Valid Inequalities for the Green Vehicle Routing Problem

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Abstract. *This work*[1] *aims to investigate the Green Vehicle Routing Problem (G-VRP), which is an NP-Hard problem that generalizes the Vehicle Routing Problem (VRP) and integrates it with the green logistics. In the G-VRP, electric vehicles with limited autonomy can be recharged at Alternative Fuel Stations (AFSs) to keep visiting customers. This research proposes MILP formulations, valid inequalities, and preprocessing conditions.*

Resumo. *Este trabalho*[2] *tem como objetivo investigar o Problema de Roteamento de Veículos Verdes (G-VRP), que é um problema NP-Difícil que generaliza o Problema de Roteamento de Veículos (VRP) integrado com a logística verde. No G-VRP, veículos com autonomia limitada podem reabastecer em Estações de Combustível Alternativo (AFSs) para visitar clientes. Esta pesquisa propõe formulações MILP, desigualdades, e condições de preprocessamento.*

1. Problem formulation

In the G-VRP proposed by [Erdoğan and Miller-Hooks 2012], a set of customers with known services times must be attended by a set of homogeneous alternative fuel vehicles, which operate from a common depot. These vehicles have a lower autonomy in terms of traveled distance, forcing them to stop and refuel in AFSs. Each customer must be visited once by exactly one vehicle and the time spent by a vehicle in a route cannot exceed a maximum operation time. The objective is to reduce the total traveled distance.

Consider a complete directed graph $G(V, E)$, where $V = C \cup F \cup \{v_0\}$, where C is the set of customers, and F is the set of AFSs. The $s_i \geqslant 0$ is the customer v_i service time, and $s_f \geqslant 0$ is the AFS v_f refueling time. Vertex $v_0 \in V$ is the depot and E is the set of edges. Edges $(i, j) \in E$ have metric costs $c_{ij} > 0$. The $\rho > 0$ is the vehicle energy consumption rate, and $e_{ij} = c_{ij}\rho$ is the required energy to traverse edge (i, j). The $\alpha > 0$ is the vehicle average speed, and $t_{ij} = \frac{c_{ij}}{\alpha}$ is the required time to traverse the edge (i, j). In the depot, there is a set $M = \{1, ..., |C|\}$ of homogeneous vehicles with an autonomy of β and maximum operation time T. The goal of G-VRP consists in defining at most $|M|$ routes with minimum cost satisfying the following constraints:

- Each route begins and ends at the vertex depot v_0;
- Each vertex $v_i \in C$ is visited once by exactly one route;
- Each vertex $v_f \in F$ can be visited multiple times by multiple routes, thus, each route might have subcycles;

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- The autonomy of a vehicle decreases in e_{ij} when it traverses edge $(i, j) \in E$. It is not possible for a vehicle to traverse an edge with a required energy higher than its autonomy. When the vehicle visits an AFS, its autonomy is restored to β;
- The time spent by a route must not exceed T.

The variables used in the MILP formulation are x_{ij}^k which equals 1 if $(i, j) \in E$ is traversed in route $k \in M$, and 0 otherwise (Theorem 2.1.1 shows that in an optimal solution each edge will be used at most once); and y_i which represents the vehicle energy level when visiting $v_i \in V$. The following MILP formulation is proposed:

$$\text{Min} \sum_{k \in M} \sum_{(i,j) \in E} c_{ij} x_{ij}^k \tag{1}$$

s.t.

$$\sum_{v_j \in V} x_{0j}^k \leqslant 1 \qquad \forall k \in M \tag{2}$$

$$\sum_{v_j \in V} x_{ij}^k = \sum_{v_j \in V} x_{ji}^k \qquad \forall v_i \in V, \forall k \in M \tag{3}$$

$$\sum_{k \in M} \sum_{v_j \in V} x_{ij}^k = 1 \qquad \forall v_i \in C \tag{4}$$

$$\sum_{(i,j) \in E : v_i \in V \setminus S \wedge v_j \in S} x_{ij}^k \geqslant \sum_{v_j \in S} x_{ji*}^k \qquad \forall k \in M, \forall S \subseteq V \setminus \{v_0\} : |C \cap S| \geq 1, \forall v_{i*} \in S \tag{5}$$

$$y_f = y_0 = \beta \qquad \forall v_f \in F \tag{6}$$

$$y_j \leqslant y_i - e_{ij} x_{ij}^k + \beta(1 - x_{ij}^k) \qquad \forall v_j \in C, \forall v_i \in V, \forall k \in M \tag{7}$$

