Reproduction Operator Sensitivity in MOEA/D under Tchebycheff Decomposition: An Empirical Study on Regular and Irregular Pareto Fronts

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Abstract. *This paper presents a controlled empirical study on the sensitivity of MOEA/D to reproduction operators under fixed Tchebycheff decomposition. Eight operator variants were evaluated in PlatEMO 4.15 on DTLZ and IDTLZ benchmark instances with regular and irregular Pareto fronts, two to five objectives, and different decision-space dimensions. The study does not propose a new algorithm; instead, it isolates the effect of replacing the reproduction mechanism while preserving the same decomposition scheme. Each configuration was executed 30 times independently and compared with the baseline MOEA/D using IGD and a Wilcoxon signed-rank test. The evidence indicates scenario-dependent behavior: ABC and simulated annealing show the strongest aggregated profile on regular many-objective fronts (9 wins, 4 losses, 3 ties each; IGD ratio $r = 0.90$), MMOPSO shows favorable aggregated IGD ratios on irregular fronts ($r = 0.64$–0.95), despite limited statistical wins, and NMS degrades performance in nearly every configuration (0–1 wins across all scenario groups). The results support operator selection based on scenario and motivate adaptive mechanisms conditioned on the Pareto-front geometry.*

1. Introduction

Multiobjective optimization problems involve conflicting objectives. Instead of a single optimal solution, the goal is to approximate a set of non-dominated solutions capable of representing different trade-offs along the Pareto front. Among multiobjective evolutionary algorithms, MOEA/D (*Multiobjective Evolutionary Algorithm based on Decomposition*) has established itself as one of the leading decomposition-based approaches, as it transforms a multiobjective problem into cooperative scalar subproblems associated with weight vectors [Zhang and Li 2007].

The performance of MOEA/D depends on algorithmic decisions that affect convergence and diversity. Two of these are particularly relevant: the scalarizing function, which defines how each subproblem evaluates a candidate solution, and the reproduction operator, which defines how new solutions are generated. The literature reports advances in scalarization, weight adaptation, adaptive operator selection, and hybrid MOEA/D variants [Li and Zhang 2009, Li et al. 2014, Wang et al. 2015, Wang et al. 2019, Almeida et al. 2018, Li 2024]. Even so, controlled empirical evidence

remains limited on the behavior of different operators when the scalarizing function is kept fixed, particularly when comparing regular and irregular Pareto fronts.

This paper investigates this specific setting. The contribution lies in the systematic analysis of the effect of replacing the reproduction mechanism in MOEA/D under Tchebycheff scalarization. This choice distinguishes the study from approaches that combine several simultaneous changes, as it allows the role of the operator to be observed while reducing confounding effects from changes in scalarization, subproblem updating, or performance metrics.

The research question is: *to what extent does the choice of reproduction operator alter the performance of MOEA/D under Tchebycheff scalarization on problems with regular and irregular Pareto fronts?* The hypothesis is that no operator is universally superior; the suitability of each operator depends on the geometry of the front, the number of objectives, and the dimensionality of the decision space.

The contributions are fourfold. First, the paper presents a systematic comparison of eight reproduction operator variants in MOEA/D under a common experimental protocol. Second, it organizes the analysis by Pareto-front geometry, separating regular and irregular instances. Third, it consolidates the results by unique configurations, avoiding improper duplication in the original tables. Fourth, it synthesizes practical recommendations for configuring MOEA/D and points to directions for adaptive operator selection.

The remainder of this paper is organized as follows. Section 2 reviews related work, while Section 3 introduces the conceptual foundations and defines the scope of the study, covering MOEA/D, Tchebycheff scalarization, the evaluated operators, the IGD metric, and Pareto-front classification. Section 4 details the experimental protocol, execution environment, benchmarks, parameter settings, and statistical procedure. Section 5 presents and discusses the aggregated and configuration-level results across the investigated scenarios. Finally, Section 6 summarizes the main conclusions, practical implications, and directions for future work.

2. Related Work

Previous research on MOEA/D relevant to this study can be broadly organized into two lines: scalarization adaptation and reproduction-operator selection or adaptation. Li and Zhang [Li and Zhang 2009] proposed neighborhood-update strategies, and it became evident that the choice of the DE reproduction operator with polynomial mutation directly affects the ability of MOEA/D to handle complicated Pareto sets, although a single fixed operator was considered throughout the entire run. Li et al. [Li et al. 2014] introduced adaptive operator selection based on multi-armed *bandits*, dynamically switching between operators during the search phase; Almeida et al. [Almeida et al. 2018] follow the same line with *bandit*-based hyper-heuristics. Wang et al. [Wang et al. 2019] proposed an *ensemble* framework that combines multiple operators simultaneously within the population. These works demonstrate benefits from making the operator adaptive, but they do not systematically isolate the effect of each individual operator under fixed decomposition, so adaptation and comparison become confounded within the same experiment.

