Modeling Chaos and Image Processing in Distributed Architectures

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Abstract -
The emergency of solving non-linear systems using chaos theory and the solution of innumerable problems using image processing approaches, leads to the use of specific machines, which explores the data and instruction parallelism. The dynamic system monitoring chaotic nonlinear systems, starting from equations or time series is quite common, but the use of combining techniques of image processing with chaos approaches isn't usual. In this paper, the combining techniques are used to characterize signals obtained from solar explosion, from which it is calculated the embedding dimension, and the power spectra. The proposed techniques is being implemented in a parallel processing machine, PAD, and the performance is discussed.

Keywords - **chaos, PAD, dimension, correlation, power spectrum, fractal**

I. INTRODUCTION

When scientists try to model problems as the flux of water, air, or some fluid, they write the Navier-Stokes differential equations. In simple cases, as a flux inside a pipe, the calculation may be exact, but in many difficult situations, the answer may be impossible analytically, depending of numerical analysis using computers. To solve these problems in a simple von Neumann machine may be impracticable, and the use of parallel or distributed architectures may be successful [HWA 93].

On the other side, many efforts have been made to the efforts to answer some questions as: what powers the sun, why does it have spots, how long will it sustains life on earth, but these questions remains to be answered. The best models of the sun's nuclear power predict a significantly higher neutrino flux than is actually observed. It is not known what causes sunspots and other solar activities or even why the sun emits x rays. The modeling of how stars evolve leads to age estimates for some stars that are greater than recent estimates of the age of the universe [HAR 95].

Many aspects as solar explosions, coronal mass ejection, magnetic storm and ionospheric phenomena are being investigated, intensively to realize the Space Weather Forecast which enables the prevention of damages to the communication systems, geological positioning satellites, energy transportation systems, and so on.

Because of rapid evolution of the space exploring systems and the expectations that the future space projects are based on the private industries partnership, increasing the number of projects in the area, the Laboratory of Communications Research, together with the Solar Terrestrial Research Center, both in Japan, started in 1989 the International Program of Space Weather Forecast [MAR 89]. The program expected the installation of several laboratories dedicated to the analysis of data stored by STEL (Solar Terrestrial Environment Laboratory) in Nagoya University, with the purpose of creation of a data base of solar explosions, coronal mass ejection, magnetic storm, and ionospheric phenomena. There are 23 laboratories, denominated SWFL (Space Weather Forecast Laboratory) Units, being installed in several institutes, and universities, around the world. The Departments of Astronomy and Applied Computer Sciences of the INPE (Brazilian National Institute of Space Research), with the creation of a Space Weather Forecast Laboratory (LPCE), accorded with STEL to the installation of Brazilian unit that will participate of the SWFP (Space Weather Forecast Program) that will use parallel computation to processing, visualization, and analysis, of solar images obtained in real time. The LPCE will be the main user of the parallel machine that will be installed at the Computation Department of Federal University of São Carlos (DC/UFSCar).

In this paper we present some aspects of the image processing, and chaos theory, used to the analysis of the data from STEL, in the related parallel machine, which is a cluster of SMP's, named PAD [BER 99], followed by a performance analysis and conclusion.

II. IMAGE PROCESSING [GON 92]

The base algorithm in the Image Processing is concerned to Fourier transform, which involves the complex parallelism of instructions and data.

A. *Fourier Transform*

Using a function $f(x)$, the Fourier transform function $F(u)$ is given as:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

A sample signal is obtained, so that the continuous signal, may be reconstructed, and the Fourier Transform and the Convolution Theorem may be used to the analysis of the sampled signal.

Considering that the function $f(x)$ varies from $-\infty$ to ∞, and given a finite value W, the Fourier transform vanishes for values of u outside $[-W,W]$. A function which its transform is in the interval defined by a finite value of W is called limited band function. To obtain the sampled version of $f(x)$, it must be multiplied by the sample function $s(x)$, which is a series of unit impulses Δx. By the convolution theorem, the multiplication in the x domain, is the same as the convolution in the frequency domain. Then, the Fourier transform is obtained by the convolution $S(u)*X(u)$.

The transform is periodical, with period $1/\Delta x$, and the repetitions of $X(u)$ will overlap. To avoid this overlap, it is selected the interval Δx so that:

$$\Delta x \leq \frac{1}{2W}$$

The inverse Fourier transform returns the initial continuous function. The complete recovery of a limited band function, in which the samples period satisfies the equation, is known as the sampling theorem of Whittaker-Shannon.

A bidimensional sampled function is a set of unitary impulses separated by Δx units in the x direction, and by Δy units in the y direction. By analogy to the unidimensional function, to avoid overlap it is selected the following intervals:

$$\Delta y \leq \frac{1}{2W_u}, \ and\ \Delta x \leq \frac{1}{2W_v}$$

To discrete signals, sampled or produced by numerical functions, the discrete Fourier Bidimensional Transfer, and its inverse may be expressed as:

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \exp[-j2\pi\,(^{ux}/_{M} + ^{vy}/_{N})]$$

with $u = 0, 1, 2, \ldots, M-1$, $v = 0, 1, 2, \ldots, N-1$, and

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) \exp[j2\pi\ (^{ux}/_{M} + ^{vy}/_{N})]$$

with $x = 0, 1, 2, \ldots, M-1$, and $y = 0,1,2,\ldots, N-1$.

B. *Fast Fourier Transform*

The number of complex products and additions required to implement the Fourier transform is proportional to N^2, but by proper decomposition, these numbers can be made proportional do $N \log_2 N$. This decomposition procedure is called fast Fourier transform (FFT) algorithm, and the computer implementation of the algorithm is straightforward. The main point to keep in mind is that the input data must be arranged in the order required for successive operations, named successive doubling algorithm. These successive operations use the arrangement known as shuffle exchange.

One important tool to the parallel the FFT algorithm, is the use of de Bruijn Graph B_n [PIT 93]. The de Bruijn graph B_n consists of $N = 2^n$ nodes. The nodes i and j have the binary representations $i_{n-1}\, i_{n-2} \ldots i_1\, i_0$ and $j_{n-1}\, j_{n-2} \ldots j_1\, j_0$, and there is an edge between i and j if and only if

$j_{n-1}\, j_{n-2} \ldots j_1\, j_0 = i_{n-2}\, i_{n-3} \ldots i_0\, 0$, or

$j_{n-1}\, j_{n-2} \ldots j_1\, j_0 = i_{n-2}\, i_{n-3} \ldots i_0\, 1$.

