Cardiac Arrhythmia Detection in ECG Signals Using Graph Convolutional Network

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Abstract. *According to the World Health Organization, by the year 2030, 23.6 million people will die from heart disease. Therefore, automatic arrhythmia detection is highly desirable. The techniques based on neural networks have obtained outstanding results for this problem. The present work explores arrhythmia detection with Graph Convolutional Networks and Dynamic Time Warping to align the heartbeats. This is the first work to address the problem as a single graph with the heartbeats as nodes to the best of our knowledge. The results indicate that the approach is promising with a Positive Prediction of 100% for Supraventricular ectopic heartbeat detection and a Sensibility of 100% for Ventricular ectopic heartbeat detection with a global accuracy of 90%.*

Resumo. *Segundo a Organização Mundial da Saúde, até 2030, 23,6 milhões de pessoas morrerão de doenças cardíacas. Portanto, a detecção automática de arritmia é desejável. As técnicas baseadas em redes neurais têm obtido ótimos resultados para este problema. Este trabalho visa explorar a detecção de arritmias com Rede Convolucional de Grafos e Dynamic Time Warping para alinhar os batimentos cardíacos. Até onde sabemos, este é o primeiro trabalho a abordar o problema como um único grafo com os batimentos cardíacos como nós. Os resultados indicam que a abordagem é promissora com Predição Positiva de 100% para detecção de batimentos ectópicos supraventriculares e Sensibilidade de 100% para detecção de batimentos ectópicos ventriculares com acurácia global de 90%.*

1. Introduction

According to the World Health Organization (WHO), heart disease is the leading cause of death on the planet, taking an estimate of 17.9 million lives each year [Organization 2021]. Besides that, approximately 75% of cases occur in low and middle-income countries. The Electrocardiogram (ECG) is the main medical exam for heart disease diagnosis due to its examination simplicity [Cohen 1986].

One of the most common heart diseases is arrhythmia - an abnormal heartbeat that changes the ECG wave's morphology. It can be sporadic and harmless, or it can indicate a severe heart problem and must be detected as soon as possible. However, the detection process is laborious and error-prone for a physician, as it consists of a beat-to-beat analysis. An alternative to speed up this process is automatic arrhythmia detection.

Automatic arrhythmia detection based on machine learning methods have presented expressive results [Acharya et al. 2017, Kachuee et al. 2018, Hannun et al. 2019, Mousavi and Afghah 2019, Hammad et al. 2020]. However, most of the methods have poor performance when dealing with imbalanced databases, which is one of the biggest obstacles to their integration with medical equipment [Luz et al. 2016, Mousavi and Afghah 2019]. Furthermore, they present low accuracy for classes with a low number of samples. The majority of these works use the conventional ECG signal as input of their approaches.

Some works in the literature [Garcia et al. 2017, Queiroz et al. 2015] explored the transformation of ECG into graphs for Arrhythmia detection as an attempt to extract more information from the non-majority ECG classes. The work presented in [Garcia et al. 2017], proposed a Temporal Vectorcardiogram (TVCG), which is a 3D signal, wherein two ECG leads are considered, along with time as the third dimension. A graph structure is built based on TVCG, in which complex network techniques perform feature extraction. The experimental results indicated that the usage of these features presents promising results (as we mention in Section 2). The TVCG is a representation based on the Vectorcardiogram (VCG) [Llamedo and Martínez 2010], a 2D signal composed of two ECG leads disregarding time. In [Queiroz et al. 2015], the proposed approach applies complex network techniques to extract features from a VCG-based graph. Next, the authors feed an SVM classifier with the extracted features. The experiments revealed that the feature extraction procedure contains relevant information for arrhythmia detection.