On the scalarization side, Wang et al. [Wang et al. 2015] proposed adaptive scalarizing functions, and Li [Li 2024] presents a survey of MOEA/D, including taxonomies of

operators and scalarizations, but without a controlled empirical comparison specifically addressing operator sensitivity on regular and irregular fronts.

The eight reproduction operators reproduced and evaluated in this paper are based on articles that proposed, in different ways, alternative mechanisms to improve the search of MOEA/D: DE follows Li and Zhang [Li and Zhang 2009]; CMA-ES follows Zapotecas-Martínez et al. [Zapotecas-Martínez et al. 2015], who inject covariance and step-size adaptation into each subproblem; the Nelder-Mead simplex search coupled with DE follows Zhang et al. [Zhang et al. 2017]; adaptive simulated annealing follows Li and Landa-Silva [Li and Landa-Silva 2011]; ACO for combinatorial problems follows Ke et al. [Ke et al. 2013]; multi-colony ABC with an external archive follows Zhong et al. [Zhong et al. 2014]; and PSO with multiple search strategies follows Lin et al. [Lin et al. 2015]. These studies provided the conceptual basis for simplified versions adapted to the continuous domain; the original mechanisms propose more elaborate components (multiple colonies, discrete pheromone matrices, persistent state across generations, cooling schedules) than the operators implemented here.

However, the literature provides evidence that adaptive operators and adaptive scalarizations improve performance, but it does not isolate, under a common protocol and fixed decomposition, the effect of eight distinct operator families on regular and irregular fronts. It is this gap that this paper seeks to fill.

3. Background and Scope

MOEA/D decomposes a multiobjective problem into N scalar subproblems. Each subproblem is associated with a weight vector $\lambda = (\lambda_1, \ldots, \lambda_m)^T$, where m is the number of objectives, and uses neighborhoods of weight vectors to share solutions. Considering x as the decision vector, $f(x)$ as the objective vector, and z^* as the estimated ideal point, the Tchebycheff scalarization is defined as

$$g^{te}(x \mid \lambda, z^*) = \max_{1 \leq i \leq m} \{\lambda_i | f_i(x) - z_i^* |\}. \tag{1}$$

This formulation is adopted to isolate the influence of the reproduction operators. Since Tchebycheff scalarization is a standard choice in MOEA/D and handles certain nonconvex fronts better than the weighted sum, it provides a suitable comparison point for a sensitivity study [Zhang and Li 2007, Li and Zhang 2009].

In this study, each variant is treated as an alternative offspring-generation mechanism within the MOEA/D update cycle. Thus, ABC, ACO, CMA-ES, DE, EDA, MMOPSO, NMS, and SA are not evaluated as complete, independent algorithms, but as reproduction operators or modules coupled to the same decomposition framework. They represent distinct search mechanisms: stochastic perturbation, probabilistic sampling, distribution adaptation, vector differences, swarms, and local search [Karaboga and Basturk 2007, Dorigo et al. 1996, Hansen and Ostermeier 2001, Storn and Price 1997, Zhou et al. 2015, Lin et al. 2015, Nelder and Mead 1965, Kirkpatrick et al. 1983]. Performance differences may therefore indicate compatibilities or tensions between reproduction dynamics, decomposition, and front geometry.

The main metric is the Inverted Generational Distance (IGD). Given a reference

set P^* and an approximation set A, IGD is given by

$$IGD(A, P^*) = \frac{1}{|P^*|} \sum_{v \in P^*} d(v, A), \tag{2}$$

where $d(v, A)$ is the smallest Euclidean distance between v and the solutions in A. Lower values indicate better proximity and coverage with respect to the reference front. IGD was chosen because it is widely used in multiobjective optimization studies and because it combines convergence and diversity aspects into a single measure [Van Veldhuizen and Lamont 1998, Tian et al. 2017]. Its limitation is that it does not replace complementary metrics, such as Hypervolume, especially when different approximations present distinct trade-offs between coverage and proximity.

The regular/irregular classification is operational and based on the geometry of the Pareto front. Regular fronts are continuous, smooth, and relatively well distributed, as occurs in DTLZ1–DTLZ4. Irregular fronts exhibit degeneracy, discontinuity, bias, disconnected regions, non-uniformity, or geometric inversion that impair the correspondence between weight vectors and well-distributed solutions. In this paper, DTLZ5–DTLZ7 and IDTLZ1–IDTLZ2 are treated as irregular instances [Deb et al. 2005, Tian et al. 2017].