$$\beta \geqslant y_i \geqslant e_{ij} x_{ij}^k \qquad \forall v_i \in C, \forall v_j \in V, \forall k \in M \tag{8}$$

$$x_{ij}^k e_{ij} \leqslant \beta \qquad \forall v_i, v_j \in V, \forall k \in M \tag{9}$$

$$\sum_{(i,j) \in E} x_{ij}^k (t_{ij} + s_i) \leqslant T \qquad \forall k \in M \tag{10}$$

$$x_{ij}^k \in \mathbb{N} \qquad \forall v_i, v_j \in V, \forall k \in M \tag{11}$$

$$y_j \in \mathbb{R}^+ \qquad \forall v_j \in V \tag{12}$$

The objective function (1) minimizes the total cost. Constraints (2) says that at most one edge will leave the depot in route k. Constraints (3) imply that the number of times a vehicle k enters vertex v_i is equals to the number of times the same vehicle exits the same vertex. Constraints 4 state each customer is visited once by exactly one vehicle. Constraints (5) are connectivity constraints. Constraints (6) guarantee that the vehicle energy level is recharged when it visits an AFS or the depot. Constraints (7) update the vehicle energy level at every customer. Constraints (8) define the maximum energy level and implies that an edge can only be traversed if the vehicle energy level is sufficient. Constraints (9) forbid edges with cost greater than the maximum energy level capacity. Constraints (10) limit the time of a route by T. Constraints (11) and (12) are variables domains. Note that there is an exponential number of connectivity constraints (5), therefore a complete enumeration of them is only possible for very small instances.

The proposed solution was to consider an iterative algorithm that adds the connectivity constraints to the formulation as they are needed to progress the optimization. First, the formulation is executed without these constraints and as soon as an integer solution x^* is obtained, the iterative algorithm is called. This method is often referred as "lazy constraint".

One of the advantages of our proposed formulation for the G-VRP, comparing to other formulations for similar electric vehicle routing problems in literature, are: (i) the ability to handle any number of visits on each AFSs; and (ii) allow solutions that have any number of consecutive visits to AFSs, despites the fact the triangle inequalities are valid for all instances, it is possible to visit two or more AFS consecutively in a route (it might be necessary depending of to the energy required to visit other far customers). For example, [Erdoğan and Miller-Hooks 2012] proposed a formulation where dummy nodes are created for each possible visit to an AFS; [Erdoğan and Miller-Hooks 2012], [Çağrı Koç and Karaoglan 2016], and [Leggieri and Haouari 2017] proposed formulations that assume a vehicle never makes two consecutive visits in AFSs; and, [Wang et al. 2018] proposed a formulation that assumed each AFS would be used at most once per vehicle.

2. Preprocessing conditions

In this subsection preprocessing conditions are proposed to potentially allow fixing some variables, and consequently, reduce the search space of the proposed MILP model. Such conditions are based on the preprocessing conditions defined by [Leggieri and Haouari 2017]. Let $F_0 = F \cup \{v_0\}$ and $G[F_0] = (F_0, \{(v_r, v_f) \in E : v_f, v_r \in F_0 \wedge e_{rf} \leqslant \beta\})$. Let $t_f^{'}$ be the shortest path time between v_0 and v_f in $G[F_0]$, $t_f^{'}$ including the refuel time of all AFSs visited by the shortest path. The preprocessing conditions are (logical notation is used):

1. If $e_{rf} > \beta \vee t_{rf} > T$, then $x^k_{rf} = x^k_{fr} = 0 \; \forall v_f, v_r \in F_0, \forall k \in M$;
2. If $\nexists v_f \in F_0 : t_f^{'} + t_{fi} + s_i + t_{i0} \leqslant T \wedge e_{fi} + e_{i0} \leqslant \beta$, then $x^k_{0i} = x^k_{i0} = 0$ $\forall v_i \in C, \forall k \in M$;
3. If $\nexists v_f, v_r \in F_0 : t_f^{'} + t_{fi} + s_i + t_{ij} + s_j + t_{jr} + t_r^{'} \leqslant T \wedge e_{fi} + e_{ij} + e_{jr} \leqslant \beta$, then $x^k_{ij} = x^k_{ji} = 0 \; \forall v_i, v_j \in C, \forall k \in M$;
4. If $\nexists v_r \in F_0 : t_r^{'} + t_{ri} + s_i + t_{if} + s_f + t_f^{'} \leqslant T \wedge e_{ri} + c_{if} \leqslant \beta$, then $x^k_{if} = x^k_{fi} = 0$ $\forall v_i \in C, \forall v_f \in F, \forall k \in M$.

