Towards Low-Cost Tangible Extended Reality for Mathematics Education

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Abstract. ***Introduction:*** *Math anxiety is a common challenge for young students, often impairing learning and engagement. Tangible and extended reality (XR) technologies offer opportunities to make learning more interactive, collaborative, and less intimidating.* ***Objective:*** *This work proposes* `XR4Math`*, a low-cost tangible XR system designed to reduce math anxiety through personalized and engaging activities.* ***Methodology:*** *The system synthesizes principles from embodied cognition, collaboration, and multimodal design.* ***Results:*** *We demonstrate its potential through two proof-of-concept applications: ExpressionXR, a tangible game for solving arithmetic expressions, and PuzzleXR, a geometric construction puzzle.*
Keywords *XR, Education, Tangible interaction, Mathematics Learning.*

1. Introduction

Mathematics education has long struggled with abstract concepts that are difficult for students to visualize and internalize. Thus, the development of mathematical skills is often associated with high anxiety and low motivation of young students, which can negatively impact their performance [Cargnelutti et al. 2017, Chang and Beilock 2016]. Traditional methods, such as textbooks and lectures, often fail to engage learners effectively. However, tangible interaction (where learners manipulate physical objects linked to digital representations) combined with extended reality (XR) technologies offers a promising solution [Belter et al. 2023].

XR is an umbrella term used for immersive technologies such as Virtual Reality (VR), which fully immerses users in a virtual environment, augmented reality (AR) that enhances our view of the world with computer generated information, and mixed reality (MR), that is somewhat in between VR and AR, combining virtual elements with physical ones in the real world [Alnagrat et al. 2022, Bulut and Borromeo Ferri 2023].

As XR technologies evolve, new paradigms develop. One such paradigm is tangible interaction, which adds another layer of immersion by enabling users to engage with digital information through physical objects, leveraging material representation and spatial awareness [Hornecker and Buur 2006]. When combined with XR, tangible interfaces allow the user to manipulate virtual objects through corresponding tangible objects, providing an intuitive ways to interact with the digital world. An example of this is an augmented reality sandbox (Figure 1), which applies tangible interaction by letting users physically shape a surface that is instantly augmented with projected maps and colors.

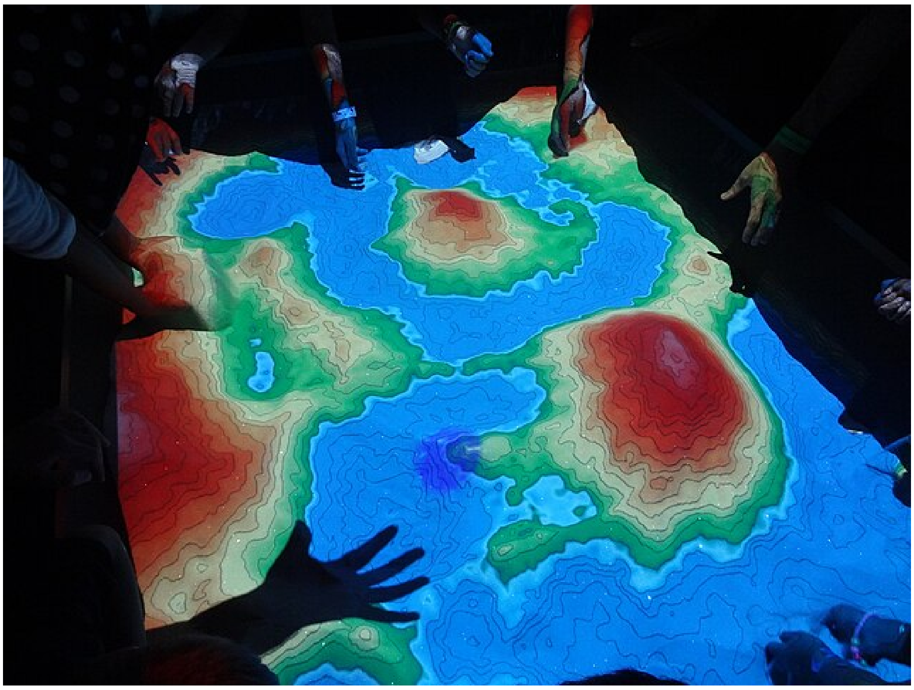

Figure 1. Tangible interaction in augmented reality. Image by Tsha333, licensed under CC BY-SA 4.0 via Wikimedia Commons [Tsha333 2016].

These experiences can be delivered through different platforms. Current XR applications can be used in mobile handheld devices, such as cell phones and tablets, or in wearable head mounted devices, sometimes called XR headsets or smart glasses. While XR research primarily focused on smart glasses with head mounted displays (HMD), the ubiquity of XR-capable smartphones has reshaped the field, making AR more widely accessible [Liao 2018].

