MIP-Heuristics for the Capacitated Lot-Sizing with Hybrid Flow Shop Integration

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Abstract. *This paper proposes a two-phase approach to integrating the capacitated lot-sizing (CLSP) and hybrid flow shop (HFS). First, we construct an initial solution exploring three heuristics: relax-and-fix, LP-and-fix, and greedy. Next, we try to improve the initial solution through a fix-and-optimize heuristic that decomposes the original MIP per period. Numerical experiments show that combining the greedy heuristic with fix-and-optimize can overcome the alternatives, obtaining small gaps for most cases, even for large-size instances.*

Resumo. *Este artigo propõe uma abordagem em duas fases para resolver o problema integrado de lot-sizing capacitado (CLSP) com flow shop híbrido (HFS). Constrói-se uma solução inicial explorando três heurísticas: relax-and-fix, LP-and-fix e gulosa. Em seguida, aprimora-se a solução inicial usando a heurística fix-and-optimize, que decompõe o MIP original em subproblemas menores, um por período. Experimentos numéricos mostram que combinar uma heurística gulosa com fix-and-optimize pode superar as alternativas, obtendo gaps menores para a maioria dos casos, mesmo para instâncias de grande porte.*

1. Introduction

Lot-sizing ([Pochet and Wolsey 2006]) and production scheduling ([Michael 2008]) are problems that arise in the context of industrial decision-making. Recent research addresses Capacitated Lot-Sizing (CLSP) ([Bitran and Yanasse 1982]) and Hybrid Flow Shop (HFS) ([Ruiz and Vázquez-Rodríguez 2010]) integration. The aim is to define a model that respects shop floor constraints and capabilities and avoids re-planning while reducing costs ([Ramezanian et al. 2013, Masmoudi et al. 2016, Qin et al. 2019, Rodoplu et al. 2020, Silva and Mateus 2022, Silva and Mateus 2023]). However, both problems are $\mathcal{NP}$-Hard ([Bitran and Yanasse 1982, Ruiz and Vázquez-Rodríguez 2010]), a fact that limits solving large instances using exact approaches. On the other hand, heuristics can overcome this limitation by obtaining good-quality solutions quickly, even for large cases.

This paper solves the integration of CLSP-HFS using MIP-heuristics in a two-phase approach that tackles an adapted mixed-integer programming (MIP) formulation derived from [Silva and Mateus 2022] that considers the original capacitated multiproduct, multiperiod, and multistage problem, but also includes sequence-dependent setup times and costs. Exact models have proven to be effective in solving small-scale cases. To address larger cases, decomposition heuristics *relax-and-fix* (R&F) and *fix-and-optimize* (F&O) have been successfully employed, respectively, to generate and improve solutions in production problems ([Pochet and Wolsey 2006, Sahling et al. 2009, Helber and Sahling 2010, Lang and Shen 2011, Toledo et al. 2015]). The R&F method decomposes the mixed-integer programming (MIP) variables, denoted as $v \in \{0, 1\}$, into Ω disjoint sets $Q^1, \ldots, Q^{\Omega}$. It then systematically solves Ω subproblems. At the ω-th iteration, the SUBMIP$^{(\omega)}_{RF}$ is solved by keeping the constraints intact and redefining the binary variables as follows: if $v \in (Q^1 \cup \ldots \cup Q^{(\omega-1)})$, v is set to $\hat{v}$ from the previous iterations; if $v \in Q^{\omega}$, v is constrained to $\{0, 1\}$; and if $v \in (Q^{(\omega+1)} \cup \ldots \cup Q^{\Omega})$, v is relaxed to $0 \leq v \leq 1$. Similarly, the F&O method solves SUBMIP$^{(\omega)}_{FO}$ by fixing v to $\hat{v}$ if $v \in (\bigcup i = 1^{\Omega} Q^i \backslash Q^{\omega})$, or keeping $v \in \{0, 1\}$ if $v \in Q^{\omega}$. Recent works also use these heuristics to solve integrated problems. [Mohammadi and Fatemi 2010] designed an R&F algorithm to solve lot-sizing with a permutational flow shop and sequence-dependent setup. [James and Almada-Lobo 2011] hybridize the R&F heuristic with local search methods to solve the integration of capacitated lot-sizing with parallel machines. Some works investigate how different decomposition strategies perform with R&F or F&O. For example, [Schimidt et al. 2019] tested six strategies that combine machines, stages, products, and periods. [Araujo et al. 2021] used information about the chronological order of periods, product and period demands, flexibility and efficiency of the available machines, and discrepancy in the processing times when decomposing the original problems in sub-MIPs. [Silva and Mateus 2023] addressed the integration of lot-sizing and hybrid flow shop by combining the R&F and F&O heuristics to solve an alternative formulation for CLSP-HFS based on precedence variables. They show that period-based decomposition achieves smaller gaps than product-based and stage-based strategies and even than the MIP model solved using a commercial solver. They start from a good-quality solution generated by the R&F heuristic that consumes $3/4$ of the total time budget (three hours). The F&O heuristic uses the remaining $1/4$ fraction of the time to improve the initial solution. Although the combined heuristic achieved gaps up to 2% for most cases, some instances could not be solved. In cases with a significant number of products (≥ 50), the R&F heuristic cannot draw an initial feasible solution since it depends strongly on solving sub-MIPs using *branch-and-bound* (B&B) algorithm – more specifically, one sub-problem by each period of the planning horizon. Given this fact, a hypothesis can be raised that competitive solutions in quality can be obtained by overcoming the combined R&F with F&O approach. For this, starting from initial solutions computed quickly, combined with an improvement by the F&O heuristic, can lead to better solutions even for large instances, assigning most of the time budget to the improvement phase.