The de Bruijn Graphs have an interesting doubling property which states that from B_n the graph B_{n+1} can be determined by identifying the edges of B_n with the nodes of B_{n+1}. There is an edge from the node i to the node j in B_{n+1} if and only if in B_n the end point of the edge i is the same as the starting point of the edge j.

C. Correlation

One of fundamental properties that states a clear connection with the chaos theory is the correlation property.

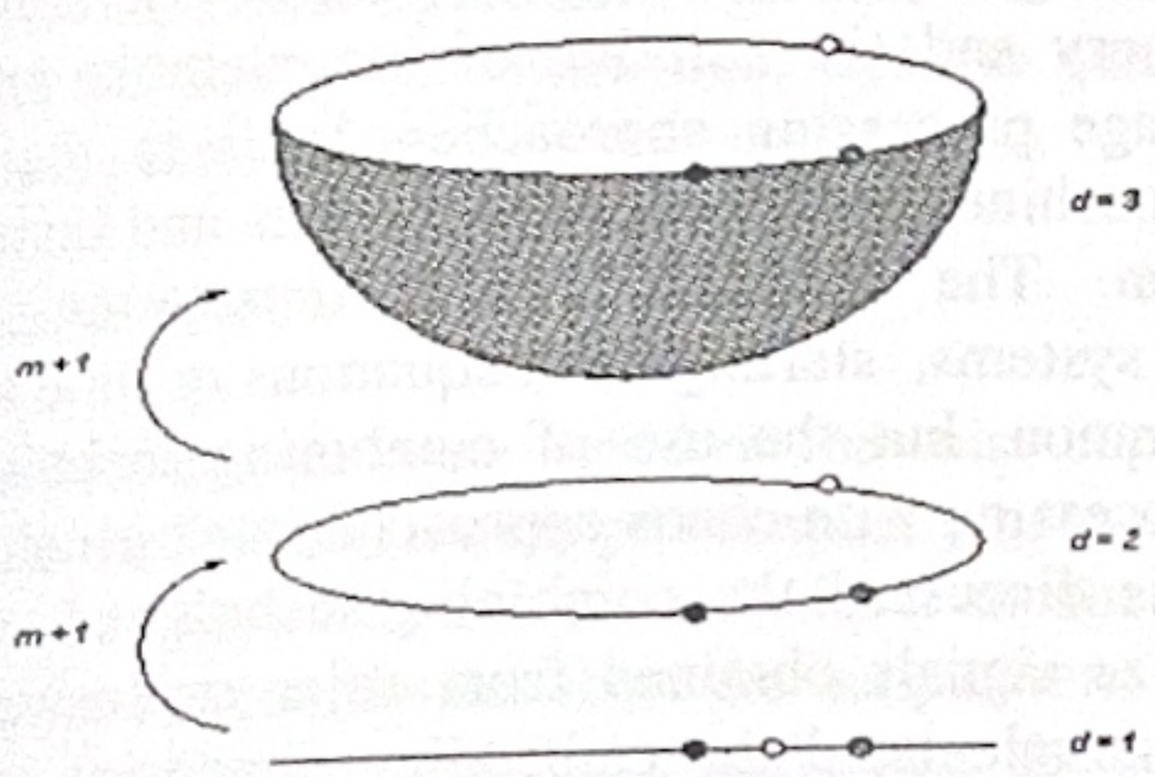
With a function $x(t)$ and delay τ, in a interval from 0 to T, its self-correlation function is given by:

$$C(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)x(t+\tau)dt.$$

In discrete terms, with samples in t_i and $t_{i+\tau}$, and calculating the summation from 1 to N-τ, where N is the sample number, the discrete correlation function is given by:

$$C(\tau) = \frac{1}{N-\tau} \sum_{i=1}^{N-\tau} x(t_i)x(t_{i+\tau}),$$

One important remark is that, in dynamic dissipative system, correlation in a space may be located in the same



line, plane or space, as can be seen in Fig.1.

Fig. 1 – The dimensions where the three points may be correlated.

III. CHAOS THEORY [PEI 92]

Chaos as a science is concerned to systems with sensitive dependence to initial conditions. Chaotic motions exhibit a very rapid growth of errors, so that despite of the existence of determinism, it turns difficult any long-term prediction. One important definition which is essential when describing chaos is the notions of strange attractors. We can describe an attractor, with a physical experiment, when using a bowl and observing how a little iron ball put into different initial conditions, come to rest at the bottom, the rest point. The attractor is this final situation, or the rest point.

A. Strange Attractors.

It is believed that long-term behavior of dissipative systems would run into simple patterns of motion such as a rest point. In contrast, strange attractors are those patterns which characterize the final state of dissipative systems that are highly complex and show all the signs of chaos. Furthermore, the geometrical patterns of strange attractors may be associated to the fractals. The reason for the popularity of chaos theory and strange attractors lies in the expectations to crack the mysteries of our planet's climate, as the solar dynamics, as well as human brain activities, through the strange attractors concept. And the chaos theory makes available a method to the reconstruction of strange attractors, using the sequences of numbers, obtained experimentally. It also make possible the computation of dimensions and Ljapunov exponents that specify the degree of strangeness of the attractor.

As an example, we describe the Rösler attractor, an elementary geometric construction of chaos in continuous systems, based on the following systems of differential equations:

$$\begin{aligned} x'(t) &= -(y(t) + z(t)) & x(0) &= x_0, \\ y'(t) &= x(t) + a.\, y(t) & y(0) &= y_0, \\ z'(t) &= b + x(t).\, z(t) - c.\, z(t) & z(0) &= z_0. \end{aligned}$$

The problem is that the understanding of natural processes does not start with a set of equations, but such models are usually obtained at the end of a long sequence of actions to the identification of the phenomena, which includes series of experiments, computer simulations, and finally mathematical analysis.

However, we must know in a given sequence of numbers whether they are concerning to an attractor, and if so, we must quantify the Ljapunov exponents and dimensions. In a simpler situation, we have a black box in which some dynamical system is present. We may probe the system at discrete time intervals and obtain the value of one of the state variables of the system. If we choose the variable z (t), and a time interval τ, we can get the sequence of values of z (t) sampled in the period of τ.