This increase in accessibility is one trend that reinforces the potential of AR in education. XR has great potential to boost motivation, increase engagement, and support experimental learning [Shaghaghian et al. 2024]. However, if not designed correctly, it could also present challenges such as cognitive overload, usability issues, and a priority of technological capabilities at the expense of pedagogical goals [Helsel 1992, Mikropoulos and Natsis 2011].

The conscious use of technology is critical for effective learning, especially for younger students. Education 5.0 is the next evolution in educational methodologies, that builds on earlier educational models that have been redefined by historical global events, and shifts the focus on the use of technology to be more centered on the students [Felcher et al. 2022]. Therefore, despite these risks, well-designed tangible XR tools can offer potential solutions in education. Tangible XR learning techniques can be used to make some mathematical problems less abstract and more intuitive and, therefore, can help decrease anxiety levels and improve math motivation in students [Chen 2019]. Furthermore, these techniques have shown to enhance the spatial ability of young students [Gecu-Parmaksiz and Delialioğlu 2020].

Despite recent significant progress with the release of modern commercial smart glasses such as the Apple Vision Pro, the Ray-Ban Meta, Lenovo ThinkReality A3, and the Microsoft Hololens (which has been already discontinued), there are still many challenges that prevent widespread adoption of XR. For example, smart glasses remain expensive, and their prolonged use can lead to discomfort, including dizziness, nausea, and eye strain [Kim and Shin 2021]. In addition, the sparse catalog of XR applications reduces its overall utility and hinders its adoption.

The next session contains a brief literature review that presents and discusses the main design issues and challenges of using tangible XR for math education. Our goal is to synthesize these issues to help with the creation of future tangible XR systems and avoid or mitigate some of the relevant current challenges. Our investigation is guided by the following research questions.

1. How has tangible XR interfaces been used for math education?
2. What empirical evidence supports their effectiveness?
3. What are the challenges and ethical concerns for their implementation?
4. How can tangible XR technologies address these challenges to improve the math learning experience?

The remainder of this paper is organized as follows. Section 2 provides an overview of related work in the field of tangible and XR for mathematics education, to help us answer the first two questions. Section 3 presents the key findings of our literature review, highlighting existing gaps, challenges, and ethical concerns. In Section 4 we address these challenges by introducing `XR4Math`, a tangible XR system designed to support mathematics learning for primary school students. Section 5 presents two proof of concept applications that showcase our system's capabilities. Finally, Section 6 discusses the implications of our work and outlines directions for future research.

2. Tangible XR for Math Education: Related Works

Although it represents a limited fraction of the overall market for XR [Alnagrat et al. 2022], the first educational opportunities for XR systems were soon discovered [Bulut and Borromeo Ferri 2023], focusing mostly on scenarios of dangerous or high-cost reproduction, i.e. piloting and surgical care. Using XR it is possible to simulate, with various degrees of realism, complex or dangerous situations without risks for the students.

As early as the 1990s, Helsel discussed concept-oriented and technology-oriented approaches to VR in education [Helsel 1992]. In a concept-oriented approach, the student is at the center of the design process, with VR serving merely as a tool used to accomplish that. In contrast, technology-oriented perspective focuses on the hardware and software's capabilities and limitations, possibly at the expense of the student's needs. Helsel argued that educational designers should prioritize conceptual orientation, ensuring that pedagogical goals are used to guide the use of technology. It was also believed that VR could shift education's reliance on textbook-based abstractions toward more experiential forms of learning.

A review study on applications of VR for education by Mikropoulos and Natsis [Mikropoulos and Natsis 2011] listed more than 50 articles published between 1999 and 2009. Most of the studies were related to science, technology, and mathematics, subjects that naturally benefit from VR's ability to explore spatial visualization and an experimental nature. However, the authors observed that the new technology was mainly used to support existing teaching practices instead of exploring new and innovative ideas provided by the new technology. In a more recent review, Bulut et al. [Bulut and Borromeo Ferri 2023] examined AR in math education. Current trends indicate that AR enhances learning and motivation, although technical limitations, usability issues, and gaps in digital literacy among teachers and students continue to hinder broader adoption.