It is important to delimit the scope of validity of the results. Since Tchebycheff scalarization was kept fixed across all experiments, the findings reflect the interaction between the reproduction operator and this specific scalarization, not a general property of MOEA/D. Other scalarizations, such as PBI and the weighted sum, may alter the magnitude or even the sign of some comparisons, especially on irregular fronts, where the choice of aggregation function strongly affects the distribution of solutions [Zhang and Li 2007, Li and Zhang 2009]. This paper does not evaluate this interaction; this is an assumed limitation and an explicit direction for future work (Section 6).

4. Experimental Methodology

The experiments were conducted in MATLAB R2026a with PlatEMO 4.15, released in May 2026 [Tian et al. 2017, BIMK Group 2026]. The computational environment used a 13th Gen Intel(R) Core(TM) i7-1360P processor, 2.20 GHz, 16 GB of RAM, a 64-bit operating system, and x64 architecture. Identifiable and unnecessary data were omitted. The scripts, the customized platform, and the raw results of the 30 runs per configuration are publicly available as a study artifact[1].

The protocol considered DTLZ and IDTLZ instances with $M \in \{2, 3, 4, 5\}$ objectives and the dimensionality and computational-budget configurations detailed in Table 1. For $M = 2$–4, the evaluation budget was $maxFE = 120{,}000$; for $M = 5$, $maxFE = 126{,}000$, with population size $N = 120$ ($M = 2$–4) and $N = 126$ ($M = 5$), following the convention of weight vectors generated by `UniformPoint`. Scenarios with $D = 100$ represent the high decision-space dimensionality condition for the problems where this configuration is applicable. Each configuration was run 30 times independently. Table 1 summarizes the protocol.

For each operator, the results were compared to the baseline MOEA/D using a Wilcoxon signed-rank test for each experimental configuration. The symbols $+$, $-$,

[1] `https://github.com/belamedeirosbl/PlatEMO-moead-variation-strategies`

Table 1. Experimental configuration.

Component	Description
Base algorithm	MOEA/D with Tchebycheff scalarization.
Operators	ABC, ACO, CMA-ES, DE, EDA, MMOPSO, NMS, and SA as reproduction variants coupled to MOEA/D.
Benchmarks	DTLZ1–DTLZ4 (regular fronts); DTLZ5–DTLZ7 and IDTLZ1–IDTLZ2 (irregular fronts).
Objectives and dimension	$M \in \{2, 3, 4, 5\}$; default configurations and $D = 100$ when applicable.
Runs	30 independent runs per configuration.
Metric	IGD, with lower values indicating better performance.
Statistical comparison	Wilcoxon signed-rank test against the baseline MOEA/D, with $\alpha = 0.05$.
Population and neighborhood	$N = 120$ ($M = 2$–4); $N = 126$ ($M = 5$); $T = \lceil N/10 \rceil$.
Weight vectors	Generated by `UniformPoint` in the MOEA/D implementation.
Stopping criterion	$maxFE = 120{,}000$ ($M = 2$–4); $maxFE = 126{,}000$ ($M = 5$).
Environment	MATLAB R2026a; PlatEMO 4.15; Intel Core i7-1360P 2.20 GHz; 16 GB RAM; 64-bit operating system; x64 architecture.

and $\approx$ indicate, respectively, statistically superior, inferior, or not significantly different from the baseline MOEA/D. Since the study involves multiple comparisons among operators, instances, and scenarios, the p-values should be reported in the complete artifacts and, when available, submitted to Holm or FDR correction [Wilcoxon 1945, Holm 1979, Benjamini and Hochberg 1995]. In this version, individual p-values were not available; therefore, the original statistical comparison symbols were preserved without reclassification by multiple-comparison correction.

During the curation of the original tables, exact repetitions were identified in part of the results for irregular problems. To avoid duplicate counting, the aggregation in Section 5 considers only unique combinations of problem, number of objectives, and dimensionality. The same deduplication procedure was applied to the detailed IGD tables, so that the complete results presented in Tables 4 and 5 document the unique configurations used in the aggregated analysis.

5. Results and Discussion

Table 2 is the main summary table of the results, while Tables 4 and 5 present the detailed IGD values per experimental configuration. Each cell of Table 2 shows the win, loss, and non-significant comparison counts ($+/-/\approx$) relative to the baseline MOEA/D and, in parentheses, the geometric ratio $r = \mathrm{IGD}_{op}/\mathrm{IGD}_{base}$, where IGD_{op} is the average IGD obtained by the variant with the respective reproduction operator, and IGD_{base} is the average IGD obtained by the baseline MOEA/D.