This simple procedure, known as Takens Reconstruction [RUE 71], allows us to get the geometric structure of any underlying attractor. In the case of Rössler system in the black box, we can choose a time delay T, multiple of τ, and look at the following sequence of vectors:

$$\begin{aligned} &(z(0), z(T), z(2T)), \\ &(z(\tau), z(\tau + T), z(\tau + 2T)), \\ &(z(2\tau), z(2\tau + T), z(2\tau + 2T)), \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ &(z(k\tau), z(k\tau + T), z(k\tau + 2T)). \end{aligned}$$

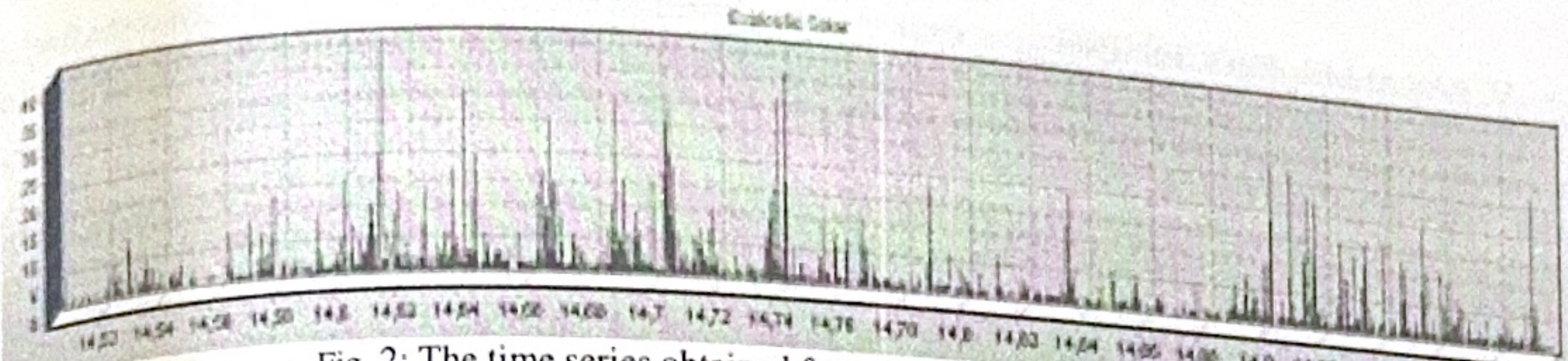


Fig. 2: The time series obtained from the solar explosion data.

- Minimum flux : *0* SFU,
- Average flux : ± *0.645* SFU, and,
- Number of points: *7198* points.

The power spectrum and fast Fourier transform algorithms (compute the self-correlation) use the extended series of *8192* points. Fig.2 illustrates the time series obtained with the above data. The horizontal coordinates show the time and the vertical coordinates show the intensity of the explosion energy.

Through the observation of the non-linearity of the phenomena, it is first calculated the self-correlation in *8192* points, of the Power Spectrum of the data. The same data is used to the Takens reconstruction, and to compute the embedding dimension.

B. Power Spectrum.

We first use the time series of the solar explosion to compute the self-correlation power spectrum, as showed in Fig. 3, that obviously is only calculated in dimension 2. The horizontal coordinates show the sample time, while the vertical coordinates show the related power spectrum. This results may be compared with the results of Takens reconstruction approach in two-dimensions.

C. Embedding Dimension.

Fig. 4 shows the energy values in vertical coordinates, and the embedding dimensions in the horizontal ones. It is showed the energy function of two solar explosion time series, as two curved lines which converges at the higher values of energy, and fractal dimension.

The results show that the solar explosion data uses dimension above 2, where the two energy values converge. The fractal dimensions convergence point obtained is *2.73*, which indicates that the suitable approach is the Takens Reconstruction of chaotic attractors, by the fact that the correlation dimension is calculated in dimension 2 [GRA 83].

