciPy Developers 2024], suitable for stiff systems. Simulations cover 160 seconds with a time resolution of 0.01 s (17,233 steps).

| Parameter | Description | Value |
|---|---|---|
| **Sea Conditions** | | |
| $H$ | Wave height | 1 m |
| $A_w$ | Wave amplitude | 0.5 m |
| $T_{\text{wave}}$ | Wave period | 6 s |
| **Floating Body** | | |
| $K_e$ | Elasticity coefficient | 32,587 N/m |
| $M$ | Mass | 17,000 kg |
| $M_r$ | Added mass | 12,716 kg |
| $R_r$ | Radiation damping | 5,379 kg/s |
| **Hydropneumatic System** | | |
| $A_p$ | Piston area | 0.5 $m^2$ |
| $g$ | Gravity | 9.81 $m/s^2$ |
| $\rho$ | Water density | 1,025 $kg/m^3$ |

**Table 1. Simulation Parameters**

### 3.4. PINN Architecture and Training Strategy

The network architecture comprises six hidden layers with 64 neurons each, activated via `tanh`. Physical parameters are modeled as trainable variables and optimized jointly with the network weights using the Adam optimizer (learning rate $10^{-3}$). No dropout or batch normalization is used to preserve the physics constraints.

To stabilize training, a two-phase strategy is employed [Ayyad et al. 2025]. In Phase I, the network is trained solely on the observational data (coefficient $w_{\text{physics}} = 0$) until the loss falls below $10^{-5}$. In Phase II, the physics loss is activated with $w_{\text{physics}} = 10^{-10}$, guiding parameter refinement. Initial values for physical parameters are set to 1.0 and scaled by 1000 in the residual to normalize gradients. Additionally, the initial values for the learnable physical parameters are set to 1.0. To accelerate model convergence, these parameters are scaled by a factor of 1000 within the residual equation.

This scaling strategy is applied because the true values of the unknown parameters typically have a magnitude of approximately $10^3$ (e.g., for $M_r$, $R_r$, and $K_e$). The normalization of this scaling is compensated for in the global loss function, specifically in the physics loss weight. Given that the physics loss function is typically the squared residual, as in Eq 8, the scaling factor $10^3$ in the residual becomes $(10^3)^2 = 10^6$ in the loss term. Thus, a $w_{\text{physics}}$ value of $10^{-10}$ results in an effective factor of $10^{-4}$ for the physics loss, balancing it with the data loss and preventing excessively large gradients from one component from dominating the optimization.

## 4. Results

This section details the experimental analysis conducted on inferring unknown parameters in the equation that describes the floating body part of a WEC. These experiments were implemented using the PyTorch machine learning framework. The corresponding code is available in a Google Colab notebook at: https://share.google/ZahqFdiEd3iD9nonK

All experiments utilize the same simulated data, depicted in Figure 2. It's noteworthy that the simulated waves (referring to the buoy displacement $y(t)$ in Figure 2) reach considerable heights, approaching five meters above sea level. This may be attributed to the use of $H_s$, representing the average of the one-third largest waves from Machado et al.'s [Machado et al. 2015] simulation, as the value for $H$ in Equation 4, which denotes the height of a single wave.

The strategy employed to determine the final inferred values for each parameter varied across scenarios. In Scenarios 4.1, the reported parameter values correspond to those obtained by the PINN at the final iteration of training. Conversely, for Scenario 4.2, the final inferred values represent the mean of the parameter values recorded over the last 100 iterations. This approach was adopted due to the increased dispersion observed in the inferred values within Scenario 4.2, which is attributed to the introduction of noise in that experimental setup.