In regular problems with few objectives, ABC obtained six wins, followed by SA with five, while MMOPSO and DE obtained four and three wins, respectively. The aggregated IGD value slightly favors MMOPSO ($r = 0.99$), whereas DE remains close

Table 2. Aggregated results by scenario. Each cell shows $+/-/\approx$ and, in parentheses, r, the geometric mean of IGD_{op}/IGD_{base} over the unique configurations of the group.

Group	ABC	ACO	CMA-ES	DE	EDA	MMOPSO	NMS	SA
Regular, 2–3 obj.	6/8/2 (1.30)	2/11/3 (4.20)	1/15/0 (2.79)	3/7/6 (1.02)	3/11/2 (4.57)	4/6/6 (0.99)	1/12/3 (10.20)	5/9/2 (1.17)
Regular, 4–5 obj.	9/4/3 (0.90)	2/9/5 (8.68)	1/14/1 (4.48)	6/7/3 (1.23)	3/11/2 (12.50)	6/8/2 (1.06)	0/16/0 (10.50)	9/4/3 (0.90)
Irregular, 2–3 obj.	1/9/2 (2.03)	2/10/0 (4.08)	0/11/1 (9.58)	2/7/3 (1.69)	2/10/0 (8.15)	2/9/1 (0.95)	0/12/0 (6.29)	1/8/3 (2.06)
Irregular, 4–5 obj.	2/8/2 (0.64)	3/8/1 (3.48)	2/10/0 (4.24)	2/8/2 (0.90)	4/8/0 (9.32)	4/6/2 (0.64)	0/12/0 (6.13)	2/7/3 (0.65)

to the baseline MOEA/D ($r = 1.02$). Since all operators accumulate losses, this scenario does not support recommending a universal direct replacement. In regular problems with four and five objectives, ABC and SA show the best aggregated profile, both with nine wins, four losses, and three ties, plus $r = 0.90$. This is the strongest finding of the study: on regular many-objective fronts, operators with stochastic diversification appear to better preserve coverage without disrupting the decomposition structure.

Results for irregular problems should be interpreted more cautiously. With two and three objectives, DE shows the best relative balance among wins, losses, and ties, although it has only two wins out of twelve unique configurations. MMOPSO exhibits the lowest aggregated IGD ratio ($r = 0.95$), but with nine statistical losses, which characterizes a localized improvement. With four and five objectives, MMOPSO shows the best aggregated profile, with four wins, six losses, and two ties, plus $r = 0.64$. ABC and SA also have favorable aggregated ratios in this group, but with few statistical wins. Thus, IGD ratios and statistical comparison symbols should be read together.

The behavior of the operators suggests plausible, though not definitive, explanations, and it is useful to compare it with what the adaptive-selection literature already reports. DE can explore useful directions when vector differences reflect the local structure of the population, a condition common in simple regular fronts, but it loses efficiency when irregular fronts distort the relationship between weight-vector neighborhoods and promising regions. This pattern is consistent with the original motivation of Li and Zhang [Li and Zhang 2009] for combining DE with repair mechanisms on complicated *Pareto sets*. MMOPSO can reach difficult regions through collective trajectories via *pbest*/*gbest*, but its advantage is unstable (few statistical wins despite favorable IGD ratios). This result echoes the observation of Lin et al. [Lin et al. 2015] that multiple search strategies help coverage but do not always guarantee statistically superior convergence. ABC and SA appear more favorable on regular many-objective fronts, possibly because they combine diversification and perturbations that are less dependent on global models and on the population's estimated covariance. This also helps explain why CMA-ES and EDA may suffer in larger spaces, where their distribution estimates become noisy with few samples per neighborhood. This behavior is consistent with adaptive-selection works based on *bandits* [Li et al. 2014, Almeida et al. 2018], which report non-uniform performance of model-based probabilistic operators when the sampling neighborhood is small. NMS almost always loses, which is compatible with the limitation of a simplex local search — isolated from the DE component of the original proposal of Zhang et al. [Zhang et al. 2017] — when MOEA/D needs to maintain diversity distributed across subproblems.

The main findings are the absence of a universal operator, the relatively good performance of ABC and SA on regular many-objective fronts, and the recurring fragility of

Table 3. Practical synthesis of the observed evidence. The recommendations are conditional and do not indicate universal superiority.