Note that the preprocessing conditions (2-4) are valid only because the edges costs satisfy the triangle inequality.

2.1. Constraints to reduce the MILP model search space

This subsection contains valid inequalities to reduce the search space of the proposed MILP formulation. The presented Lemmas and Theorems use the following notations. Let:

- R be a feasible route;
- $p(R) \geqslant 0$ be the number of AFSs visited by the route R;
- $q(R) \geqslant 1$ be the number of customers visited by the route R;

- $R_F = (r_1, r_2, ..., r_{p(R)-1}, r_{p(R)})$ be the sequence of AFSs visited by route R sequence;
- d_i be the amount of fuel spent between r_{i-1} and r_i by R for $1 < i \leqslant p(R)$, d_1 be the amount of fuel spent between v_0 and r_1, and $d_{p(R)+1}$ be the amount of fuel spent between $r_{p(R)}$ and v_0;
- $c(R) = \sum_{i=1}^{p(R)+1} d_i$ the amount of fuel spent by R;
- $\lambda = min\{min_{v_i \in V \setminus \{v_0\}}\{e_{i0}\}, min_{v_f, v_r \in F : v_f \neq v_r}\{e_{fr}\}\}$ be the minimum energy required to visit two different AFS or to visit the nearest vertex starting from the depot.

The inequalities are presented below.

Theorem 2.1.1. *There exists an optimal solution in which route will use each edge at most once, i.e., $x_{ij}^k \in \mathbb{B}\ \forall v_i, v_j \in V, \forall k \in M$.*

Theorem 2.1.2. *Let $v_f \in F$ be visited by route R. If R belongs to an optimal solution, then v_f will be visited at most $q(R)+1$ times by R, i.e., $\sum_{v_j \in V} x_{jf}^k \leqslant \sum_{v_i \in C} \sum_{v_j \in V} x_{ji}^k + 1$ $\forall v_f \in F, \forall k \in M$.*

Theorem 2.1.3. *$c(R) \leqslant (p(R)+1)\beta$, i.e., $\sum_{v_i \in V} \sum_{v_j \in V} e_{ij} x_{ij}^k \leqslant (\sum_{v_f \in F} \sum_{v_j \in V} x_{fj}^k + 1)\beta\ \forall k \in M$.*

Lemma 2.1.4. *If a route R belongs to an optimal solution, then $d_i + d_{i+1} > \beta$ for $1 \leqslant i \leqslant p(R)$.*

Theorem 2.1.5. *If a route R belongs to an optimal solution, and $p(R)$ is odd, then $c(R) > (p(R)+1)\frac{\beta}{2}$, i.e., $\sum_{v_i \in V} \sum_{v_j \in V} e_{ij} x_{ij}^k > (\sum_{v_f \in F} \sum_{v_j \in V} x_{fj}^k + 1)\frac{\beta}{2}\ \forall k \in M$, when $p(R)$ is odd.*

Theorem 2.1.6. *If a route R belongs to an optimal solution, and $p(R)$ is even, then $c(R) > p(R)\frac{\beta}{2} + \lambda$, i.e., $\sum_{v_i \in V} \sum_{v_j \in V} e_{ij} x_{ij}^k > (\sum_{v_f \in F} \sum_{v_j \in V} x_{fj}^k)\frac{\beta}{2} + x_{ij}^k \lambda\ \forall k \in M, \forall v_i, v_j \in V : v_i \neq v_j$, when $p(R)$ is even.*

Theorem 2.1.7. *If a route R belongs to an optimal solution, and $p(R)$ is odd, then $c(R) > p(R)\frac{\beta}{2} + \lambda$, i.e., $\sum_{v_i \in V} \sum_{v_j \in V} e_{ij} x_{ij}^k > (\sum_{v_f \in F} \sum_{v_j \in V} x_{fj}^k)\frac{\beta}{2} + x_{ij}^k \lambda\ \forall k \in M, \forall v_i, v_j \in V : v_i \neq v_j$, when $p(R)$ is odd.*

Theorem 2.1.8. *If a route R belongs to an optimal solution, then $c(R) > p(R)\frac{\beta}{2} + \lambda$, i.e., $\sum_{v_i \in V} \sum_{v_j \in V} e_{ij} x_{ij}^k > (\sum_{v_f \in F} \sum_{v_j \in V} x_{fj}^k)\frac{\beta}{2} + x_{ij}^k \lambda\ \forall k \in M, \forall v_i, v_j \in V : v_i \neq v_j$.*

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