Expected Emergent Algorithmic Creativity and Integration in Dynamic Complex Networks

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Abstract. *We present a theoretical investigation of the emergence of complexity or irreducible information in networked computable systems when the network topology may change over time. For this purpose, we build a network model in which nodes are randomly generated Turing machines that obey a communication protocol of imitation of the fittest neighbor. Then, we show that there are topological conditions that trigger a phase transition in which eventually these networked computable systems begin to produce an unlimited amount of bits of expected emergent algorithmic complexity, creativity and integration as the network size goes to infinity.*

1. Introduction

We present a theoretical investigation of systemic properties in networked complex systems, focusing on the emergence of complexity or irreducible information when computable systems are connected by communication channels. For this purpose, we define a general mathematical model based on computability theory, information theory, and graph theory. This model enables one to represent and build definitions as well as theorems on networked computable systems where the set of nodes (or vertices) is defined as a population of Turing machines (or programs) and edges or arrows (i.e., connections or communication channels) are defined by a MultiAspect Graph (MAG) [Wehmuth et al. 2016]. We call this model as *algorithmic networks* [Abrahão 2016, Abrahão et al. 2017].

As pointed and exemplified in [Abrahão et al. 2017], the motivations for our definitions and problems as well as the range of our intended applications of the present study belongs to an intersection of complex networks, distributed computing, evolutionary game theory, algorithmic and statistical information theory, theory of computation, complex systems theory. Thus, the present work is developed as an interdisciplinary theoretical investigation. In order to start tackling this broad subject, we narrow our scope to study a toy model and the general problem of how much more algorithmic complexity, creativity or integration a complex network of randomly generated computable systems produces when playing a game of optimizing the average fitness (or payoff) of the population through diffusion. Given certain topologies and information-sharing (or communication) protocols we aim at mathematically investigating the expected amount of emergent irreducible information. Comparing the algorithmic complexity of networked nodes with the algorithmic complexity of isolated nodes gives a direct formal way to measure this amount of emergent irreducible information. In this work, we will show that the results in [Abrahão et al. 2017] can be directly extended in a way such that there is also a lower bound for the amount of expected emergent algorithmic creativity or (behavior) integration.

Therefore, in order to be able to build definitions and prove theorems, we study these properties of complex systems by simplifying or making an abstraction of the intricate and vast set of different behaviors of networked complex systems. As discussed in [Abrahão et al. 2017], the present work achieves results that may be related to questions ranging, for instance, from the problem of symbiosis, cooperation, and integration to biological, economic, and social networks. Moreover, it follows the pursuit of a universal framework for the problem of complexity in complex networks as supported by [Barabási et al. 1999, Barabasi 2009]. While it is conjectured in network science that different types of real-world networks are related by graph properties (e.g. a small diameter compared to the network size or the small-world phenomenon), the theoretical approach we are developing suggests that these relations may be sound. Our results go in the direction of answering the problem of why a network topology and an information-sharing (or communication) protocol become relevant from an emergent open-ended evolutionary point of view that takes into account synergy, complexity, irreducible information, and computational power in solving problems.

2. Model

We defined in [Abrahão et al. 2017] a particular model of algorithmic networks with a randomly generated population of nodes (i.e., Turing machines) that plays the Busy Beaver Imitation Game (BBIG). This is a class of synchronous dynamic algorithmic networks $\mathfrak{N}_{BB}$ that is based on an information-sharing protocol of the simple imitation of the fittest neighbor. A network Busy Beaver Game is a general game in which each player attempts to calculate the largest integer it can using the information shared by its neighbors. In this sense, the BBIG is a special case of the network Busy Beaver Game in which every node only propagates or diffuses the largest integer, taking into account the one produced by itself and the ones from their neighbors.

Thus, as presented in [Abrahão et al. 2017], these algorithmic networks $\mathfrak{N}_{BB}$ can be seen as playing an optimization procedure where the whole pursues the synergistic increase of the average fitness through diffusing over the network the best randomly generated solution to a problem. This is so, assuming that this problem is Turing equivalent to the halting problem and assuming the Busy Beaver function as a measure of fitness. Moreover, the network topology may vary over time, which comes from the formalism of Time-Varying Graphs (TVG) as in [Costa et al. 2015]. While optimization through selection in a random sampling may refer to evolutionary computation or genetic algorithms, for instance, in which the best solution eventually appears and remains sufficiently stable over time, in our model optimization is obtained in a manner that the best solution also eventually appears, but it is diffused over time in order to make every individual as averagely closer to the best solution as they can.