2. Numerical Experiments

To check the hypothesis, we execute numerical experiments using a set of randomly generated instances adapted from [Silva and Mateus 2022]. They were carried out on a computer system with Intel(R) Core(TM) i7-4790K CPU @ 4.00GHz 16 GB RAM DDR3 1333 MHz running Linux Ubuntu with codes implemented using Python 3 and Gurobi 9.1.2 to solve the MIP models. The benchmark comprises instances with 30, 40, 50 and 70 products that can be processed in 3 heterogeneous machines per stage, in 3 different stages of production, to meet the demand in the horizon of 8 periods. The benchmark is separated into instances with high (**H**) and low (**L**) setup costs to evaluate the impact of these costs on algorithm performance. These instances are solved along three hours of time budget using three alternatives: (i) the combined R&F with F&O heuristic as proposed by [Silva and Mateus 2023]; (ii) a combined LP-and-Fix ([Pochet and Wolsey 2006]) with F&O heuristic; and (iii) a combined Greedy with F&O heuristic, proposed in this work. In particular, the greedy heuristic is simple and works as follows. Assuming that we will produce the required demand for each product and period, the corresponding lots of products are scheduled in the available machines by stage, one by one. It computes the most economical way to append a given lot of product in an existing sequence of products reducing the costs involved in the corresponding setup to prepare the chosen machine. For details about how to compute the setup cost, see the original formulation in [Silva and Mateus 2022]. Regarding the budget, R&F uses $3/4$ of the time to construct an initial solution because it demands enough time to solve each sub-problem. The other alternatives are free to split the time between the construction and improvement phases. After drawing an initial solution, the F&O uses the remaining time to improve it.