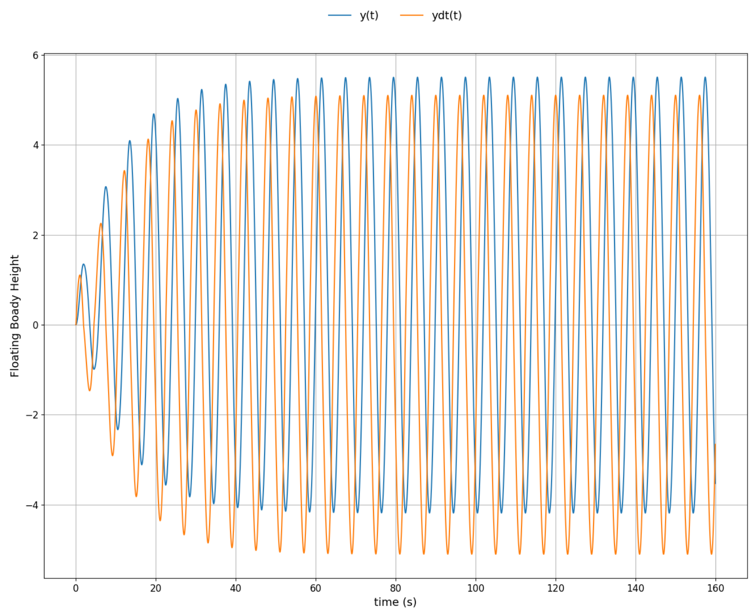


**Figure 2. Simulated vertical displacement and velocity of a Wave Energy Converter (WEC) buoy. The function $y(t)$ represents the vertical position of the buoy in meters, while $\dot{y}(t)$ denotes its velocity in meters per second.**

### 4.1. Scenario 1: Inferring Physical Parameters $R_r$, $M_r$, and $K_e$ with Varying Training Data Time Window

This scenario was designed to evaluate whether the PINN is capable of simultaneously inferring three physical parameters: $R_r$, $M_r$, and $K_e$. These coefficients were selected due to their analogy with the parameters of an underdamped harmonic oscillator, which were previously estimated using PINNs [Mamlankou 2025]. The model was trained using the first 700 time steps of the simulation data and optimized for 100,000 epochs. No experimental variable was altered across runs; the same configuration was repeated for 10 independent trials to assess consistency.

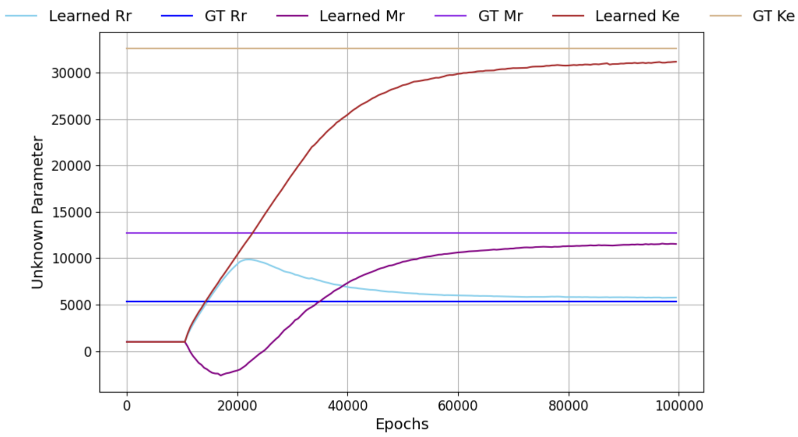


**Figure 3. Evolution of the inferred $R_r$, $M_r$ and $K_e$ parameter during a single training session**

Interestingly, Figure 3 shows that during the first epochs in which the unknown parameters are being learned, the $M_r$ starts decreasing and the $R_r$ value grows beyond its ground truth value. After a small number of epochs they start to converge in the right direction. This could be because $K_e$ becomes closer to its ground truth and possibly bringing the gradient for $R_r$ to the right direction.

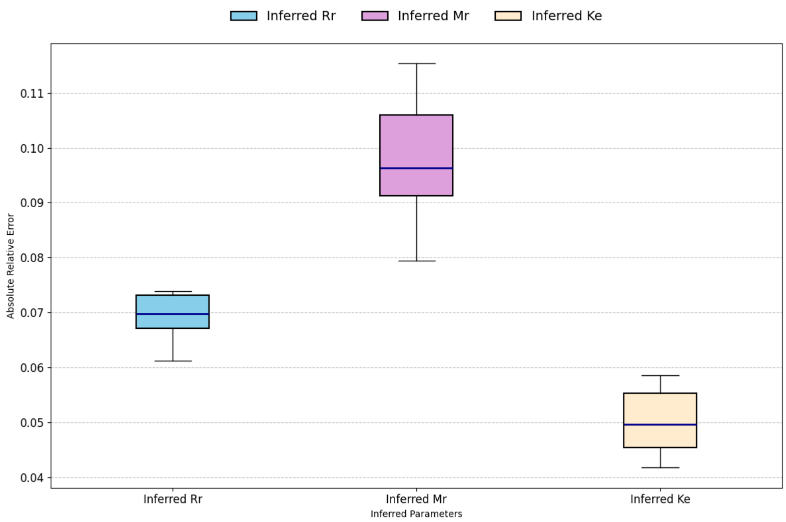


**Figure 4. Boxplot of the Absolute Relative Error of PINN in inferring the $R_r$, $M_r$ and $K_e$ parameters.**

## 4.2. Scenario 2: Inferring Physical Parameters $R_r$ and $M_r$ under Varying Noise Levels

This scenario evaluates the robustness of Physics-Informed Neural Networks (PINNs) in solving inverse problems when the training data is corrupted by noise. To simulate realistic measurement conditions, zero-mean Gaussian noise was added to the data, making the task of inferring the unknown parameters $R_r$ and $M_r$ more challenging. The experimental variable is the standard deviation of the additive noise, set to 0.001, 0.01, and 0.1. Training was performed on a fixed time window comprising 200 simulation steps.