Scenario	Most promising operators	Interpretation
Regular fronts, 2–3 objectives	ABC, SA, and MMOPSO	Possible gains, but sensitive to increasing dimensionality and without clear dominance; instance-level validation is recommended.
Regular fronts, 4–5 objectives	ABC and SA	Best aggregated profile; good initial candidates for sensitivity analysis.
Irregular fronts, 2–3 objectives	DE and MMOPSO, with caution	Localized improvements; statistical comparison symbols and IGD ratio do not indicate broad dominance.
Irregular fronts, 4–5 objectives	MMOPSO, with caution	Best aggregated balance, but still with enough losses to avoid a universal conclusion.
High dimensionality	Avoid NMS as a direct replacement	Simple local search tends to harm diversity and coverage in MOEA/D.

NMS. The localized findings include gains from DE on some irregular problems with few objectives and gains from MMOPSO on irregular problems with many objectives. This distinction avoids overinterpretation and preserves the main message: operator choice should be conditioned on the scenario.

5.1. Detailed Results by Configuration

Table 2 summarizes the results by problem group, operator, and experimental scenario. To make the analysis auditable within the manuscript itself, Tables 4 and 5 present the detailed IGD values used in the aggregation, consolidating the objectives $M = 2$–5 and the evaluated dimensionalities into a single table. Table 4 reports the problems with a regular Pareto front; Table 5 presents the problems with an irregular Pareto front, considering only unique configurations. In both, each cell shows the average IGD and the statistical comparison symbol relative to the baseline MOEA/D ($+$: significantly better; $-$: significantly worse; $\approx$: no statistically significant difference); the complete standard deviations are available in the study artifacts.

6. Conclusion

This paper presented a controlled empirical study on the sensitivity of MOEA/D to reproduction operators under Tchebycheff decomposition. The results show that reproduction-operator choice affects MOEA/D performance, with the magnitude and direction of this effect depending on Pareto-front geometry, the number of objectives, and decision-space dimensionality.

The results do not support the existence of a universally superior operator. ABC and SA were more competitive on regular many-objective fronts; DE and MMOPSO

Table 4. IGD results for problems with a regular Pareto front ($M = 2$–5; default D and $D = 100$). Each cell shows the average IGD and the statistical sign relative to the baseline MOEA/D ($+$: significantly better; $-$: significantly worse; $\approx$: no statistically significant difference); shaded cells indicate the lowest average IGD in the row, including the baseline MOEA/D, with ties broken by decimal places not shown; complete standard deviations are available in the study artifacts.