While early XR applications focused on recycling teaching methods, more recent research has explored different XR modalities. Huang et al. investigated knowledge retention in different XR tools by comparing VR and AR versions of a mobile solar system application [Huang et al. 2019]. The app presented three-dimensional representations of the planets accompanied by an audio commentary. In AR mode, participants viewed the digital content overlayed on the real world using the smartphone's camera. In the VR mode, an HMD was used to offer a fully immersive experience without real world content. The results showed that the VR participants reported greater enjoyment and paid more attention to the environment, while the AR participants retained more auditory information. The study concludes that the VR's immersion overwhelms attention, reducing focus on other modalities of information such as auditory. In contrast, as AR conditions are not as immersive as VR, it leaves more attentional resources for auditory input. The study highlights the importance of aligning the XR modality with specific learning goals when designing educational applications.

Rosseto et al. [de Moraes Rossetto et al. 2023] proposed an application for teaching about the solar system based on both VR and AR technologies for students from public schools in impoverished neighborhoods in Brazil. Both AR and VR modules were developed with the same functionalities, the AR module used fiducial markers to display the content in the correct place, the user had to manually point the smartphone to read and identify the marker first before the virtual information was loaded. The VR module used Google's Cardboard acting as an wearable HMD, in this mode, skyboxes were used as a background to give the impression of an immersive environment. The data collected from the experiments showed that the VR module was more effective in teaching the concepts than the AR module. VR was also rated higher in terms of immersion interactivity, while AR was perceived as more intuitive. The authors noted that the students showed greater interest in the VR experience, likely due to the novelty of a wearable headset, as most of these students had never experienced such technologies before.

2.1. XR for math education

Several AR applications have also been suggested specifically for teaching math-related subjects, particularly geometry, where the change from two-dimensional representations to immersive three-dimensional visualization is natural.

For example, Khan et al. [Khan et al. 2018] created *Mathland*, a mixed reality application that combines mathematical concepts with the physical world, incorporating elements of collaboration and constructionist learning.

Geogebra, a popular mathematics software that combines visualizations of geometry, algebra and calculus, also offers an AR version, Geogebra AR [GeoGebra 2025]. This app allows users to explore 3D mathematical models and interact with them in the real world through augmented reality, enabling them to manipulate, rotate, and reposition the models in real-time.

Koparan et al. [Koparan et al. 2023] implemented an AR geometry-related mobile application to assist secondary school students with spatial intelligence. User studies showed that AR-supported teaching methods improved student performance. Similarly, Shaghaghian et al. [Shaghaghian et al. 2022] created an AR application, BRICKxAR/T, designed to teach geometric concepts and spatial transformations using an iPad. The application was later tested in workshops, where it was found that students using the AR system scored significantly higher and had a lower overall task load compared to those in non-AR workshops [Shaghaghian et al. 2024]. However, students also experienced significant physical strain and effort while using AR applications on a tablet, compared to traditional learning methods.

2.2. Tangible XR

Tangible interaction aligns with embodied cognition, which posits that learning is enhanced through physical engagement [Zaman et al. 2012]. Studies suggest that manipulating physical objects (e.g., blocks, tokens) helps students internalize abstract math concepts like geometry, algebra, and spatial reasoning [Rodić and Granić 2022].

Vygotsky's Zone of Proximal Development (ZPD) emphasizes peer interaction in learning. Tangible XR systems, such as ARcane Tabletop [Leonidis et al. 2024], facilitate collaborative problem-solving by allowing students to interact with shared digital-physical objects. Research shows that tangible apps improve multimodal communication (verbal, gestural, symbolic) compared to traditional paper-and-pencil methods [Rodić and Granić 2022, Zaman et al. 2012].

2.3. Applications of Tangible XR in Math Education

Tangible XR systems have been used to improve mathematics education through several forms of immersive and embodied interaction.

Li et al. [Li et al. 2019] identified that while AR games have motivational effects, many focused on overlaying digital content without much concern for user interaction. To address this, they proposed an AR game with tangible interaction designed to improve the learning experience of children. Digital animals were displayed on a printed background, and answers were displayed across the paper digitally. In the tangible interaction mode, children physically tilted or lifted the paper to move the animal to the correct answer, simulating gravity. In contrast, the touchscreen-based interaction, children simply clicked on the screen. During user studies, children understood that moving the paper would make the animal move, but they had difficulty controlling how much they should lift the paper. Despite those challenges, tangible interaction mode was preferred.

In another study, Kang et al. developed ARMath [Kang et al. 2020], a mobile AR system for children aimed to bridge the gap between everyday life activities and mathematical concepts. The system is equipped with a perception engine that recognizes real objects with semantic understanding. A problem generator module gets the semantic context of the scene and creates math problems such as equations based on the objects detected. ARMath provides both a tangible and a touchscreen mode for interaction. In the tangible mode, the child can move the physical object to solve arithmetic operations.

Later, user studies showed little difference in preference between the two modes; however, children were more single-step oriented in tangible mode, whereas as virtual mode adopted a slow and multi-step approach.