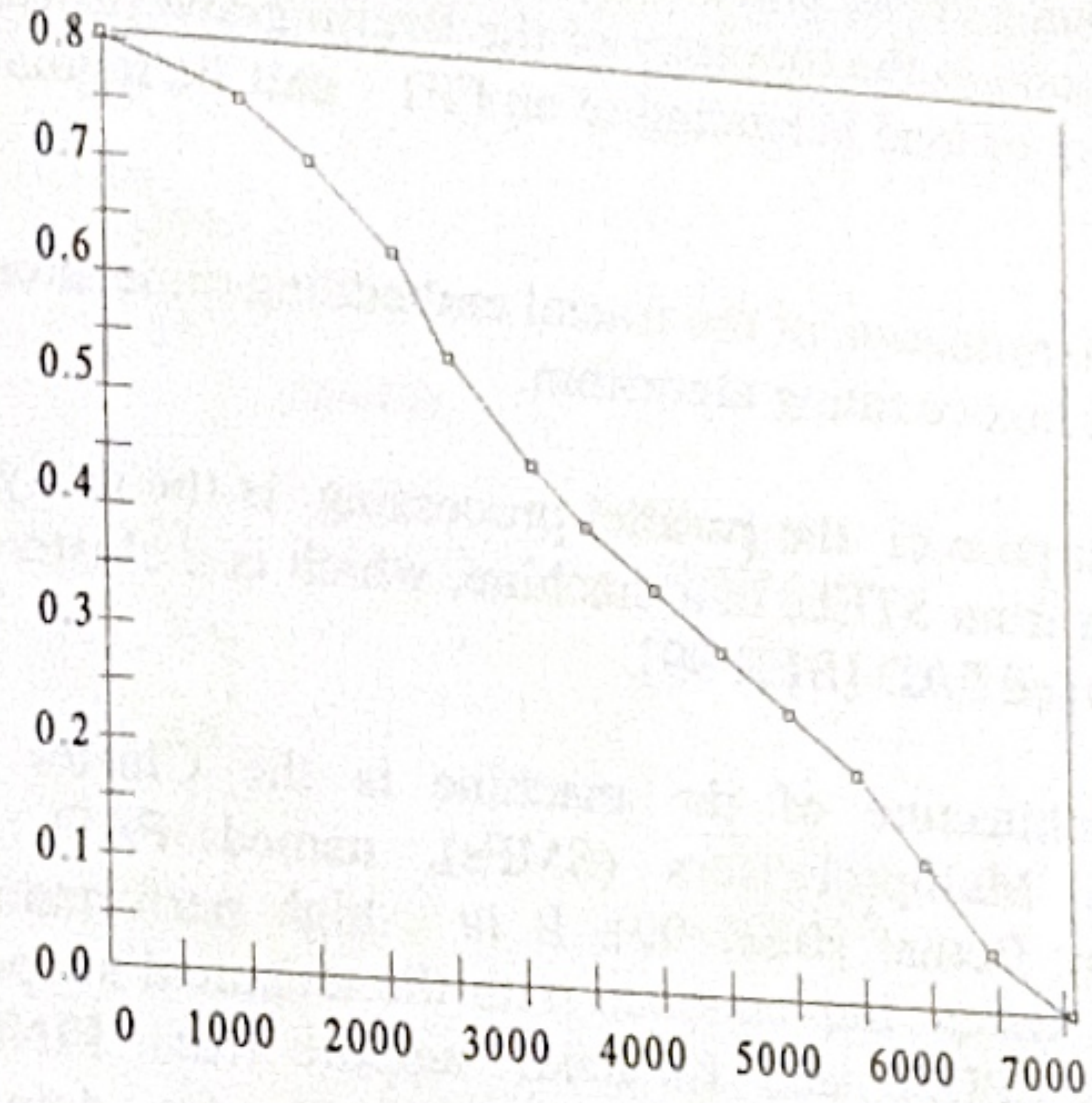


Fig. 3 – Power Spectrum (vertical) versus time series (horizontal), obtained by self-correlation algorithm.

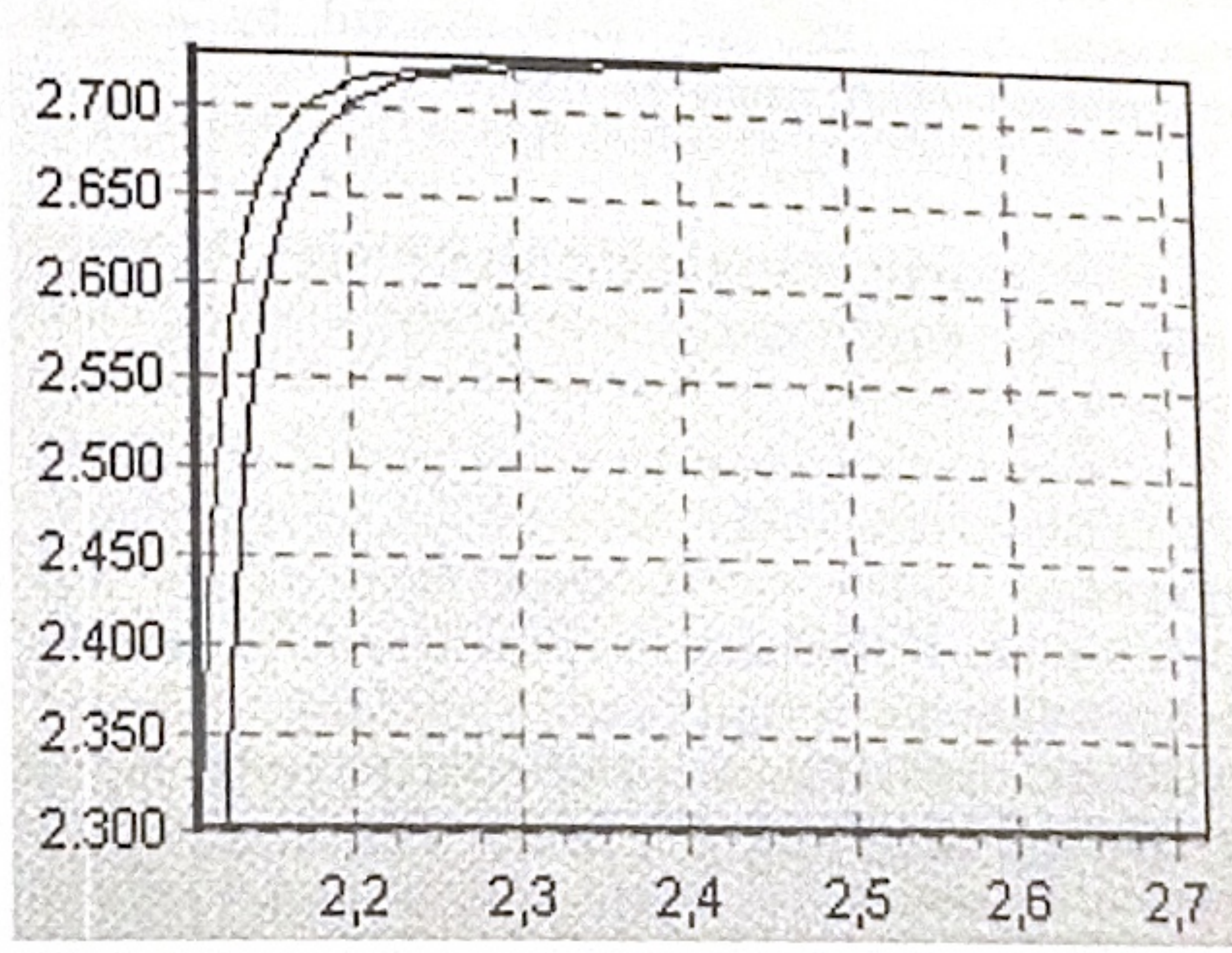


Fig. 4 – Energy values (vertical) versus embedding dimensions (horizontal).

Plotting these points in three-dimensional space with connecting line segments, we obtain the Rössler attractor.

B. Ljapunov Exponents.

The Ljapunov exponent characterizes the average logarithmic growth of relative error per iteration, expressed by:

In practice a small error in the initial point will be scaled

$$\lambda = \lim_{0\to\infty} \lim_{E_0\to\infty} \frac{1}{n}\sum_{k=1}^{n} \log(|E_k/E_{k-1}|)$$

by the factor of e^{λ} in each iteration. A positive exponent means that nearby orbits move away, while a negative one means approximation. So, the negative exponent is not expected for a chaotic attractor.

C.Fractals.

Traditional geometry defines the point as with dimension zero. If this point is moved, and we get all the generated points, we have a line, with dimension one. If this line is moved vertically to itself then we have a square, a figure with dimension two. In the same way, we can obtain a cube, with dimension three. It is possible continue this procedure, moving the cube to obtain another figure with superior dimension, although we couldn't see the final figure. As an example, we suppose a line which spreads over a surface without crossing itself. At the bound, this line will be filling the total plane, but in the traditional geometry this line is still a figure in one dimension, but we can perceive that it is near two-dimensional. Using this idea, Felix Hausdorff created the idea of fractionated dimension. Benoit B. Mandelbrot introduced the fractal dimension [MAN 82]. According to these ideas, it is possible to imagine dimensions as *1.58*, and *1.95* [MAT 97].