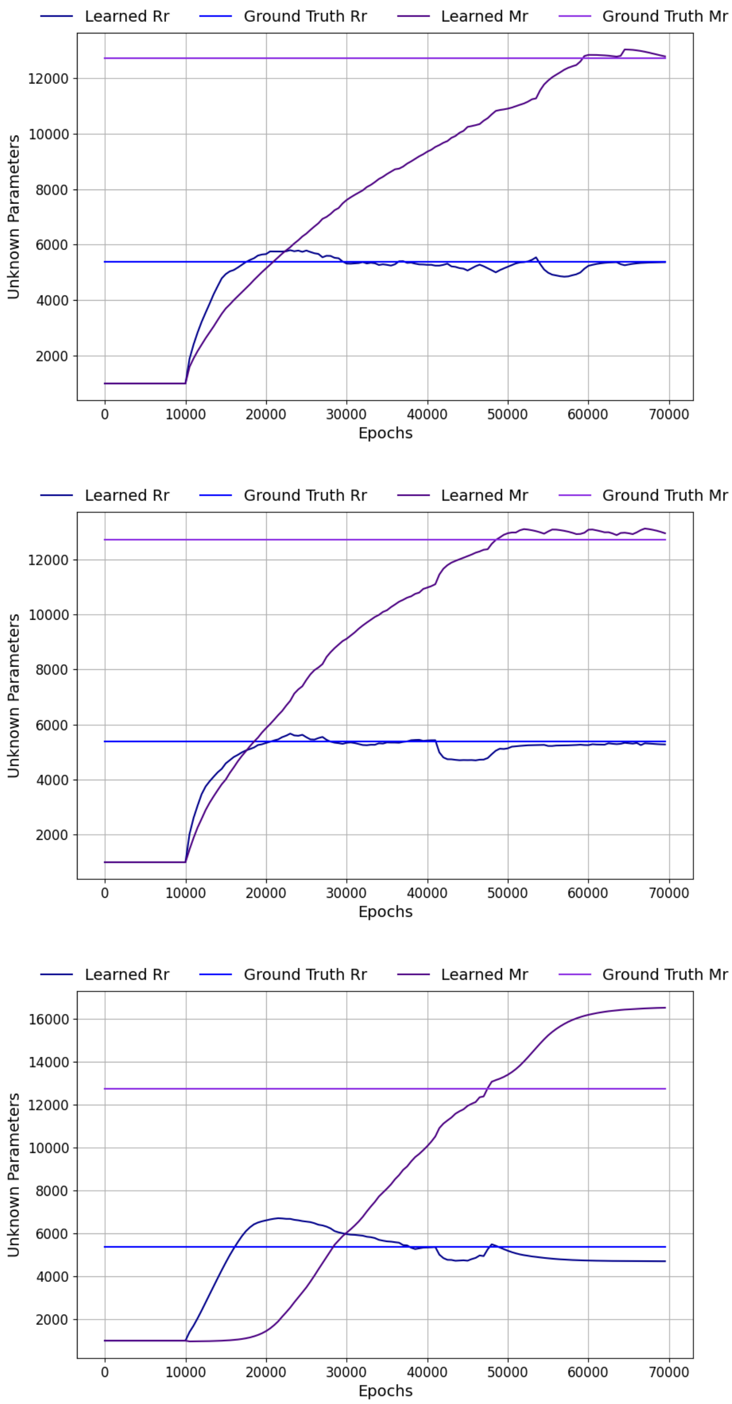


**Figure 5. Evolution of the inferred $R_r$ and $M_r$ parameters during a single training session for varying standard deviation of added noise: (top) 0.001, (center) 0.01, (bottom) 0.1.**

Figure 5 illustrates how the value of each of the two parameters vary throughout the training process across epochs. Note that under noise generated by a standard deviation of 0.001 and 0.01 the parameter estimation is similar and accurate for both parameters. However, under the stronger noise with standard deviation of 0.1, the estimation accuracy is compromised, particularly when estimating $M_r$, which consistently exceeds its ground truth value.