Problem	M	D	ABC	ACO	CMA-ES	DE	EDA	MMOPSO	NMS	SA	Base
DTLZ1	2	6	1.49e-3 $\approx$	3.69e-2 $-$	3.68e-3 $-$	1.49e-3 $\approx$	5.33e-2 $-$	1.49e-3 $\approx$	7.61e+0 $-$	1.49e-3 $\approx$	1.49e-3
	3	7	2.61e-2 $+$	2.63e+0 $-$	1.02e-1 $-$	2.60e-2 $+$	1.16e+1 $-$	2.62e-2 $+$	1.60e+1 $-$	2.62e-2 $+$	2.63e-2
	2	100	1.84e+2 $-$	2.35e+2 $-$	5.95e+1 $-$	3.60e+2 $-$	4.25e+2 $-$	2.21e+2 $-$	8.15e+2 $-$	1.90e+2 $-$	3.77e+1
	3	100	1.70e+2 $-$	2.14e+3 $-$	1.56e+3 $-$	1.06e+2 $\approx$	1.77e+3 $-$	2.74e+2 $-$	7.09e+2 $-$	1.74e+2 $-$	9.50e+1
	4	8	8.99e-2 $+$	7.33e+0 $-$	2.19e-1 $\approx$	9.85e-2 $-$	1.40e+1 $-$	1.05e-1 $-$	1.52e+1 $-$	9.00e-2 $+$	9.02e-2
	5	9	1.14e-1 $\approx$	8.71e+0 $-$	3.82e-1 $-$	1.86e-1 $-$	1.59e+1 $-$	1.55e-1 $-$	1.59e+1 $-$	1.14e-1 $\approx$	1.13e-1
	4	100	1.08e+2 $-$	2.05e+3 $-$	1.65e+3 $-$	5.30e+1 $+$	1.66e+3 $-$	2.47e+2 $-$	7.84e+2 $-$	1.07e+2 $-$	6.51e+1
	5	100	7.52e+1 $\approx$	1.89e+3 $-$	1.73e+3 $-$	9.01e+1 $\approx$	1.55e+3 $-$	1.20e+2 $-$	8.41e+2 $-$	8.15e+1 $\approx$	7.74e+1
DTLZ2	2	11	3.30e-3 $-$	3.30e-3 $-$	3.36e-3 $-$	3.30e-3 $-$	3.31e-3 $-$	3.30e-3 $-$	3.45e-3 $-$	3.30e-3 $-$	3.30e-3
	3	12	6.40e-2 $+$	6.38e-2 $+$	6.12e-2 $+$	6.38e-2 $+$	6.37e-2 $+$	6.36e-2 $+$	9.87e-2 $-$	6.40e-2 $+$	6.44e-2
	2	100	4.81e-3 $-$	3.49e-3 $-$	1.74e-2 $-$	5.84e-3 $-$	7.45e-3 $-$	3.69e-3 $-$	1.45e-2 $-$	4.76e-3 $-$	3.32e-3
	3	100	6.53e-2 $-$	6.46e-2 $-$	1.19e-1 $-$	9.63e-2 $-$	1.59e-1 $-$	6.39e-2 $+$	4.35e-1 $-$	6.51e-2 $-$	6.43e-2
	4	13	2.38e-1 $+$	2.35e-1 $+$	1.84e-1 $+$	2.36e-1 $+$	2.35e-1 $+$	2.33e-1 $+$	3.39e-1 $-$	2.38e-1 $+$	2.39e-1
	5	14	3.21e-1 $+$	3.18e-1 $+$	4.18e-1 $-$	3.22e-1 $-$	3.21e-1 $+$	3.21e-1 $\approx$	5.85e-1 $-$	3.21e-1 $+$	3.21e-1
	4	100	2.37e-1 $\approx$	2.38e-1 $\approx$	3.07e-1 $-$	2.60e-1 $-$	2.43e+0 $-$	2.34e-1 $+$	7.63e-1 $-$	2.37e-1 $\approx$	2.38e-1
	5	100	3.28e-1 $-$	3.63e-1 $-$	5.25e-1 $-$	5.38e-1 $-$	4.57e+0 $-$	3.30e-1 $-$	9.65e-1 $-$	3.29e-1 $-$	3.22e-1