3. Design Issues for `XR4Math`

Future development of tangible XR systems for math education must deal with the challenges and explore the opportunities as presented in the previous session. This session discusses such challenges and opportunities to help us design `XR4Math`.

While early XR applications largely repurposed traditional educational methods within interactive environments [Mikropoulos and Natsis 2011], more recent work has taken a ground-up approach, leveraging XR technologies from the outset to develop entirely new pedagogical strategies. In particular, tangible XR has demonstrated significant potential to enhance engagement, comprehension, and collaborative learning, especially in mathematics. Mirza et al. [Mirza et al. 2025] emphasize that effectively integrating AR requires new pedagogical approaches, yet most educators are not trained in these methods. Their study reveals that only 27% of surveyed teachers use mobile apps for teaching, and just 24% incorporate AR-based mobile applications.

These findings are aligned with the Brazilian educational scenario presented in the TIC Educação 2022 survey [CETIC.br 2023], which reports that 17.9% of teachers expressed uncertainty about how to use digital technologies with students, while 37.8% cited a lack of technical support staff at their schools. Furthermore, 49.6% observed that students often become overly distracted by these technologies, and 14.5% noted that integrating them into lessons requires excessive preparation time. Most notably, 83.9% of teachers reported insufficient access to computers, indicating that access to more advanced devices, such as tablets or XR equipment, is likely even more constrained. Existing mobile applications for education face a number of limitations, including pedagogical misalignment, hardware requirements, poor usability, lack of interactivity, and limited adaptability.

Choosing whether to develop an XR application for AR, VR, or MR platforms presents a complex challenge. As highlighted by Huang et al. [Huang et al. 2019], VR offers a high level of immersion and environmental focus, but may overwhelm users and reduce their attention to other sensory modalities, such as auditory input. In contrast, AR supports integration with the physical world, enabling multitasking and preserving attentional bandwidth for audio and collaboration. This makes AR particularly well-suited for educational contexts where guidance and social interaction are key components of the learning experience.

In addition, high-end XR hardware such as the Apple Vision Pro, remains costly and less accessible. As highlighted by Rosseto et al. [de Moraes Rossetto et al. 2023], impoverished neighborhoods face technological restrictions. Wearable devices can increase engagement, but accessibility must remain a key concern.

For these reasons, `XR4Math` adopts a wearable AR setup using smartphones and a Google Cardboard (as seen in Figure 2).

The literature mentions the effectiveness and preference of tangible interfaces by the participants. Inspired by embodied cognition [Zaman et al. 2012], systems

Figure 2. Google Cardboard VR headset. Image by Evan-Amos, public domain, via Wikipedia [Evan-Amos 2015].

that allow users to physically manipulate objects have shown improved certain aspects of learning compared to traditional methods [Rodić and Granić 2022]. Studies like BRICKxAR/T [Shaghaghian et al. 2024] and ARMATH [Kang et al. 2020] have demonstrated the benefits of tangible XR: higher preference and performance. On the other hand, some students reported ergonomic issues or physical strain when using handheld devices like tablets. `XR4Math` offers a hands-free interaction via a head-mounted AR device where users can manipulate physical markers directly without the need of holding the display device.

ARcane [Leonidis et al. 2024] and others employ collaborative learning, based on Vygotsky's Zone of Proximal Development (ZPD), by using shared digital-physical objects. `XR4Math` supports shared physical interaction where multiple students can interact and solve problems while working together.

While there are several AR applications developed to support math learning, many fall short in scope. Current applications offer target specific concepts such as geometry or arithmetic, relying on touchscreen input or constant device handling. Others may prioritize XR's technology instead of focusing on a student-centered approach, such as social and ethical skills for Education 5.0. The use of technology can also be concerning since it might hinder student's communication and interaction. Lehtonen et al. [Lehtonen et al. 2023] have shown that, when working in pairs, students using tangible apps interacted with with each other much more often and in more ways than their paper-and-pencil peers.

Kourtesis [Kourtesis 2024] presents a review of the risks and ethical challenges in the Metaverse using XR. Though not directed related to math education, similar issues might arise in applications that require collaboration between multiple users, such as data privacy risks, cybersecurity vulnerabilities, cybersickness, addiction, dissociation, harassment, bullying, and misinformation.

Furthermore, tangible interaction is not generalized, most proposed systems are tied to a single application or concept. `XR4Math` is designed as a flexible system with reusable physical markers that map to digital components. This makes it possible to develop different educational activities, including social and collaborative ones, with minimal changes to the physical side.