Tabela 1. Results obtained from running the alternatives over the *benchmark*

		R&F + F&O			Greedy + F&O			LP&Fix + F&O		
Inst.	L.B	Gap^{RF} (%)	Gap^{FO} (%)	Time (s)	Gap^{GR} (%)	Gap^{FO} (%)	Time (s)	Gap^{LF} (%)	Gap^{FO} (%)	Time (s)
30L.1	39.231	—	—	—	14,32	0,71	8.818	5,36	0,71	10.805
30L.3	41.021	0,96	0,86	10.800	16,58	0,74	10.800	3,31	0,74	10.800
30L.5	42.672	—	—	—	15,68	* 0,39	4.216	2,14	2,02	441
30L.7	42.025	1,12	0,73	10.800	17,99	* 0,70	8.022	3,30	2,90	1.492
30L.9	40.905	2,11	1,23	10.800	17,60	* 0,98	10.800	6,46	5,06	3.142
30H.1	98.762	—	—	—	34,73	* 1,30	10.800	7,24	1,58	10.800
30H.3	97.534	2,64	1,51	10.800	31,73	* 1,39	10.005	7,35	1,57	10.800
30H.5	100.505	3,08	1,54	10.800	36,88	10,20	8.091	6,38	* 1,57	10.800
30H.7	99.357	2,10	1,03	9.992	24,31	* 0,88	10.539	4,96	3,62	3.777
30H.9	98.220	3,44	1,79	10.800	19,94	15,40	2.707	9,36	* 1,23	10.482
40L.1	54.129	1,84	0,82	10.800	12,22	* 0,78	10.800	2,37	1,44	7.356
40L.3	55.689	2,14	0,82	10.800	14,95	* 0,59	10.800	3,82	0,96	10.800
40L.5	52.799	2,21	1,28	10.800	16,52	1,05	10.800	3,82	* 0,89	10.800
40L.7	52.490	2,01	1,13	10.800	16,71	0,94	10.800	4,68	* 0,91	10.800
40L.9	53.441	2,68	* 1,22	10.800	13,92	8,58	4.062	5,36	1,34	10.800
40H.1	129.099	7,05	2,58	10.800	19,13	* 2,01	10.800	8,52	2,17	10.800
40H.3	129.602	6,76	2,24	10.800	23,01	* 1,86	10.800	—	—	10.816
40H.5	127.942	6,16	3,01	10.800	18,33	14,09	2.715	9,32	* 2,19	10.800
40H.7	127.501	5,81	1,74	10.800	27,07	* 1,64	10.800	5,82	1,94	10.800
40H.9	128.743	7,93	2,94	10.126	26,93	2,20	10.800	9,53	* 2,19	10.800
50L.1	65.990	4,46	1,92	10.800	15,23	* 1,34	10.800	—	—	10.827
50L.3	65.691	3,75	1,55	10.800	11,84	1,24	10.800	4,12	* 1,09	10.800
50L.5	68.414	4,29	1,73	10.800	11,20	* 1,40	10.800	—	—	10.827
50L.7	67.531	3,75	2,18	10.800	12,66	* 1,29	10.800	5,23	1,49	10.800
50L.9	67.072	4,44	1,82	10.800	15,54	* 1,11	10.800	—	—	10.825
50H.1	158.923	—	—	—	18,95	* 2,43	10.800	—	—	10.827
50H.3	157.415	4,38	2,35	10.800	18,82	* 1,54	10.800	—	—	10.830
50H.5	160.847	—	—	—	21,38	* 1,79	10.800	—	—	10.829
50H.7	160.499	7,69	2,78	10.800	18,67	* 1,90	10.800	4,24	2,41	10.800
50H.9	160.653	—	—	—	51,24	* 2,12	10.800	—	—	10.827
70L.1	94.577	—	—	—	10,02	* 1,84	10.800	—	—	—
70L.3	92.551	—	—	—	10,63	* 1,80	10.800	—	—	—
70L.5	89.920	—	—	—	12,62	* 2,21	10.800	—	—	—
70L.7	96.017	—	—	—	12,29	* 1,77	10.800	—	—	—
70L.9	91.753	—	—	—	10,94	* 2,03	10.800	—	—	—
70H.1	219.015	—	—	—	18,31	* 2,81	10.800	—	—	—
70H.3	219.640	—	—	—	17,69	* 4,14	10.800	—	—	—
70H.5	220.610	—	—	—	14,58	* 2,87	10.800	—	—	—
70H.7	218.991	—	—	—	15,45	* 2,67	10.800	—	—	—
70H.9	22.1975	—	—	—	16,04	* 2,96	10.800	—	—	—

Table 1 shows the results. Column 'L.B.' presents the best bound computed by B&B. Columns 'Gap (%)' presents, respectively, the gap obtained from the initial and final solution of the combined approaches compared to the 'L.B.'. Finally, 'Time (s)' reports the total runtime. Void cells indicate that the constructive heuristics found no initial feasible solution, while cells with ($\star$) indicate the alternative with the best performance for a given instance. First, the combined Greedy with F&O heuristic obtained smaller gaps than the alternatives for 75% of the cases. Also, it computed feasible solutions for all cases, independent of the problem size. Alternatives R&F with F&O e LP-and-fix with F&O sometimes fail to find feasible solutions, especially when more products are involved. The time allocated for the R&F is fixed, and the subproblems with 50 products or more contain a high number of binary variables and constraints, so this time may be insufficient to find a feasible solution. The time assigned to the LP-and-fix is flexible. However, it can also consume the whole budget to draw an initial solution if the linear relaxation of the MIP model returns fractional values for most of the binary variables that decide machine allocation and sequencing. Since CLSP-HFS is a minimization problem, scheduling constraints derived from [Silva and Mateus 2022] tend to fractions most of these binary variables to reduce the total setup cost in the relaxation. So, the LP-and-fix fixes a few integral variables from the linear relaxation and solves the MIP to determine the other ones. It can be time-consuming for large instances and can return no initial solution if the budget expires without computing a feasible solution in the B&B. Otherwise, R&F or LP-and-fix draws a feasible solution within the budget, and F&O improves it using the remaining time. With less time dedicated to the improvement phase, it results in gaps larger than the Greedy with F&O alternative. Second, if we evaluate only the constructive methods, we find that the simple Greedy heuristic returns low-quality gaps. Given the complexity of the scheduling combinatorics, the schema used to schedule a lot of a given product cannot compute more economical choices. Despite it, it consumes less time to draw an initial solution than the alternatives (e.g., $\sim$ 60 seconds to compute a solution with 70 products) and releases almost the total time budget to the improvement phase via F&O. Finally, we conclude that **H**-instances are more complicated to solve than **L**-instances because the larger gaps obtained for them.

3. Concluding Remarks

Experimental results suggest that coupling the F&O heuristic as an improvement method to simple greedy or MIP-heuristics can potentially find good-quality solutions even for large-size instances, as hypothesized. In future works, we suggest designing a better greedy heuristic and adopting stronger alternative formulations with LP-and-fix to allow fixing more binary variables from the corresponding linear relaxation.

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