DTLZ3	2	11	3.67e-3 $-$	5.81e+0 $-$	2.25e-1 $-$	5.10e-3 $-$	1.65e+1 $-$	3.55e-3 $\approx$	1.64e+2 $-$	3.64e-3 $-$	3.45e-3
	3	12	6.38e-2 $+$	4.59e+1 $-$	4.80e+0 $-$	6.52e-2 $-$	1.21e+2 $-$	6.40e-2 $\approx$	1.81e+2 $-$	6.37e-2 $+$	6.42e-2
	2	100	4.18e+2 $-$	5.30e+2 $-$	1.82e+2 $-$	6.51e+2 $-$	9.38e+2 $-$	3.67e+2 $-$	2.22e+3 $-$	4.20e+2 $-$	8.82e+1
	3	100	3.75e+2 $-$	4.57e+3 $-$	2.46e+3 $-$	2.82e+2 $\approx$	4.13e+3 $-$	3.80e+2 $-$	2.06e+3 $-$	3.65e+2 $-$	1.49e+2
	4	13	2.36e-1 $+$	1.41e+2 $-$	5.34e+0 $-$	2.60e+0 $-$	1.64e+2 $-$	2.49e-1 $-$	1.87e+2 $-$	2.35e-1 $+$	2.37e-1
	5	14	3.57e-1 $-$	1.85e+2 $-$	7.43e+0 $-$	1.47e+0 $-$	1.73e+2 $-$	3.93e-1 $-$	1.77e+2 $-$	3.49e-1 $-$	3.21e-1
	4	100	2.10e+2 $-$	6.97e+3 $-$	6.02e+3 $-$	1.98e+2 $\approx$	5.43e+3 $-$	2.89e+2 $-$	2.20e+3 $-$	2.23e+2 $-$	1.81e+2
	5	100	1.47e+2 $+$	7.07e+3 $-$	6.24e+3 $-$	2.08e+2 $+$	5.48e+3 $-$	1.86e+2 $+$	2.09e+3 $-$	1.31e+2 $+$	2.51e+2
DTLZ4	2	11	2.00e-1 $+$	1.26e-1 $+$	4.47e-1 $-$	2.80e-2 $+$	2.79e-2 $+$	7.72e-2 $+$	3.58e-3 $+$	2.99e-1 $-$	2.25e-1
	3	12	2.40e-1 $+$	1.98e-1 $\approx$	7.25e-1 $-$	1.16e-1 $\approx$	1.61e-1 $\approx$	9.61e-2 $\approx$	3.26e-1 $\approx$	1.82e-1 $+$	3.25e-1
	2	100	4.23e-1 $+$	2.00e-1 $\approx$	7.18e-1 $-$	1.80e-1 $\approx$	9.33e-3 $\approx$	3.98e-1 $\approx$	8.79e-3 $\approx$	3.73e-1 $+$	4.47e-1
	3	100	4.54e-1 $\approx$	3.96e-1 $\approx$	9.25e-1 $-$	3.88e-1 $\approx$	1.90e-1 $+$	3.30e-1 $\approx$	4.64e-1 $\approx$	2.96e-1 $\approx$	6.21e-1
	4	13	2.63e-1 $+$	3.81e-1 $\approx$	7.77e-1 $-$	2.81e-1 $\approx$	2.94e-1 $\approx$	3.01e-1 $\approx$	5.74e-1 $-$	2.91e-1 $+$	4.37e-1
	5	14	3.25e-1 $+$	4.96e-1 $\approx$	8.28e-1 $-$	3.61e-1 $+$	3.77e-1 $+$	3.42e-1 $+$	8.55e-1 $-$	3.25e-1 $+$	5.29e-1
	4	100	4.02e-1 $+$	5.13e-1 $\approx$	9.68e-1 $-$	4.18e-1 $+$	6.70e-1 $\approx$	4.53e-1 $+$	8.63e-1 $-$	3.77e-1 $+$	6.72e-1
	5	100	4.88e-1 $+$	5.97e-1 $\approx$	1.09e+0 $-$	5.06e-1 $+$	1.39e+0 $-$	4.48e-1 $+$	1.64e+0 $-$	4.57e-1 $+$	7.05e-1
$+/-/\approx$			15/12/5	4/20/8	2/29/1	9/14/9	6/22/4	10/14/8	1/28/3	14/13/5	