Based on these insights, the following design principles were established for `XR4Math`:

1. **Affordable**: Use of affordable hardware (e.g., smartphones and Google Cardboard) to enable wider adoption.
2. **Tangible**: User manipulates physical markers to interact with digital content.
3. **Wearable**: Head-mounted displays allows hands-free interaction, enabling full use of both hands for tangible tasks.
4. **Modularity**: The system supports multiple educational applications through a modular design, allowing the adaption of the same physical components for different lessons.
5. **Low-Tech Alternatives**: An "offline" or *after-the-fact* rendering mode enables use even in contexts with limited real-time processing capabilities, such as using printed boards and post-evaluation.
6. **Ethical**: Though access to students' data should be granted to tutors to help the development of the students, access should be limited to preserve privacy. Students should also be monitored constantly to prevent cybersickness and addiction.

4. `XR4Math` Implementation Issues

Following the design principles presented in the previous session, this session introduces `XR4Math`, a system designed to be a low-cost, wearable XR platform for primary school students. The system can be used with a smartphone as a head-mounted display (HMD) such as in Google Cardboard [Google 2025b]. By relying on a smartphone and an affordable Cardboard extension instead of a high-end AR headset, our system lowers the barrier to entry and enhances accessibility for students. Clearly, more expensive AR platforms could be used as well.

The system is designed to support two setups:

1. **Real-Time Rendering** requires a device capable of keeping up with the task of tracking several targets and real-time interaction. When used as a Cardboard display, it maximizes immersion and leaves user's hands free for the physical interaction. In scenarios in which the Cardboard is not available, including when multiple users share a single device for visualization, it is still possible to interact normally with the system, holding the device on one hand;
2. **After the Fact Rendering** in case real-time rendering is not available, students can used a less powerful device to process some events of the game. For example, users can complete the activity using a printed board instead of the AR projected