In this work, we propose a new ECG graph representation, using only one lead and Dynamic Time Warping for heartbeat alignment. We model each heartbeat as a node on a graph and use a Graph Convolutional Network (GCN) to classify the nodes (a heartbeat). To the best of our knowledge, this is the first work in the literature to explore one lead of ECG classification with a GCN in a node classification fashion. We believe that this approach has advantages that justify the investigation: (i) we use only one ECG lead, (ii) The Dynamic Time Warping (DTW) technique applied provides a great measure to heartbeats' alignment, (iii) The GCN allows us to explore message passing through the nodes.

In the present work, the ECG is transformed into a graph and feed-forwarded to a GCN [Kipf and Welling 2016]. It is also verified if a GCN is able to extract more information from an ECG-based graph. Different from the approaches presented in [Queiroz et al. 2015] and [Garcia et al. 2017], we investigate if a specialized graph classifier can improve the ECG classification for non-majority classes, such as the Supraventricular ectopic beat (S) and, mainly, the Ventricular ectopic (V), which has the smallest number of samples in the dataset used in this work.

The experimental results indicated a global accuracy of 90%, a Positive Prediction of Supraventricular ectopic arrhythmia type (class S) of 100%, and a Sensibility of Ventricular ectopic arrhythmia type (class V) of 100%.

The remainder of this work is divided as follows. In Section 2, the related works are presented. The Dynamic Time Warping is described in Section 3 and the Graph Convolutional Network in Section 4. The proposal of representing a heartbeat as a graph is presented in Section 5. The experiments are described in Section 6 and the discussion of the results in Section 7. Finally, the conclusions reached with the proposed approach are presented in Section 8.

2. Related Works

Several authors have effectively contributed to the automatic arrhythmia detection problem.

In [Lin and Yang 2014], the authors proposed an automatic heartbeat classification method for arrhythmia detection and classification based on normalized RR intervals and morphological features. Their approach consists of signal preprocessing, feature extraction, and linear discriminant classification. First, the high-frequency noise and baseline drift is removed from the ECG input signal. Then, the feature extraction applies wavelet analysis and linear prediction modeling to derive the normalized RR intervals and two types of morphological features. Finally, the linear discriminant classifier combines the extracted features to the classification test. The authors applied their method to the MIT-BIH database and achieved a global accuracy of 93.00% under the inter-patient paradigm.

In [Garcia et al. 2017], it is developed a new ECG representation called Temporal Vectorcardiogram (TVCG), which is a three-dimensional signal composed of two distinct ECG leads and the time. The TVCG is turned into a complex network for feature extraction. Their approach employs an optimization stage based on particle swarm optimization to select the best features of the complex network. This stage also enables the fine-tuning of the Support Vector Machine (SVM) classifier for arrhythmia detection. The experimental results adopting the MIT-BIH database and under the inter-patient paradigm presented a global accuracy of 92.40%.

It is presented in [Mousavi and Afghah 2019] an automatic heartbeat classifier for arrhythmia detection applying the sequence-to-sequence model along with a Convolutional Neural Network (CNN). The sequence-to-sequence model architecture is composed of a Recurrent Neural Network (RNN) encoder and decoder. The encoder consists of Long Short-Term Memory (LSTM), which encodes the input signal. The decoder, on the other hand, computes the category of each beat of the input signal. The authors applied their approach to the MIT-BIH database and achieved a global accuracy of 99.53% under the inter-patient paradigm.

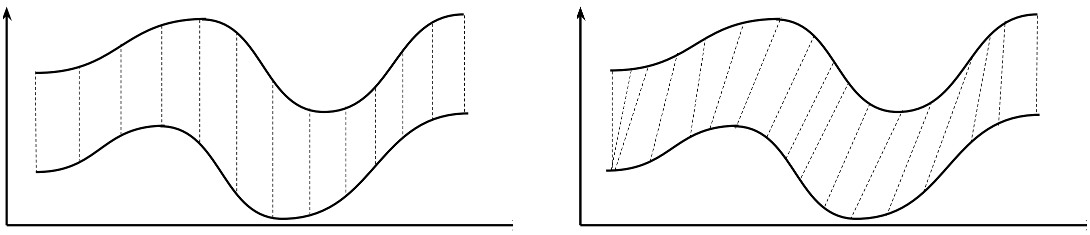
In [Hammad et al. 2020], the authors presented a multi-tier Deep Learning Model suffused with Machine Learning and optimization based on Genetic Algorithm (GA) for arrhythmia detection. The Deep Neural Network model extracts features of each patient. Then, the Genetic Algorithm determines the optimum combination of these features. Next, several classifiers, such as K-Nearest Neighbors (KNN), SVM, Multilayer Perception (MLP), are utilized to classify the features. The experimental results adopting the MIT-BIH database and under the inter-patient paradigm presented a global accuracy of 87.20% when the method used KNN along with GA.