3. Results

We have proved in [Abrahão et al. 2017] that there is a lower bound for the expected emergent algorithmic complexity in algorithmic networks $\mathfrak{N}_{BB}$. It depends on how much larger is the average diffusion density in a given time interval compared with the cycle-bounded conditional halting probability. This lower bound also depends on the parameter for which one is calculating the number of cycles. In fact, we have proved a corollary showing that this parameter can be calculated, for example, from the cover time, as defined in [Costa et al. 2015]. Thus, our results hold even in the case of spending a computably larger number of cycles compared to the cover time. Furthermore, we have proved that there are asymptotic conditions on the increasing power of the cover time as a function of the population size such that they ensure that there is a central time to trigger *expected emergent open-endedness* [Abrahão et al. 2017]. That is, there is a central time to allow communication between nodes with the purpose of generating unlimited expected emergent algorithmic complexity with the least amount of cycles (i.e., communication rounds).

We have also made a small modification on the family of Time-Varying Graphs of $\mathfrak{N}_{BB}$ with the purpose of investigating what would happen if the networks had a relative small-diameter. We have replaced the cover time with the temporal diffusion diameter (i.e., the number of time intervals to reach every node from any node)—or the classical diameter (i.e., the maximum shortest path length) in the static case—in the definition of this family of graphs. Indeed, we have proved in [Abrahão et al. 2017] that, in this case, a small diameter (i.e., dominated by $\mathbf{O}(\lg(N))$, where N is the network size) is sufficient for existing a central time to trigger expected emergent open-endedness.

These proofs stems from the main idea of combining an estimation of a lower bound for the average algorithmic complexity/information of a networked node and an estimation of the expected algorithmic complexity/information of an isolated node. While the estimation of the former comes from the very BBIG dynamics through diffusion of the biggest sent partial outputs, the estimation of the latter comes from the strong law of large numbers, Gibb's inequality, and algorithmic information theory applied to the randomly generated population of Turing machines.

As indicated in [Abrahão 2016] for static algorithmic networks only, if one defines the emergent creativity of a node as

$$I_A(\mathbf{U}(p^b_{net}(o_i, c))|\mathbf{U}(p_{iso}(o_i, c)),$$

where $I_A(y|x)$ is the conditional (prefix) algorithmic complexity/information (program-size complexity, Kolmogorov complexity, or Solomonoff-Kolmogorov-Chaitin complexity) necessary to return the string y as output given the string x as input, then our results also hold for replacing the expected emergent algorithmic complexity/information with expected emergent algorithmic creativity. Thus, there would also be an emergent open-endedness on creativity from the studied conditions, and not only on algorithmic complexity. To this end, since we were estimating lower bounds for the expected emergent algorithmic complexity/information in [Abrahão et al. 2017], note that from algorithmic information theory we have that

$$I_A(\mathbf{U}(p^b_{net}(o_i, c))|\mathbf{U}(p_{iso}(o_i, c)) \geq I_A(\mathbf{U}(p^b_{net}(o_i, c)) - I_A(\mathbf{U}(p_{iso}(o_i, c)) + \mathbf{O}(1).$$

Therefore, $I_A(\mathbf{U}(p^b_{net}(o_i, c))|\mathbf{U}(p_{iso}(o_i, c))$ defines the amount of algorithmic information necessary to produce the same output that an isolated node would do if networked. In other words, it gives a formal measure on how much more algorithmically creative a node becomes when networked compared with when isolated.

Additionally, if one defines the measure of how much integrated with the network the behavior of node is as $Int(o_i, c)$, where $Int(o_i, c)$ is the information that one would need to actually inform as partial inputs in order to make an isolated node o_i behaves like it was networked during c cycles, then it directly follows from algorithmic information theory that

$$Int(o_i, c) + \mathbf{O}(1) \geq I_A(\mathbf{U}(p^b_{net}(o_i, c))|\mathbf{U}(p_{iso}(o_i, c)).$$

Thus, our results in [Abrahão et al. 2017] also hold for replacing the expected emergent algorithmic complexity/information with expected (behavior) integration of nodes.

4. Conclusions

We have shown in [Abrahão et al. 2017] that there are topological conditions that trigger a phase transition in which, eventually, algorithmic networks $\mathfrak{N}_{BB}$ begin to produce an unlimited amount of bits of expected emergent algorithmic complexity as the population size goes to infinity. In this work, we have also shown that the same statement holds for expected emergent algorithmic creativity and for expected emergent (behavior) integration. These topological conditions come from a positive trade-off between the average diffusion density and the number of cycles. Therefore, the diffusion power of a dynamic network has proven to be paramount with the purpose of optimizing the average fitness of an algorithmic network that plays the Busy Beaver Imitation Game (BBIG). Further, this diffusion power may come either from the cover time, as in [Costa et al. 2015], or from a small diameter compared to the network size.

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