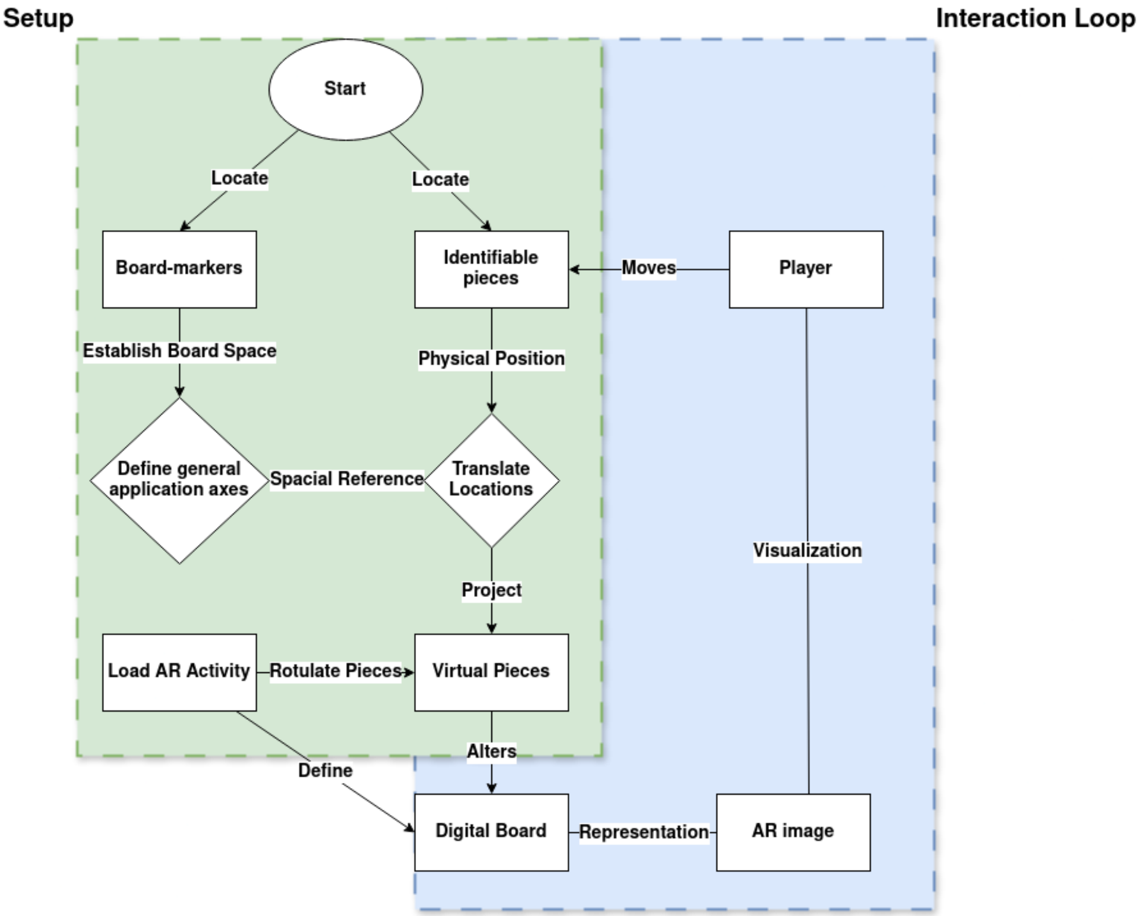

Figure 3. Block diagram of XR4Math. The setup involves locating and mapping each one of the physical components, to create the coordinate references, and loading the information of the specific application to be played. Afterwards, the user's movement on the marked pieces is tracked and the virtual table state altered to reflect the interaction, including moving the digital object assigned to the specific marker.

one, and take a picture of it, so that the verification can be done, and some AR can be visualized offline.

An overall block diagram of the system is shown in Figure 3. During setup of the application, all markers, including the board, are located, labeled, and tracked within the coordinate system of the application overlaid with the physical scene for spacial reference. The corresponding digital models are loaded. Within the interaction loop, the models are registered to their corresponding markers and spatially rendered, generating the AR image that is visualized bu the user.

4.1. Tangible Interaction

To bridge the physical and virtual worlds, `XR4Math` employs tangible physical objects that users are required to manipulate to complete the game (or task). Each tangible object is also an AR marker, that can be used to render different components of the game. For the design of the markers, two categories can be considered as follows:

- Spatial Markers: Two L-shaped objects placed on the surface to define the space

where to display the virtual content. This centralized reference allows for a higher spatial dependency[Wither et al. 2009], so that the virtual position of each object better follows its assigned referent [Lee et al. 2023].
- Activity Markers: Color or pattern coded objects recognized and processed by the camera. These are manipulated by the user depending on the activity.

Markers might change in size, weight, and other properties for some activities, since they might depend on the application. For example, tasks that allow rotation will require markers that enable rotation estimation, which can be accomplished by visual features drawn on each piece as to break symmetry, with eventual tasks that recognize spins on other axes having more demanding requirements.

By mapping each marker to a unique virtual identifier (glyph) and defining a flat interaction surface, we create a general and flexible interactive board-game platform. This design minimizes computational load, as AR tracking is simplified through predefined, easily recognizable markers.

Several AR frameworks for mobile devices facilitate the development of AR-based applications. Google's ARCore [Google 2025a], Apple ARKit [Apple 2025] are some of the most complete feature-wise [Nowacki and Woda 2020] frameworks, which provide functionalities such as motion tracking, marker recognition (anchors), depth understanding, and more, which are essential for `XR4Math`.

4.2. Expected Benefits and Remaining Challenges

Tangible XR systems such as `XR4Math` can provide several pedagogical advances, especially related to math education. However, its widespread adoptions depend on tackling relevant technical, cognitive and ethical challenges.

Among the main benefits, tangible XR systems offer improved engagement, with students often reporting increased motivation and curiosity [Kang et al. 2020, Li et al. 2019]. These interactions will help sustain focus and result in a more playful, exploratory learning environment. 3D visualization also aids in comprehending complex topics such as calculus and topology, while shared tangible interfaces promote teamwork and discussion.

On the other hand, cost and accessibility are major barriers of entry: the development of XR software and providing XR-capable devices to schools, especially in impoverished regions, can be expensive and resource intensive [de Moraes Rossetto et al. 2023, Upadhyay et al. 2024]. Without affordable solutions, many schools, particularly in poor regions, may be excluded from these innovations. The risk of cognitive overload, while more predominant in VR [Huang et al. 2019], can also happen in XR applications if the interface design is not constructed carefully. Finally, ethical concerns must also be addressed, including data privacy issues, equity issues, and addiction risks.

5. Application examples of the `XR4Math` platform

In this section, we present two potential applications as a proof-of-concept of the `XR4Math` platform. Our first example, ExpressionXR, is a tangible application that can be extended to any kind of multiple-choice questionnaire. The second example illustrates