Given a self-similarity structure, which is concerned to the fractal structure, there is a relation between the reduction factor s and the number of pieces α into which the structure can be divided:

$$\alpha = 1/s$$

and

$$D = \log \alpha / \log(1/s)$$

where D is the dimension of self-similarity structure, or the fractal dimension.

There are several approaches to compute the fractal dimensions, and one of them is the Box Counting [FED 88].

In this method the fractal is covered by a grid of N_i^2 squares, and it is determined the number of squares needed to cover the fractal. In other words, it is verified the number of squares that include at least a pixel of the fractal image. In sequence, it is chosen finer grids composed by $N_1^2 < N_2^2 < N_3^2 <\ldots< N_m^2$ squares and calculated the respective number of squares $S(N_1)$, $S(N_2)$, $S(N_3)$, ..., $S(N_m)$ necessary to cover the fractal image. If $S(N) = N^D$, we can get the fractal dimension D as the angular coefficient of the line constructed by $\log S(N)$ against $\log S(1/N)$ plotting.

IV. APPLICATION TO THE SOLAR CORONAL DYNAMIC ANALYSIS

As stated earlier, in this paper we present some aspects of the image processing, and chaos theory, used to the analysis of the data from STEL (Solar Terrestrial Environment Laboratory) in Nagoya University, in a parallel machine.

The emergency of solving non-linear systems using chaos theory and the solution of innumerable problems using image processing approaches, leads to the use of specific machines, which explores the data and instruction parallelism. The dynamic system monitoring chaotic nonlinear systems, starting from equations or time series is quite common, but the use of combining techniques of image processing with chaos approaches isn't usual. In this paper it is discussed the combining techniques used to characterize signals obtained from solar explosion, from which it is calculated the embedding dimension, and the power spectra.

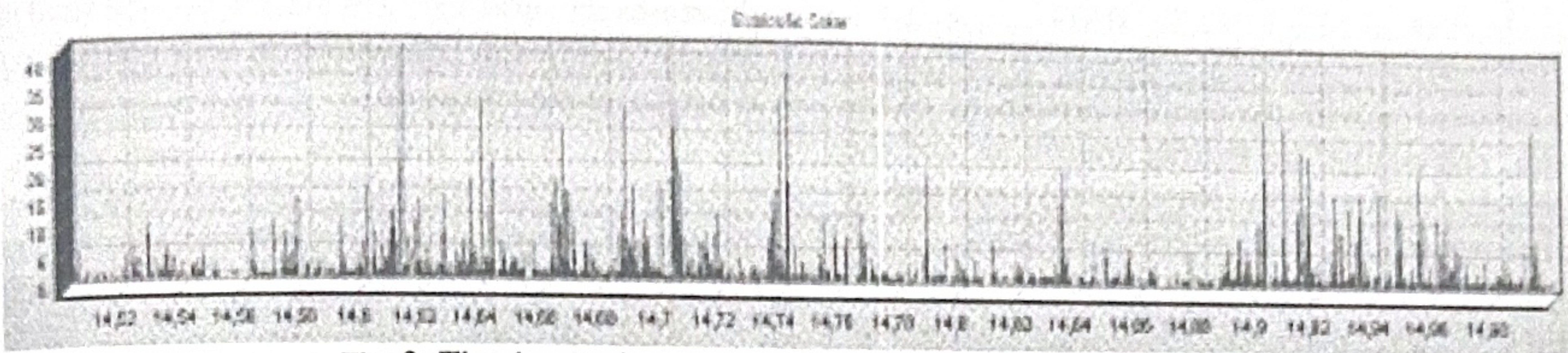
A very simple approach in linear systems is the use of filters [BRO 92], but the use of these methods depends on the signals in phase space in low dimensions. Our measurement of fractal dimensions to the solar explosion is *2.73* [MUC 99].

Algorithms of the reconstruction methods used originally to attractors as [LOR 63] and Rössler [RÖS 76], was used here to the solar explosion signals.

A. The Time Series of Solar Explosions.

The specific application developed in this work, used a time series associated to the explosion radiation, with the following specifications:

- Frequency: *164* MHz,
- Maximum flux : *9.92* SFU (Solar Flux Unit),



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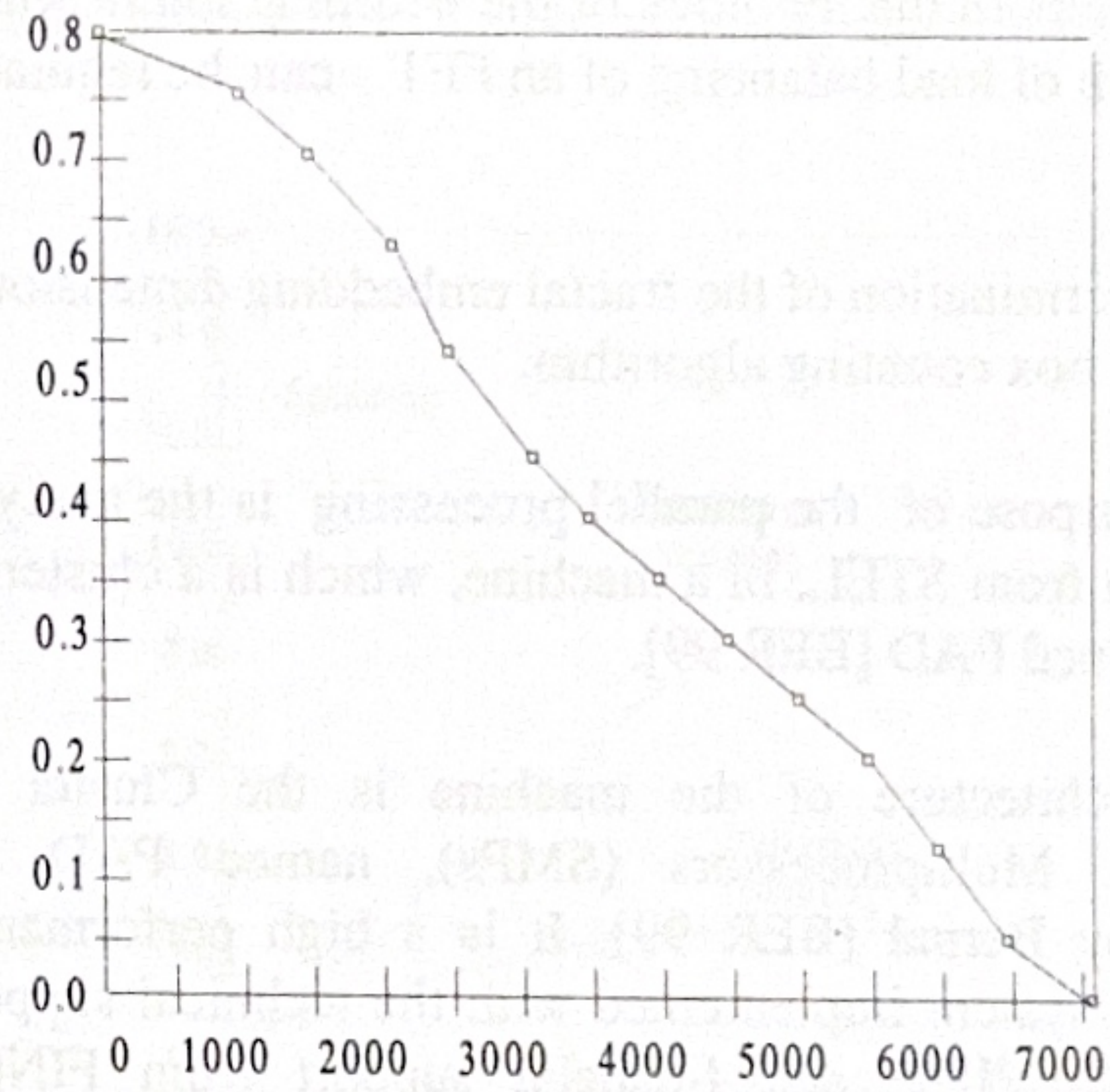