To the best of our knowledge, no work in the literature represents each heartbeat as a node of a network and applies GCN for ECG classification, which makes our approach a novelty. Besides, it is advantageous to use only one lead because we do not always have access to more than one, combined with the fact that we have a reduced computational cost.

3. Dynamic Time Warping (DTW)

The Dynamic Time Warping is an algorithm to find an optimal alignment between two time dependent sequences by measuring the similarity between them [Müller 2007]. It detects similar shapes with different phases and thus minimizes the effects of shifting and distortion in the time-series [Senin 2008].

The Euclidean distance metrics and its variants are highly sensitive to temporal axis distortion, as can be seen in Figure 1(a). The DTW algorithm, on the other hand, addresses the time axis distortions issue through non-linear time-normalisation [Riedel et al. 2007], as can be seem in Figure 1(b).



(a) Euclidean distance **(b) Dynamic Time Warping**

Figure 1. Difference between the alignment used when comparing the time series using the Euclidean Distance and Dynamic Time Warping.

In order to find the best alignment between two time-series: A and B, the DTW searches a path through a cost matrix of dimensions i_A and j_B that minimizes the total distance between them, where i_A is the number of points of time-series A and j_B the number of points of time-series B.

The cost matrix is composed of the distances between each point of A in relation to every point in B and vice versa. The distances can be calculated by Euclidean distance, Manhattan distance or other distance metrics.

Equation 1 presents the time-normalized distance between A and B:

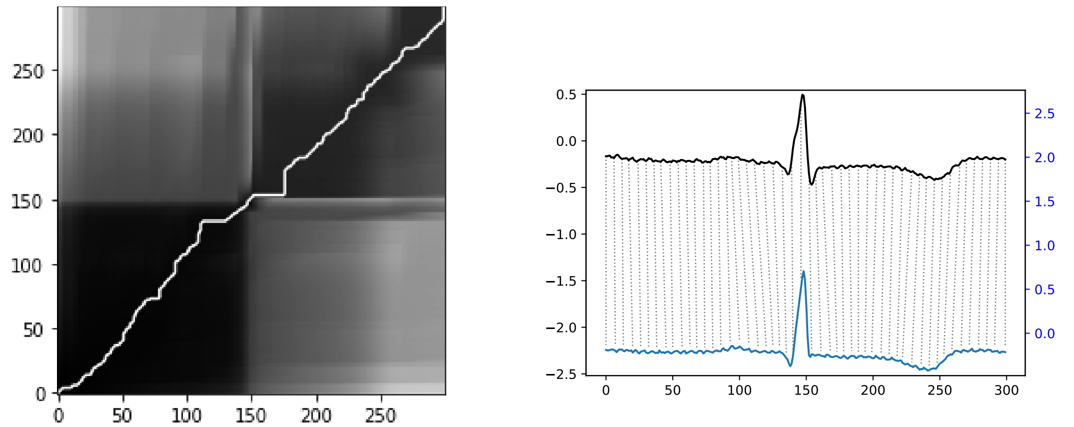
$$G(A,B) = \left[\frac{\sum_{s=1}^{k} d(i_k, j_k) \cdot w_s}{\sum_{s=1}^{k} w_s} \right], \tag{1}$$

where $d(i_k, j_k)$ is the distance between point i_k, from the time-series A, and point j_k, from the time-series B. w_s is a weighting coefficient.

