Post-Synthesis Optimization of Reversible Circuits

Edinelço Dalcumune[1,3]**, Luis A. B. Kowada**[2]**, Celina M. H. de Figueiredo**[3]**,**
Franklin de L. Marquezino[3]

[1]DCEX – Universidade Federal dos Vales do Jequitinhonha e Mucuri (UFVJM)
Teófilo Otoni, MG – Brazil

[2]Instituto de Computação – Universidade Federal Fluminense (UFF)
Niterói, RJ – Brazil

[3]PESC/COPPE – Universidade Federal do Rio de Janeiro (UFRJ)
Rio de Janeiro, RJ – Brazil

`edcomune@ufvjm.edu.br`,`luis@ic.uff.br`,`{celina,franklin}@cos.ufrj.br`

Abstract. *One of the main motivations for reversible computing is that quantum computing has as one of its foundations the reversibility of all gates, that is, quantum computing circuit models are reversible. An important problem in reversible computing that has been intensively studied for the last decades is the synthesis of reversible circuits. The extended abstract considers optimization rules aiming to a new algorithm for post-synthesis optimization of reversible circuits composed of generalized Toffoli gates.*

1. Introduction

The synthesis of reversible circuits is an important problem in reversible computing and has received considerable attention due to the possibility of applications in many areas of science such as cryptography, quantum computing, low-power design, and bioinformatics [5, 8, 11]. The synthesis depends on the number of gate types (gate library) used in realizing the circuits, and the reversible circuit is defined as a sequence of reversible gates.

Post-synthesis optimization techniques for reversible circuits using Toffoli gates with positive controls (also known as Multiple Control Toffoli or MCT gates) have been used in several studies [7, 8, 13]. Generalized Toffoli gates (also known as MCT gates with mixed controls) were introduced for the synthesis of reversible circuits by [6], and in [3] they were used for the first time for post-synthesis optimization of reversible circuits. In recent years, there has been a growing interest in post-synthesis optimization of reversible circuits using Toffoli gates with negative controls via substitution rules and template matching [1, 2, 3, 4, 9, 10, 12, 14, 15].

In this extended abstract, we shall focus on reviewing the rules of [1, 2, 4, 10, 14], in order to propose a new algorithm for post-synthesis optimization of reversible circuits using generalized Toffoli gates. The number of gates or Gate Count (GC) is the metric used to evaluate our post-synthesis optimization process.

2. Optimization rules

Simplification rules are given to optimize reversible circuits using a post-synthesis algorithm. We give two elimination, as well as merge, swap, move and replacement rules.

Rule 1 (Elimination of Identical gates) *Two identical generalized Toffoli gates g and h cancel each other [1, 2, 8, 10].*

Rule 2 (Elimination of NOT gates) *Assume that two NOT gates on a line y, with no target placed on line y in any of the gates in between, are given. We can remove the NOT gates and complement the polarities of all control connection on line y between the two NOT gates [1, 3, 4].*

Definition 1 *Two generalized Toffoli gates g and h are adjacent if they have the targets on the same line, and differ in only one line with respect to the controls (positive, negative or "don't care").*

Rule 3 (Merge for Adjacent gates) *Two adjacent gates can be merged into one gate in the reversible circuit. The resulting gate will have the control connection (positive, negative or "don't care") which does not appear on the line in which they differ [1, 2, 3, 4, 10].*

Rule 4 (Swap) *Two generalized Toffoli gates can be swapped if one of the following conditions occurs: (i) the target line of one gate is not a control line of the other gate; (ii) the two gates have a reverse control connection on one line, and in all other lines on which both gates have control connections, these connections are identical [1, 2, 10, 14].*

Rule 5 (Move) *Let g and h be two generalized Toffoli gates and suppose the following conditions are satisfied: (i) the target of gate g is on a line where gate h has a control connection, and (ii) every line on gate g with a control connection also has a control connection on gate h, and in all lines on which both have control connections these are the same. Then g and h can be switched if we invert the control connection of the h gate positioned on the same line as the target of the g gate [2].*

Definition 2 *Two generalized Toffoli gates g and h are distance-2 gates if they have the targets on the same line, and differ in two lines with respect to the controls (positive, negative or "don't care"). We refer to Table 1 where we depict the possible scenarios.*

Rule 6 (Replacement for Distance-2 gates) *A pair of distance-2 gates g and h can be replaced by other pairs of distance-2 gates according to Table 1, which shows the possibilities for replacement, where the previously known cases [4] are referenced. In each set of three pairs of gates, we can choose the most appropriate pair of gates.*

Table 1 depicts the six possible different sets with three equivalent pairs of gates. In each set of gates we can choose the most appropriate pair of gates for the reversible circuit, i.e., a pair of gates that leads to a new application of rules, possibly providing a decrease in the gate count.

Lemma 1 *There are eighteen types of pairs of distance-2 gates, divided into six sets of three elements each. The three pairs of gates in each set are equivalent.*

3. Proposed algorithm

To describe Algorithm 1 that explores for the first time the six above described rules simultaneously, we need the concept of *segment*. In a given reversible circuit of generalized Toffoli gates $G = \{g_1, g_2, \ldots, g_p\}$, a segment is defined as a set of consecutive gates $G_{seg} = \{g_{k_1}, g_{k_1+1}, \ldots, g_{k_2}\}$, where $G_{seg} \subseteq G, 1 \leq k_1 \leq k_2 \leq p$, such that any pair of gates in G_{seg} can be swapped without affecting the functionality of the circuit.

Table 1. Scenarios for distance-2 gates

Case	Line	Pair 1		Pair 2		Pair 3	
		g	h	g	h	g	h
$a[4]$	i	1	0	1	-	0	-
	j	0	1	-	1	-	0
$b[4]$	i	1	0	1	-	0	-
	j	0	-	1	-	1	0
$c[4]$	i	1	-	0	-	1	0
	j	0	1	0	-	-	1
$d[4]$	i	1	-	0	-	1	0
	j	1	0	1	-	-	0
e	i	1	0	1	-	0	-
	j	1	-	0	-	0	1
f	i	1	-	1	0	0	-
	j	-	0	1	0	-	1

'1': positive control,
'0': negative control,
'-': "don't care"

Algorithm 1: Post-synthesis optimization

Input: A reversible circuit R.
Output: An equivalent reversible circuit R' of R.

1 Identify the segments $S_1, S_2, ...$;
2 **for** *each segment S* **do**
3   Try to apply rule 2;
4   **for** *each pair of gates g, h of S* **do**
5   **if** *g and h are identical or g and h are adjacent* **then**
6   **if** *g and h are not consecutive* **then**
7   Apply rule 4;
8   Apply rule 1 or rule 3;
9 **for** *each pair of gates g, h such that $g \in S_i$ and $h \in S_{i+1}$* **do**
10   **if** *there exists a pair g', h' equivalent to g, h (according to rules 5 and 6) such that g' is identical or adjacent to a gate in S_{i+1} or h' is identical or adjacent to a gate in S_i* **then**
11   Apply rule 4 to move g to the end of S_i and h to the beginning of S_{i+1};
12   Apply rules 5 and 6;
13   Apply rule 1 or rule 3;

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