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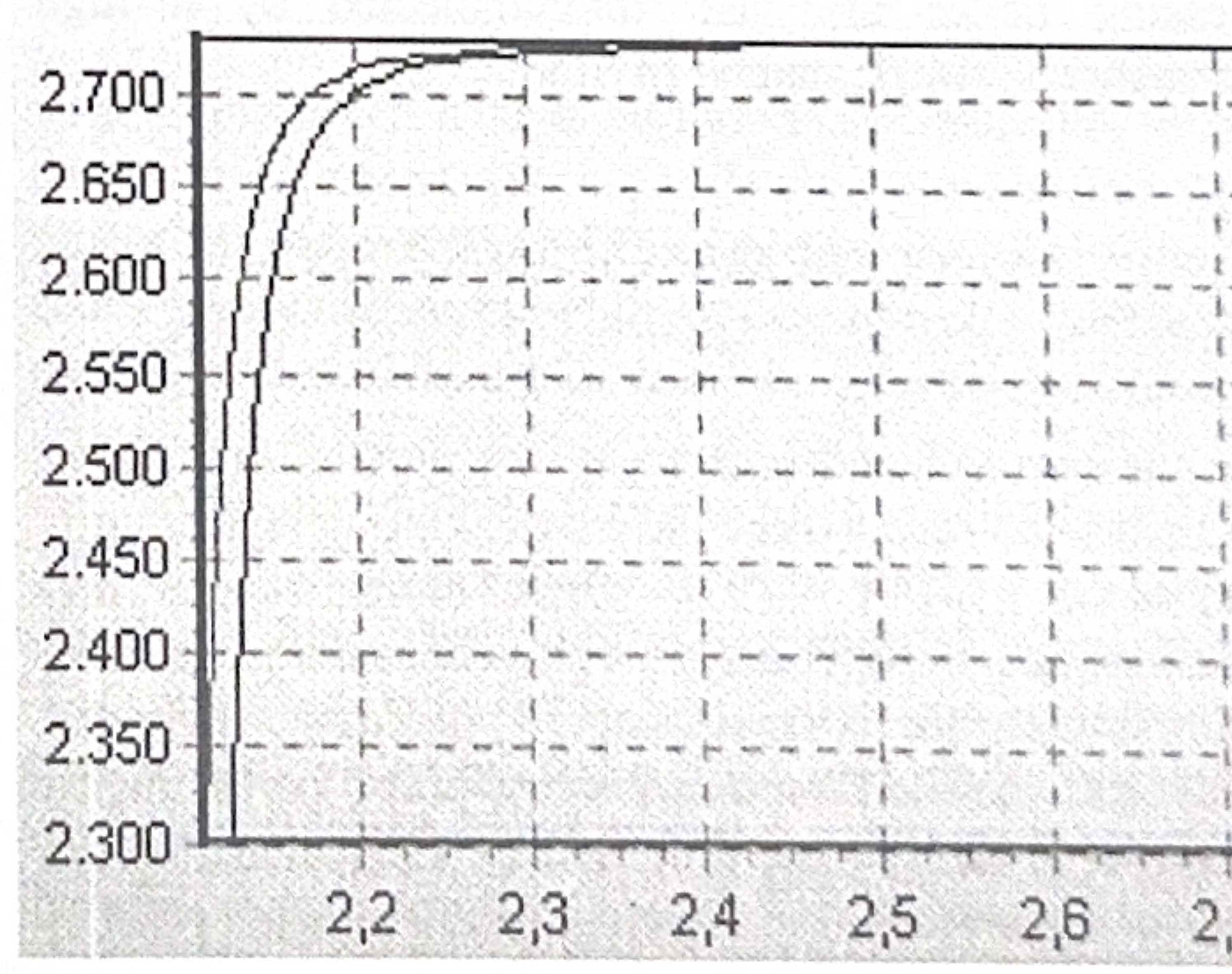

Fig. 4 – Energy values (vertical) versus embedding dimensions (horizontal).

V. PARALLEL PROCESSING

We are concerning with the parallel processing of the power spectra, calculating the Fourier transform, and the chaos approach, determining the fractal embedding dimension, followed by the Takens reconstruction.

In parallel computations the fast Fourier transform (FFT) can be well parallelized using the shuffle exchange network, but a clever combining of the nodes on the perfect shuffle results in the topology of the Bruijn graph in which the problem of load balancing of an FFT can be regulated [PIT 93].

The determination of the fractal embedding dimension it is used the box counting algorithm.

The purpose of the parallel processing is the analysis of the data from STEL, in a machine, which is a cluster of SMP's, named PAD [BER 99].