The best alignment path between A and B is given by:

$$P = arg\, min(G(A,B)). \tag{2}$$

The Figure 2(a) presents a heat-map of the cost function of two different heartbeats, where the regions of low cost are indicated by dark colors and regions of high cost by light colors. The white line in Figure 2(a) is the path with lowest cost, which indicates the corresponding points of the first time-series in the second time series, as can be seem in the Figure 2(b).



(a) Heat-map of the cost function of two different heartbeats

(b) Corresponding points of the first time-series in the second one

Figure 2. DTW corresponding points of two heartbeats.

4. Graph Convolutional Network (GCN)

The Graph Convolutional Networks were firstly proposed in [Defferrard et al. 2016] to generalize Convolutional Neural Networks (CNNs) from regular Euclidean space domain data, such as image, video, and speech, to high-dimensional irregular domains, like data represented by graphs such as social networks and brain connections. In [Kipf and Welling 2016], the authors proposed a semi-supervised GCN, with message passing operations inspired by a linear approximation to spectral graph convolutions, followed by a non-linear activation function. The authors consider a multi-layer GCN with the layer-wise propagation rule presented by

$$H^{(l+1)} = \sigma(\tilde{D}^{-\frac{1}{2}}\tilde{A}\tilde{D}^{-\frac{1}{2}}H^{(l)}W^{(l)}) \ , \tag{3}$$

in which $\tilde{A} = A + I_N$ is the matrix A (adjacency matrix of the undirected graph G) with added self-connections, I_N is the identity matrix, $\tilde{D_{ii}} = \sum_j \tilde{A}_{ij}$ and $W(l)$ is a layer-specific trainable weight matrix. $\sigma(.)$ denotes an activation function, such as the $ReLU() = max(0,)$. $H(l) \in \mathbb{R}^{N \times D}$ is the matrix of activations in the l^{th} layer and $H^{(0)} = X$.

The convolution of a signal x with a filter $g_{\theta'}$ is given by

$$g_{\theta'} \star x \approx \sum_{k=0}^{K} \theta'_k T_k(\tilde{L})x \ , \tag{4}$$

in which $\tilde{L} = \frac{2}{\lambda_{max}}L - I_N$, λ_{max} denotes the largest eigenvalue of L. L is the normalized graph Laplacian defined as $L = I_N - D^{-\frac{1}{2}}AD^{-\frac{1}{2}}$, where D is the diagonal matrix with the degrees of each vertex, i.e., $D_{ii} = \sum_j A_{ij}$. $\theta' \in \mathbb{R}^K$ is a vector of Chebyshev coefficients. The Chebyshev polynomials are recursively defined as $T_k(y) = 2y\ T_{k-1}(y) - T_{k-2}(y)$, with $T_0(y) = 1$ and $T_1(y) = y$ [Hammond et al. 2011].

A GCN can be constructed by stacking multiple convolutional layers of the form of Equation 4, each layer followed by a point-wise non-linearity.

In the present work, it is considered a two-layer GCN for semi-supervised node classification on a graph. First, in a pre-processing step, we calculate $\hat{A} = D^{-\frac{1}{2}}\tilde{A}D^{-\frac{1}{2}}$. Then the forward model takes the form:

$$Z = f(X, A) = softmax(\hat{A}\ ReLU(\hat{A}XW^{(0)})W^{(1)})\ , \tag{5}$$

in which $W^{(0)} \in \mathbb{R}^{CxH}$ is an input-to-hidden weight matrix for a hidden layer with H feature maps. $W^{(1)} \in \mathbb{R}^{HxF}$ is a hidden-to-output weight matrix. The softmax activation function is applied row-wise and it is defined as $softmax(x_i) = \frac{1}{\mathcal{Z}}exp(x_i)$ with $\mathcal{Z} = \sum_i exp(x_i)$. For semi-supervised classification, the cross-entropy error is evaluated over all labeled examples presented in Equation 6:

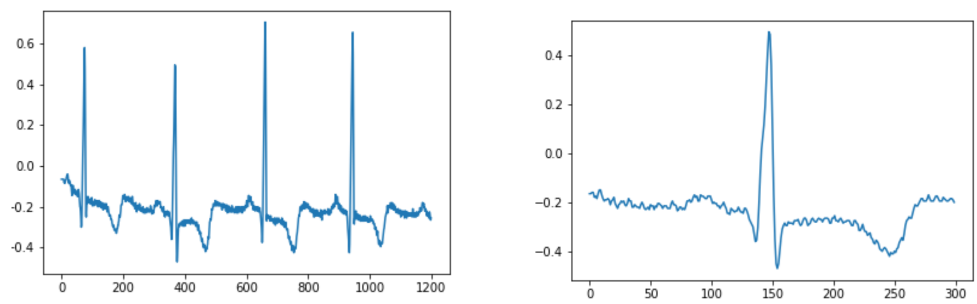
$$\mathcal{L} = -\sum_{l\,\epsilon\mathcal{Y}_L}\sum_{f=1}^{F} Y_{lf}\, ln\, Z_{lf}\ , \tag{6}$$

in which $\mathcal{Y}_L$ is the set of node indices that have labels and Y_{lf} is the set of labeled examples.

The neural network weights $W^{(0)}$ and $W^{(1)}$ are trained using gradient descent.

5. A Graph of Heartbeats

We convert the ECG signal into a graph, to apply the GCN to detect cardiac arrhythmia through ECG classification. The first step is to define the graph nodes. Therefore, a single node is a heartbeat like the one in Figure 3(b), which comes from the segmentation of an ECG signal like the one in Figure 3(a).



(a) ECG sinal **(b) Segmented heartbeat**

Figure 3. ECG segmentation.

Each heartbeat was segmented with 300 samples, centered on the R peak.

The node features is a list of the heartbeat values are represented as a list of heart-heartbeat raw samples (Figure 4).

We use Dynamic Time Warping (DTW) as the similarity algorithm to link nodes due to the optimal alignment of the heartheartbeats it provides. Two nodes are connected if the DTW value is less than a global threshold, which is empirically computed as we discuss in Section 6. The obtained undirected graph G provides the matrix A, mentioned in Section 4.

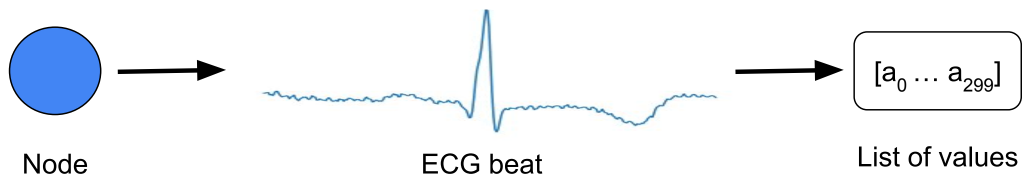


Figure 4. ECG node representation.

6. Experiments

6.1. The MIT-BIH database

The MIT-BIH database provides real clinical situations/scenarios. For this reason, this database is used in most works found in the literature. It consists of 48 annotated records obtained from 47 patients studied by the Beth Israel Hospital Arrhythmia Laboratory in Boston, USA, between the years of 1975 and 1979. Each record has 30 minutes ECG acquisition of two leads sampled at 360 Hz. The database has a total of more than 109.000 heartbeats. Each R peak of these heartbeats is labeled as a heartbeats type.

The experiments are conducted with the MIT-BIH database, following the inter-paradigm protocol proposed in [De Chazal et al. 2004], in which the database is split into a group called DS1, which is used for training and DS2, which is used for evaluation.