Table 5. IGD results for problems with an irregular Pareto front ($M = 2$–5; $D = 100$, except DTLZ7 at default dimension). Each cell shows the average IGD and the statistical sign relative to the baseline MOEA/D ($+$: significantly better; $-$: significantly worse; $\approx$: no statistically significant difference); shaded cells indicate the lowest average IGD in the row, including the baseline MOEA/D, with ties broken by decimal places not shown. Presents only unique configurations, after removal of duplicate rows; complete standard deviations are available in the study artifacts.

Problem	M	D	ABC	ACO	CMA-ES	DE	EDA	MMOPSO	NMS	SA	Base
DTLZ5	2	100	4.75e-3 $-$	3.49e-3 $-$	1.60e-2 $-$	5.84e-3 $-$	7.51e-3 $-$	3.69e-3 $-$	1.45e-2 $-$	4.79e-3 $-$	3.32e-3
	3	100	1.14e-2 $-$	1.17e-2 $-$	5.79e-2 $-$	3.27e-2 $-$	5.30e-2 $-$	1.17e-2 $-$	2.06e-1 $-$	1.11e-2 $\approx$	1.10e-2
	4	100	2.89e-2 $-$	3.64e-2 $-$	9.91e-2 $-$	6.94e-2 $-$	2.10e+0 $-$	2.76e-2 $\approx$	4.65e-1 $-$	2.89e-2 $-$	2.75e-2
	5	100	5.20e-2 $-$	6.21e-2 $-$	1.71e-1 $-$	6.52e-2 $-$	2.63e+0 $-$	4.65e-2 $+$	6.09e-1 $-$	5.16e-2 $-$	4.72e-2
DTLZ6	2	100	1.84e-1 $-$	5.84e+1 $-$	4.54e+1 $-$	3.30e-3 $+$	3.18e+1 $-$	3.30e-3 $+$	8.85e-2 $-$	1.49e-1 $-$	6.72e-2
	3	100	1.52e-2 $+$	6.52e+1 $-$	8.55e+1 $-$	1.09e-2 $+$	4.59e+1 $-$	1.09e-2 $+$	2.70e+1 $-$	1.89e-2 $+$	2.19e-1
	4	100	2.82e-2 $+$	7.59e+1 $-$	8.80e+1 $-$	2.63e-2 $+$	5.08e+1 $-$	2.62e-2 $+$	3.87e+1 $-$	2.78e-2 $+$	1.03e+1
	5	100	4.97e-2 $+$	7.59e+1 $-$	4.87e+1 $-$	5.16e-2 $+$	4.96e+1 $-$	4.63e-2 $+$	4.03e+1 $-$	4.99e-2 $+$	2.06e-1
DTLZ7	2	21	9.29e-2 $-$	2.01e-2 $-$	1.09e-1 $-$	1.08e-1 $-$	1.22e-1 $-$	9.29e-2 $-$	3.64e-2 $-$	9.29e-2 $-$	2.00e-2
	3	22	3.03e-1 $\approx$	1.79e-1 $+$	3.00e-1 $-$	3.34e-1 $\approx$	1.66e-1 $+$	2.96e-1 $\approx$	2.93e-1 $-$	3.81e-1 $\approx$	2.37e-1
	2	100	4.46e-1 $-$	6.06e-3 $-$	5.71e-1 $-$	4.43e-1 $-$	3.50e-2 $-$	5.75e-3 $-$	2.33e-1 $-$	4.45e-1 $-$	5.47e-3
	3	100	4.01e-1 $\approx$	1.61e-1 $+$	7.16e+0 $-$	3.64e-1 $\approx$	1.56e-1 $+$	2.44e-1 $-$	9.00e-1 $-$	3.66e-1 $\approx$	1.92e-1
	4	23	5.64e-1 $\approx$	4.86e-1 $+$	7.40e-1 $-$	5.57e-1 $\approx$	4.64e-1 $+$	5.31e-1 $-$	9.33e-1 $-$	5.39e-1 $\approx$	5.06e-1
	5	24	7.00e-1 $\approx$	5.95e-1 $+$	1.24e+0 $-$	7.96e-1 $-$	5.61e-1 $+$	6.62e-1 $\approx$	1.45e+0 $-$	6.70e-1 $\approx$	6.43e-1
	4	100	5.48e-1 $-$	4.87e-1 $+$	1.20e+1 $-$	8.65e-1 $-$	4.83e-1 $+$	4.98e-1 $+$	2.06e+0 $-$	6.19e-1 $-$	5.07e-1
	5	100	6.98e-1 $-$	6.14e-1 $\approx$	8.98e+0 $-$	1.54e+0 $-$	6.05e-1 $+$	6.47e-1 $-$	3.89e+0 $-$	7.73e-1 $\approx$	6.16e-1
IDTLZ1	2	100	1.81e+2 $-$	2.35e+2 $-$	6.49e+1 $-$	3.10e+2 $-$	4.26e+2 $-$	2.28e+2 $-$	7.82e+2 $-$	1.89e+2 $-$	3.96e+1
	3	100	3.69e+2 $-$	2.03e+3 $-$	1.34e+2 $\approx$	3.56e+2 $\approx$	3.31e+3 $-$	4.40e+2 $-$	1.30e+3 $-$	3.62e+2 $-$	1.24e+2
	4	100	4.61e+2 $-$	4.89e+3 $-$	1.45e+2 $+$	5.12e+2 $-$	4.28e+3 $-$	5.69e+2 $-$	1.68e+3 $-$	4.60e+2 $-$	2.23e+2
	5	100	5.10e+2 $-$	6.15e+3 $-$	1.48e+2 $+$	6.16e+2 $\approx$	4.99e+3 $-$	5.86e+2 $-$	1.98e+3 $-$	4.97e+2 $-$	2.55e+2
IDTLZ2	2	100	1.22e-2 $-$	1.02e-2 $-$	1.53e-2 $-$	1.27e-2 $-$	2.14e-2 $-$	1.10e-2 $-$	1.93e-2 $-$	1.19e-2 $-$	9.96e-3
	3	100	8.02e-2 $-$	7.92e-2 $-$	9.31e-2 $-$	9.79e-2 $-$	4.43e-1 $-$	8.09e-2 $-$	1.22e-1 $-$	8.04e-2 $-$	7.62e-2
	4	100	1.49e-1 $-$	1.69e-1 $-$	2.61e-1 $-$	2.81e-1 $-$	2.54e+0 $-$	1.55e-1 $-$	2.58e-1 $-$	1.51e-1 $-$	1.46e-1
	5	100	2.16e-1 $-$	2.69e-1 $-$	4.19e-1 $-$	4.53e-1 $-$	3.69e+0 $-$	2.31e-1 $-$	3.41e-1 $-$	2.17e-1 $-$	2.11e-1
$+/-/\approx$			3/17/4	5/18/1	2/21/1	4/15/5	6/18/0	6/15/3	0/24/0	3/15/6	

showed localized improvements on irregular fronts; and NMS showed unfavorable performance in most comparisons. As a practical implication, reproduction operators should be treated as choices conditioned on the scenario, rather than as global replacements for the default MOEA/D operator.

These findings motivate adaptive operator selection. A natural extension is to switch operators during the run based on search signals, such as solution dispersion, IGD progress, subproblem difficulty, and estimates of front geometry. Future work should also compare Tchebycheff with PBI and the weighted sum, report Hypervolume, apply Holm or FDR correction when p-values are available, evaluate WFG, and include real-world engineering problems.

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