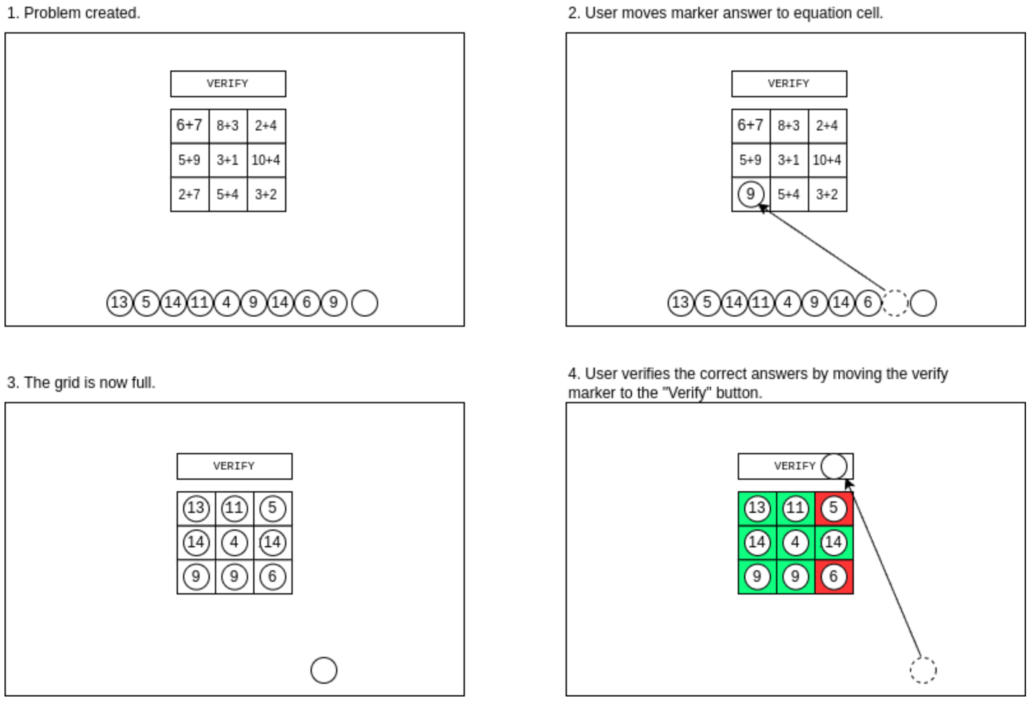

Figure 4. Application flow diagram - ExpressionAR. 1. Show the initial screen. 2. The user can drag pieces to answer questions on the board. 3. Each piece correctly responds one expression. 4. Depicts the board once all places are filled, allowing the evaluation to be made

a tangible puzzle game that can be extended for other kinds of questions, such as sentence-completion and mix-and-match.

5.1. ExpressionXR

This section presents ExpressionXR, an application designed to help students practice solving arithmetic expressions through tangible interaction. As illustrated in Figure 4, the system projects a virtual grid of mathematical expressions onto the user's physical space. The student interacts using tangible markers, which the system recognizes and associates with specific numerical solutions.

The primary task requires the student to place each tangible marker onto the grid cell containing the corresponding expression. After arranging all markers, the student can initiate a check by placing a special "verify" marker on a virtual *VERIFY* button. The system provides immediate feedback by coloring each cell's border red (incorrect) or green (correct). Lifting the verify marker concludes the feedback cycle and generates a new set of expressions, allowing for continuous practice.

5.1.1. Inherent Flexibility and Extensibility

The ExpressionXR design showcases the framework's significant flexibility. The application's logic and content can be easily extended and modified:

- **Content Inversion**: The roles of tangible and virtual elements can be reversed,

with numerical solutions displayed on the virtual grid and the expressions printed on the tangible markers.
- **Difficulty Scaling**: The challenge can be dynamically adjusted by incorporating more complex operators (e.g., multiplication, division), multi-step expressions, or algebraic functions.
- **Domain Adaptability**: The application is not limited to mathematics. It can be readily adapted for any domain requiring matching pairs, such as vocabulary definitions, historical dates, or scientific symbol identification.

5.1.2. Alternative Modes of Interaction

Beyond the core real-time experience, the framework supports other alternative modes such as:
- **Asynchronous Offline Mode**: To support scenarios without constant device use, ExpressionXR features an "after-the-fact" rendering mode. A worksheet with the expression grid can be printed, and students can solve it physically. Afterwards, they can use a smartphone to capture a photo of the completed board, allowing the app to digitally evaluate the answers and provide rich feedback.
- **Multi-User Collaboration and Competition**: The application is designed for multi-user interaction. Multiple students, each with their own device, can engage with the same board simultaneously. This enables various collaborative or competitive scenarios. For example, a competitive game could be implemented where players take turns placing markers, claiming a grid spot only after a correct placement, similar to tic-tac-toe. This multi-user model can also extend to remote players in a shared tangible VR environment.

A functional prototype of the ExpressionXR graphical interface has been developed using WebGL. We are currently integrating this interface with the tangible marker recognition system via Google's ARCore.

5.2. PuzzleXR

Our second application serves as a toy model to foster spatial reasoning skills, specifically designed for primary and middle school students, but that can be easily extended to sentence completion and mix-and-match questions, using simpler shapes or just rectangles.