The signals from DS1 used for training are: 108, 114, 116, 118, 124, 201, 203, 205 and 207. and the ones used for testing (DS2) are: 208, 209, 215 and 223.

Among the classes of ECG presented in the MIT database, the present work focus on the classification of the 3 main classes: Normal heartbeat (N), Supraventricular ectopic heartbeat (S) and Ventricular ectopic heartbeat (V).

The number of samples used for training and testing are indicated in Table 1 and 2, respectively.

Table 1. Data used for training

Class	samples
(N)	300
(S)	300
(V)	300
Total	900

Table 2. Data used for testing

Class	samples
(N)	69
(S)	14
(V)	17
Total	90

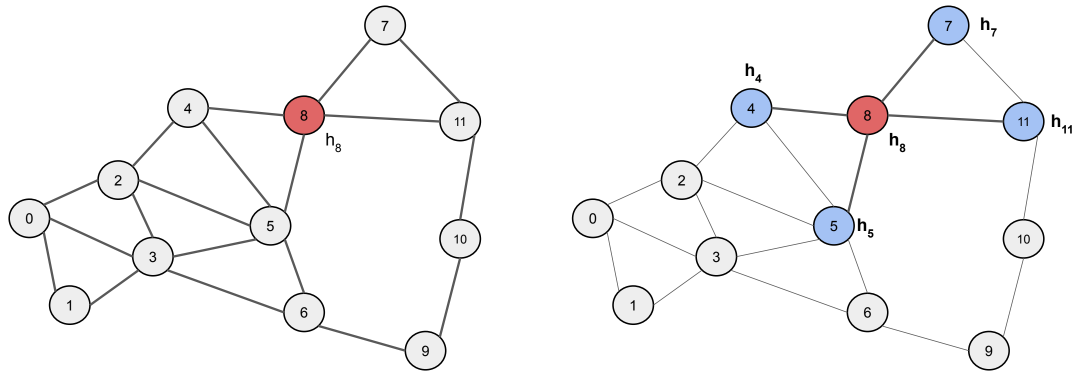
6.2. Graph Convolutional Network algorithm

The implemented Graph Convolutional Network has two layers. The model is trained in 100 epochs. The first hidden layer generates 100 hidden features and the second layer generates 3 output features corresponding to the number of classes of the classification problem. The Adam optimizer is used to optimize the weights with the learning rate of 10^{-4}.

6.3. Neighborhood Sampling for Node classification

The proposed GCN approach receives a single graph G to perform the training and testing, as discussed in Section 5. In some cases, the graph is so large that it makes its entire processing unfeasible because it consumes all the memory available. That is the case in this work, since the graph generated with all the ECG signals has more than a hundred thousand nodes with 300 features of float type. For this reason, a Neighborhood Sampling approach is implemented. This technique first takes a group of nodes, where the message passing (convolution) will be performed. The number of nodes of this group is defined according to the batch size. For example, considering batch size of 1, a single node is selected. Then, the graph G, which contains all the nodes, is transformed into a sub-graph containing only the selected node and its neighbors. Finally, it is sent to the GCN, where the convolution is performed.

Figure 5(a) shows this step with batch size equals to 1, in which node 8 is selected.



(a) Selecting node 8 for neighborhood sampling

(b) Selecting Neighbors of node 8 for neighborhood sampling

Figure 5. Neighborhood Sampling.

The Neighborhood Sampling approach used in the experiments results applies a batch size of 200.

6.4. Evaluation metrics

The metric used to evaluate the GCN is the Overall accuracy (Acc), and a confusion matrix for a better understanding of the results. We also evaluate the Positive Prediction $(+P)$ and Sensibility (Se) for the class S and class V,

$$Acc = \frac{TP_N + TP_S + TP_V}{\#samples}$$

$$+P_S = \frac{TP_S}{TP_S + FP_S} \qquad +P_V = \frac{TP_V}{TP_V + FP_V} \tag{7}$$

$$Se_S = \frac{TP_S}{TP_S + FN_S} \qquad Se_V = \frac{TP_V}{TP_V + FN_V}$$

in which TP_N is the number of normal samples correctly classified; TP_S is the number of class S samples correctly classified; TP_V is the number of class V samples correctly classified; FN_S is the number of class S samples classified as normal or class V; FN_V is the number of class V samples classified as normal or class S; FP_S is the number of normal and class V samples classified as class S; and FP_V is the number of normal and class S samples classified as class V.