The architecture of the machine is the Cluster of Symmetric Multiprocessors (SMPs), named PAD, as describe by Bernal [BER 99]. It is a high performance computing system implemented with the technical support from LSI-EPUSP and financial support from FINEP (Financiadora de Estudos e Projetos). In its default configuration, the architecture is composed of eight processing nodes, one access workstation and one administration workstation, as shown in Fig. 5. Each processing node is composed by a SMP of two Pentium processors. All processing elements (workstations and nodes) are interconnected by a Fast-Ethernet network. The processing nodes are also interconnected by a high performance network, named Myrinet.

VI. PERFORMANCE ANALYSIS

To the performance analysis of the parallel processing we consider the case, where the Box Counting algorithm is implemented [MAT 97], where block transfer and processing time, of the fractal image, is considered. In these algorithm, there are two main steps: 1) the image is divided in grids, and verified the existence of pixels pertaining to the fractals, in each square unit; and 2) it is calculated the number of square units, in which at least one such pixel is present. The two steps described have the complexity of $O(n^2)$, considering the image with n^2 points. To each grid size, it is necessary to repeat the steps described, until the sufficient number m is obtained to the calculation of angular coefficient in the space of $log\ S(N)$ versus $log\ S(1/N)$, remembering that increasing the number m decreases the grid size. In the analysis the final step to the angular coefficient determination isn't considered, since it is used a single processor.

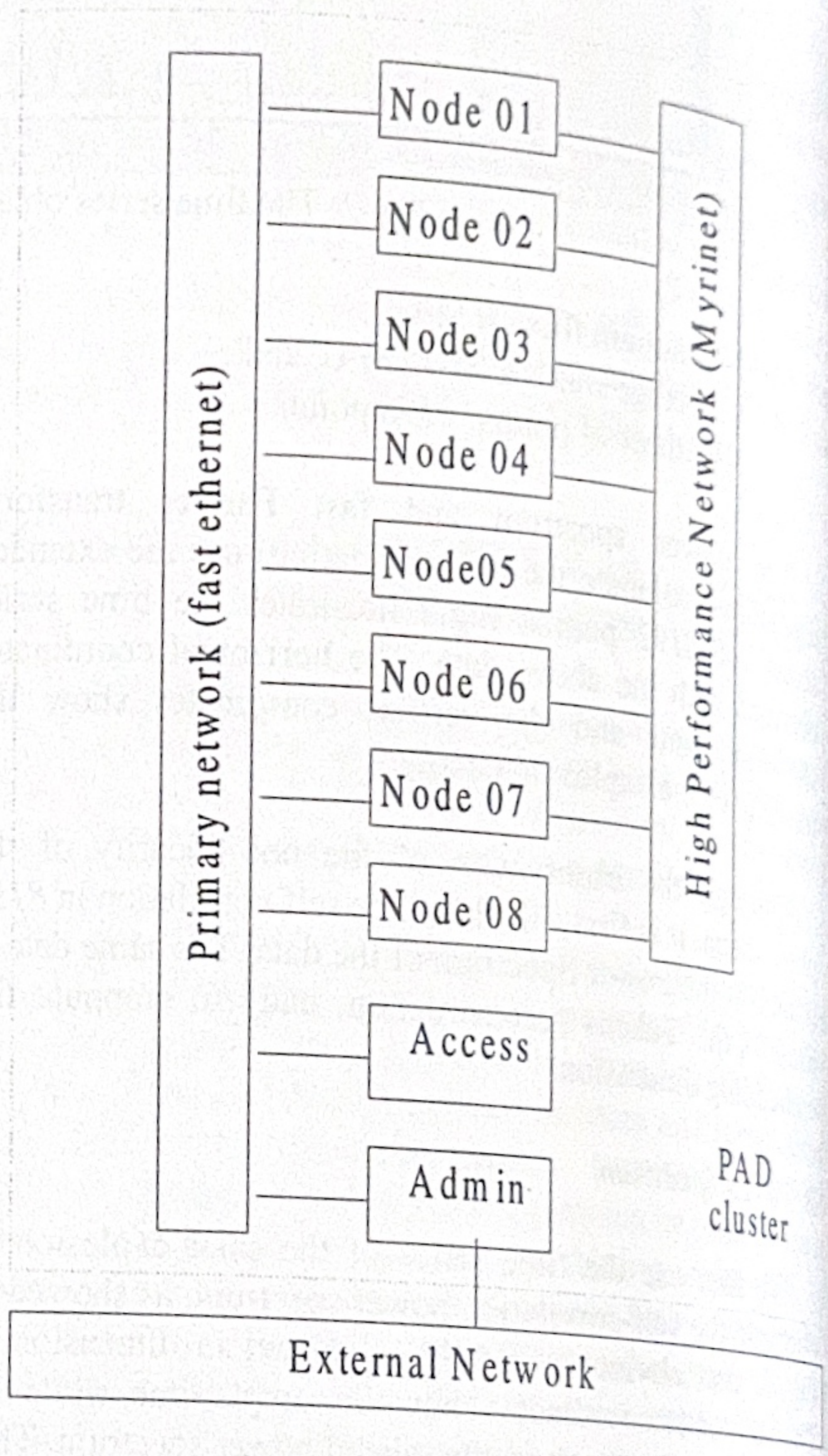

Fig. 5. PAD cluster.

The analysis is made varying the number of processors, with a square image with *256x256* points. There are two possibilities to apply the box counting algorithm : full image box counting and sub-image box counting, described as follows.

In Table I, it is showed the result of the first case, full image box counting, where the algorithm considers the transference of complete image to all p available processors, and each processor calculating the number of points in some grid size. The first column indicates the number of processors used; the next four columns corresponds to the processing time, when it is varied the grid number m; and the final column represents the speed-up, when $m = 16$. For example, when $p = 1$ and $m = 2$, a single processor takes care of the box counting with grid size 1 and 2; and when $p = 2$ and $m = 2$, one processor takes care of the box counting with grid size *1* and the other processor, with the grid size *2*. There is no reason to use $p = 4$ processors and the box counting with only $m = 2$, because in this case two processors will be idle, so the table I doesn't consider these cases. The speed-up is small in this case because of the high transference time of the full image to the processors.