In PuzzleXR, each marker is associated with a distribution of board positions, forming unique shapes that can be freely rotated and moved (see top left of Figure 5). Each black spot represents a marker, and the complete shape is visible only through XR visualization.

The goal for the user is to correctly place all markers on the grid, aligning each piece with its appropriate position and orientation to fully cover the board, as illustrated in Figure 5. The game's difficulty can be adjusted by modifying the grid size, the number of pieces, and the complexity of their shapes, making it adaptable to different age groups and learning levels.

Designed to support hands-on and visual learning, PuzzleXR encourages students to explore and understand geometric transformations—an essential aspect of spatial

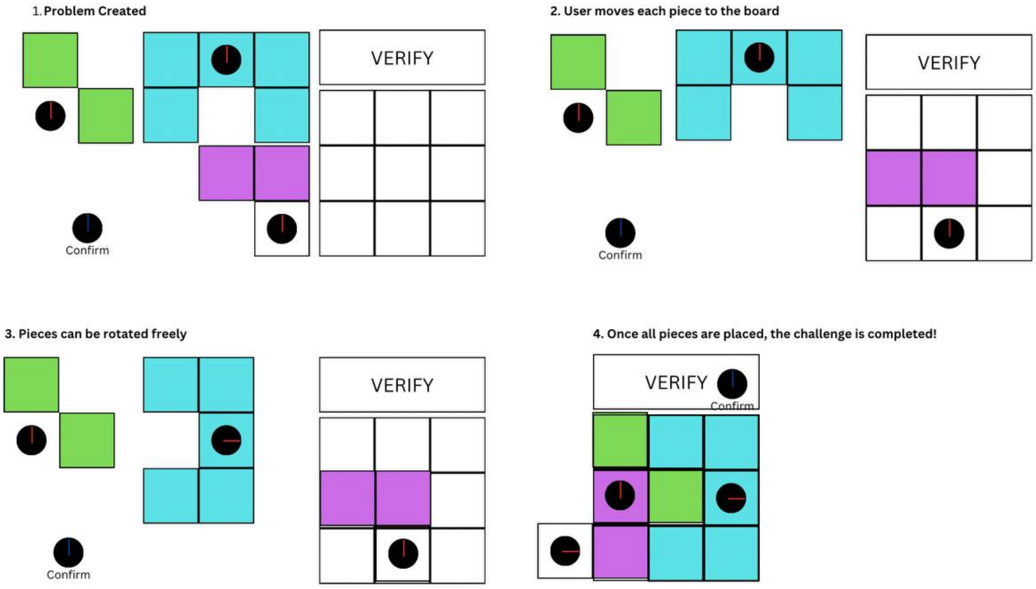

Figure 5. Application flow diagram - PuzzleXR. The game is completed when all slots on the grid are occupied by the generated pieces.

cognition. When real-time rendering is not available, PuzzleXR can be played using a printed board and physical marker pieces. Students manually arrange the shapes on the grid to complete the puzzle, then use XR to visualize the arrangement. Once finished, a photo of the board is taken using a smartphone, and the system analyzes the solution and provides feedback.

PuzzleXR also supports multiplayer modes, either collaborative or competitive, in both AR and VR environments.

As with ExpressionXR, a prototype of PuzzleXR has been developed using WebGL, and a tangible ARCore-based interface is currently in development.

6. Conclusion

Tangible extended reality (XR) technologies have great potential to revolutionize math education by bridging abstract concepts with physical interaction. A quick review of the literature points out several design issues about the use of tangible XR for education, such as cost, usability, and ethical risks. To address such challenges, the article proposes `XR4Math` as a potential low-cost solution to improve student performance and engagement when solving math exercises. `XR4Math` leverages smartphones and low-cost head mounted extensions such as Google Cardboard, offering a more accessible alternative to modern commercial headsets. Additionally, `XR4Math` includes a mode that does not require real-time rendering, allowing students to continue to participate in activities without a smartphone that processes real-time AR.

Beyond accessibility, `XR4Math` provides a flexible system for the development of multiple educational applications, addressing the limited number of XR applications. This is demonstrated through two proof of concepts: an associative game to solve algebraical expressions and a toy model for geometrical constructions.

In future work, we will evaluate both tangible XR games with math teachers to ascertain their effectiveness and help refine and improve our initial prototypes, such as the alternatives to tangible objects and their properties, such as color, size, and shape, reassess the need for fiducial board markers, and better understand the cognitive load of additional features and markers. Different applications will also be developed to address subjects other than mathematics (such as chemistry and physics).

Ethical Considerations

This research did not involve human participants, and therefore did not require approval from an ethics committee.

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