7. Results and Discussion

The proposed approach has some limitations, such as the fact that the construction of the graph is computationally expensive. For this reason, it is not possible to use all the data available in the database to carry out the experiments. Therefore, the current evaluation is not ideal. However, we aim to investigate the feasibility of exploring the ECG classification problem as a single graph, considering each of its nodes as a heartbeat, which is challenging and unprecedented in the literature. We do not intend to compare our results with state-of-the-art methods, but rather to evaluate the robustness and feasibility of the proposed approach.

The global accuracy of the testing was 90.00%. Hence, a confusion matrix was built to observe the performance of the GCN in the classification of each ECG class. The confusion matrix of the testing results is shown in Figure 6.

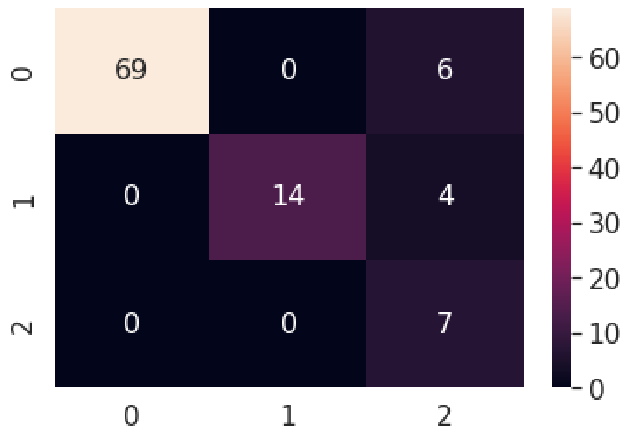


Figure 6. Confusion Matrix of the testing. 0 represents the Normal Class, 1 the S class and 2 the V class.

Through the Figure 6, we observe that all samples classified as classes N and S by the proposed approach were correct. However, the samples of class V were not fully correctly classified. The percentage of correct answers for the class V samples was 41.2%. Since the data was not normalized and the threshold was empirically defined as 0.8, the nodes of class V may have been impacted by data instability.

The analysis of the results indicates that the new representation of ECG through a graph is promising. It is noteworthy the quality of the connections of the nodes of classes N, that obtained 100.0% of positive prediction, compared to the classes S and N, that obtained accuracy of 100.0% $(+P_S)$ and 41.2% $(+P_V)$, respectively in the tests. Furthermore, the use of Neighborhood Sampling allowed not only the use of more samples, but also a faster execution. The sensibility was 92.0% for the class N, 77.8% (Se_S) for the class S and 100.0% (Se_V) for the class V.

8. Conclusion

Automatic arrhythmia detection is a complex process with room for improvement. GCN has emerged as a promising approach to several pattern recognition problems, however, it has not yet been explored for arrhythmia detection.

In this work, a new GCN is proposed for the task and Neighborhood sampling is also applied to the ECG graph. Besides that, it is important to measure two heartbeats' alignment to know if they can connect or not. For this task, we used the DTW algorithm.

Through the results, we verified that the implemented GCN algorithm got promising results for the classes N and S, both 100.0% of positive prediction. However, the algorithm underperformed for class V - 41.2% of positive prediction but 100.0% of sensitivity. We observed that class V nodes did not have good connections within the generated ECG graph, though.

In future work, we intend to implement approaches that allow the use of the entire database without overloading memory. In this way, it will be possible to explore the full potential of the proposed approach. In addition, future works include the definition of a better threshold and the normalization of the nodes features.

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