TABLE I

PROCESSING TIME AND SPEED-UP, USING FULL IMAGE TRANSFER TO ALL PROCESSORS (FULL IMAGE COUNTING).

p	m=2	m=4	m=8	m=16	Speed-up m = 16
1	5.24	10.48	20.97	41.94	1.00
2	14.39	17.01	22.25	32.73	1.28
4		14.39	17.01	22.25	1.88
8			14.39	17.01	2.46
16				14.39	2.91

In Table II, it is showed the results of the sub-image box counting algorithm where each processor is responsible by the processing of all *m* grid size, considered to a specific sub-image of the fractal. For example, when $p = 1$ and $m = 2$, the single processor takes care of the grid dimension *1* and *2* as the case of the Table I; but when $p = 2$ and $m = 2$, one processor takes care of the box counting with grid size *1* and *2* of half of the image, for instance the left half, while the other processor, with the grid size *1* and *2* of the remaining right half. In the case of $p = 4$ processors, and $m = 2$, the image is divided in four sub-images, of *1/4* size, and each processor will be responsible by the processing of the box counting, with the grid size *1* and *2*. In this case, the image transfer overhead is reduced increasing the number *p* of processors, and the result is better than the first case.

TABLE II

PROCESSING TIME AND SPEED-UP, USING SUB-IMAGE TRANSFER TO THE PROCESSORS (SUB-IMAGE COUNTING).

p	m=2	m=4	M=8	m=16	Speed-up m = 16
1	5.24	10.48	20.97	41.94	1.00
2	8.50	11.12	16.36	26.85	1.56
4	4.25	5.56	8.18	13.42	3.12
8	2.12	2.77	4.07	6.67	6.28
16	1.05	1.38	2.03	3.33	12.59

In this analysis it is considered the processor execution time of 40 ns, and the typical 1024 bytes block transfer time of 0.184 ms.

We conclude that all the algorithms concerning parallel image processing, and Takens reconstruction of attractors, would have better performance if we could distribute the sub-images to the processors. In this case, the speed-up will be better than, using algorithms, which distribute the full images to the processors. Fig. 6 shows the speed-up of the parallel processor to the box counting algorithm.

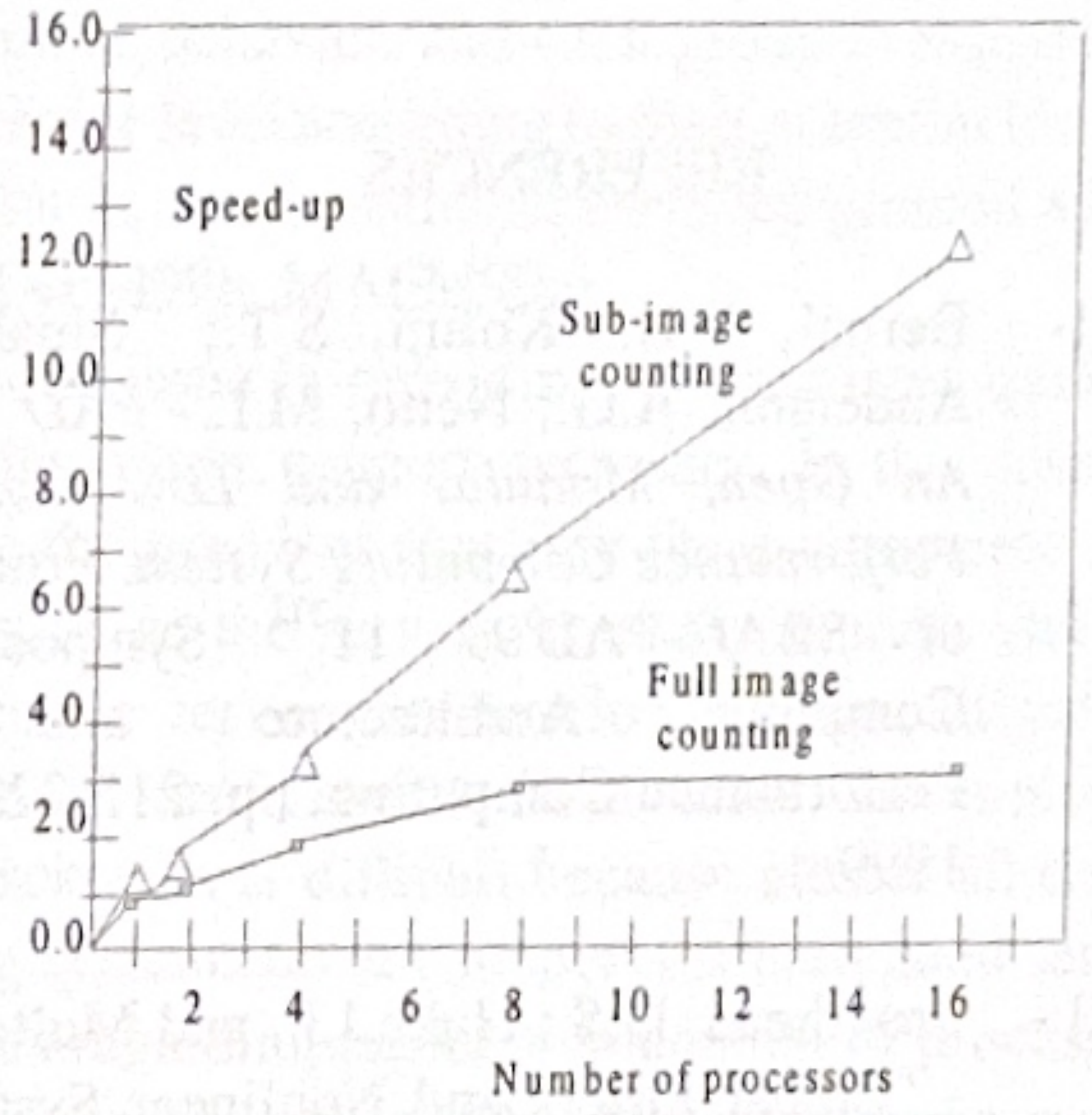

Fig. 6. Speed-up of parallel box-counting algorithm.

VII. CONCLUSIONS

It is presented a high performance computing using a cluster architecture. The methods uses conventional image processing algorithms to determine the self-correlation, and chaos theory to determine the strange attractors, of the solar explosion data. The use of combining techniques of image processing with chaos approaches isn't usual. In this paper, the combining techniques are used to characterize signals obtained from solar explosion, from which it is calculated the embedding fractal dimension, and the power spectra. The proposed techniques are being implemented in a parallel processing machine, PAD, and the performance is discussed. It is concluded that all the algorithms concerning parallel image processing, and Takens reconstruction of attractors, would have better performance if we could distribute the sub-images to the processors. In this case, the speed-up will be better than, using algorithms, which distribute the full images to the processors. This final conclusion is based on the performance analysis algorithm of the box counting algorithm, where both types of implementations is tested.

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