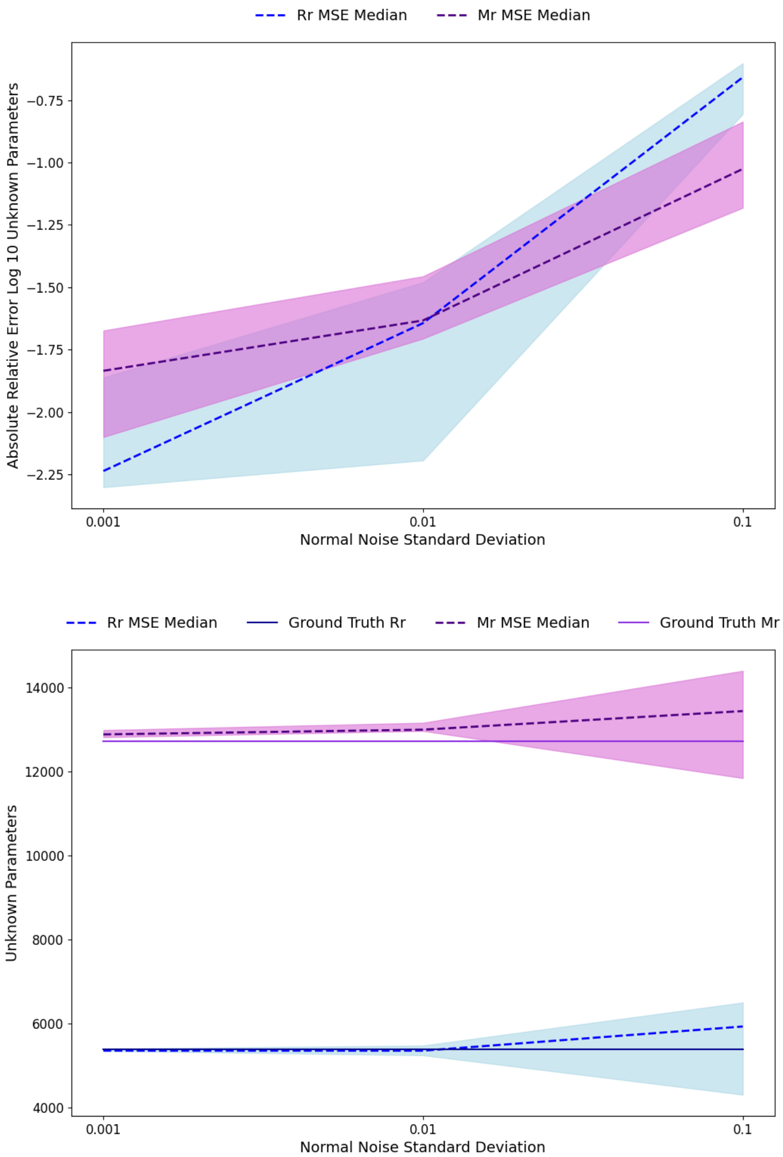


**Figure 6. Absolute Relative Error (ARE) of the PINN when inferring the parameters $R_r$ and $M_r$ under varying levels of additive noise (top). Inferred parameter values for the same experiment are shown below (bottom). The dotted line indicates the median ARE across 10 independent runs, while the shaded area represents the interquartile range for each experimental configuration.**

Figure 6 (bottom) shows that, for noise standard deviations of $10^{-3}$ and $10^{-2}$, the inferred values remain close to the ground truth and exhibit minimal variability across runs. However, as the noise standard deviation increases to $10^{-1}$, the median of the 10 runs significantly deviates, rising above the ground truth, and the dispersion of results substantially increases.

## 5. Conclusion

PINNs exhibit superior robustness and effectiveness for physics-based simulations, particularly under data scarcity. These findings establish PINNs as valuable tools for simulation and optimization in engineering applications.

The experiments conducted in this study yielded several key insights. While PINNs are generally data-efficient, it is crucial to test and tune the quantity of training data employed for inverse problem-solving. Data scarcity can lead to convergence problems, resulting in the model converging to values significantly distant from the ground truth [Berardi et al. 2025]. A further noteworthy observation underscored the importance of calibrating the relative weights of the physics-based and data-based components in the total loss function. This is particularly relevant in inverse problems because, as the introduction of the physics-based loss can initially cause a sharp increase in the total loss, potentially steering the gradient descent toward a suboptimal or misleading directions.

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