Computational Optimization Using Spherical Geometric Modeling for Time-Domain Wave Propagation

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Abstract. *This study proposes a method to reduce computational cost in time-domain wave modeling. By applying spherical geometric modeling, the wave equation is solved only in dynamically defined regions. The approach was evaluated using mean squared error (MSE) on 3D acoustic models with varying source positions and propagation times. Results showed up to 25% runtime reduction, highlighting the method's potential for seismic imaging and reservoir analysis.*

1. Introduction

Finite-difference modeling has been widely studied to improve accuracy and computational efficiency. Foundational works have enhanced numerical precision, spectral response, and performance through methods like REM and optimized FDTD schemes [Alford et al. 1974, Nihei and Li 2007, Chu and Stoffa 2012, Furse 2000]. Additionally, recent approaches aim to reduce memory and runtime in seismic modeling. Techniques such as wavefield reconstruction, empirical wavefront filtering, and dynamic domain adaptation (e.g., OHEX) have achieved significant gains [Tan and Huang 2014, Nogueira et al. 2020, Nogueira and Porsani 2021, Delfino 2024]. Building on these ideas, the present work introduces a dynamic 3D spherical strategy that restricts computations to the wavefront region, reducing computational cost by 24.90%.

2. Method

The radius of the sphere is computed as a function of maximum wave velocity (v_{max}), iteration time (t_i), and time sampling (t_s). With a constant velocity model, the radius grows linearly ($radius = v_{max} \cdot t_i \cdot t_s$). Only relevant points inside the growing sphere are updated, avoiding unnecessary computations. Coordinates are computed iteratively. At each time step, x-limits define the horizontal extent, and circular projections in xy and xz planes provide y and z values using the Pythagorean theorem. This creates a dynamic 3D spherical mask applied at each iteration. The method was implemented in Mamute (C++), available at: `https://gitlab.com/lappsufrn/seismic/ufrn-fwi/mamute`. The experiments were conducted at the Núcleo de Processamento de Alto Desempenho (NPAD/UFRN). The algorithm ran sequentially. Each test was executed 15 times on the AMD-512 partition, using 64 CPUs and 1 exclusive node to ensure consistent performance evaluation. The objective was to evaluate the difference in runtimes between both methods for various source positions, while maintaining the same receiver position.

Table 1. Propagation time for the wave to reach the receiver (s)

Source localization	Config I	Config II
Center of the grid	0.60	3.07
Center of the grid face	0.92	3.71
Center of the grid edge	1.15	5.08
Vertex of the grid	1.20	6.15

Table 2. Runtime difference between sphere method and conventional method (%)

Source localization	Config I	Config II
Center of the grid	22.04	24.90
Center of the grid face	18.90	22.45
Center of the grid edge	19.45	10.24
Vertex of the grid	0.73	1.17

3. Results

Configurations I (100×200×200) and II (500×500×300) were selected to represent different scenarios for 3D grids. This setup allows for evaluating wave propagation time without the wavefront exceeding the grid boundaries. A spatial discretization of 10 meters and a temporal sampling interval of 0.5 ms enable the gradual expansion of the spherical region to be monitored over short time steps, thereby avoiding a sudden increase in computational load. The acoustic wave equation was discretized using an eighth-order finite difference scheme in space and a second-order scheme in time. The 0.5 ms time step was defined based on stability requirements (Courant–Friedrichs–Lewy criterion) and appropriate temporal resolution. CPML boundaries with 50 grid points were applied to minimize artificial reflections at the domain edges. Table 1 shows wave propagation times to the receiver for different source positions in configurations I and II. These times were calculated based on the source-receiver distance and maximum propagation velocity (D/v_{max}). The receiver is fixed, and only the source location varies (center, face center, edge center, and vertex). Table 2 shows the percentage difference between the conventional method (no restriction) and the proposed one (with spherical restriction), for both configurations.

4. Conclusion

The mean squared error (MSE) was calculated relative to the full-domain solution, with the source at the center. The values were 2.026×10^{-58} and 5.026×10^{-46} for configurations I and II. The method computes wave propagation only within the spherical region, significantly reducing runtime. However, in this case, RAM usage remains the same as in the conventional method, since memory allocation still considers the full 3D domain.

References

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