Patient Referral Trajectories in Acute Myocardial Infarction: A Complex Network Perspective

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Abstract. *This work investigates the application of complex network-based methods to improve the understanding of clinical trajectories for patients with Acute Myocardial Infarction (AMI). By modeling ambulatory care data from São Paulo as a graph, the analysis of patient referral patterns through the perspective of social network science indicates that referral dynamics are not strictly governed by a 'hierarchy of importance' or node degree, but highly concentrated spatially, typically organized in a regionalized manner or internally within specific health units. These results suggest that while major hubs coordinate AMI care, referral decisions may be driven more by geographical proximity and immediate availability than by institutional hierarchy. This research provides a baseline for future work in identifying specialist communities and analyzing the impact of socioeconomic factors on cardiovascular care pathways.*

1. Introduction

The study of patient referral trajectories through the healthcare system is essential not only for understanding of the progression of diseases, but also for a more profound analysis of the complex contextual factors that shape patient care. Elements such as workflow efficiency and the activity level of a health unit can directly impact time-sensitive scenarios, such as Acute Myocardial Infarction (AMI) [Noshad et al. 2021]. With that in mind, the present work investigates the application of complex network-based methods to AMI care, focusing on patient trajectory patterns within specialist networks.

Given the complexity of real-world healthcare systems, traditional indicators often rely on single aggregated measures that fail to capture the full care trajectory [Kelman and Friedman 2009, Noshad et al. 2022]. While such metrics provide valuable summaries, they may overlook the dynamic and relational dimensions along which patients move across specialist networks [An et al. 2017]. Moreover, AMI care is inherently time-sensitive and the sequence of referrals and clinical decisions can significantly influence myocardial recovery and long-term survival [An et al. 2018].

Representing healthcare systems as graphs has emerged as a promising alternative for modeling patient trajectories as interconnected, evolving processes rather than isolated events [Lomi et al. 2014, Lee et al. 2011], allowing the identification of clinical outcomes based on trajectory configurations. Building on this perspective, this study aims to investigate whether distinct patterns of clinical paths can be identified within specialist networks and how these patterns relate to AMI treatment outcomes, aiming to contribute to the literature by characterizing patient trajectories from the perspective of social network science. Specifically, our study is driven by the following Research Question (RQ): *How can social networks be used to improve our understanding of the clinical trajectory of patients with Acute Myocardial Infarction (AMI)?* Considering this RQ, the main goal of this work is to characterize and identify patterns in AMI patient trajectories, from a social network perspective.

2. Fundamental concepts and related work

Several studies have explored network-based and process-oriented approaches to represent and analyze healthcare systems. In this context, a network (or graph) is a mathematical representation composed of nodes (e.g., physicians, hospitals, or clinical events) and edges representing referrals, interactions, or temporal transitions, enabling the explicit modeling of complex relationships [Lee et al. 2011, Lomi et al. 2014, An et al. 2018]. In healthcare trajectory networks, patient referrals have been widely adopted as a measurable proxy for communication and collaboration among physicians across specialties. [An et al. 2018] constructed large-scale referral networks by connecting pairs of physicians whenever a patient visited both within a month, with no intermediate visits, extending earlier investigations of referral pathways [An et al. 2017] in a study that indicates that geographic factors, particularly distance, strongly shape referral patterns [An et al. 2018].

Complementary work uses process mining on event logs to reconstruct and evaluate care pathways. However, real-world healthcare data is often too complex for standard mining tools, frequently supported by summary metrics [Noshad et al. 2022]. To address this, [Rosa et al. 2023] proposed a complex network framework to model patient trajectories as event-based graphs and perform detailed process analysis, effectively handling high-volume data and capturing real-world care variability.

Taken together, these studies illustrate complementary perspectives on healthcare systems, spanning macro-level referral structures, micro-level process analysis, and event-based representations of care pathways. [An et al. 2018] capture the structure of physician collaboration; [Noshad et al. 2022] focus on event-level process analysis; and [Rosa et al. 2023] propose flexible network-based representations of patient trajectories. However, these approaches remain only partially integrated and are often applied to large-scale datasets with complete provider information or general clinical contexts.

In this work, we address these limitations by modeling AMI care trajectories using real-world data at the city level. Our approach adapts referral network construction to settings without explicit physician identifiers, representing providers as (specialty, unit) tuples, and combines structural and temporal analyses to investigate how referral patterns, particularly inter-unit transitions, relate to pathway duration and complexity.

3. Methodology

In this study, we used public ambulatory care data from the Brazilian Unified Health System (SUS) obtained through the *SIGA-Saúde* system of the São Paulo city government, consisting of clinical visit records collected throughout the year 2014 in São Paulo/SP. The dataset provides, among other attributes, a unique identifier for each patient, the attending physician's specialty, the health unit, the International Classification of Diseases (ICD) code, and the log date. Although ICD codes were not available for all records, we found 618 instances with code "I21" (AMI diagnosis). We then extracted all logs corresponding to these patients, yielding a subset of cardiac patients. After removing patients with only a single record (irrelevant for studying referral paths), the final subset comprised 550 patients, 58 unique physician specialties, and 408 unique ICD codes.

The methodology of this study is built upon the analysis of patient trajectories using a network model. We decided to follow a modeling approach similar to that of [An et al. 2018], as explained in Section 2. Therefore, we too implemented a referral network, although on a smaller scale – [An et al. 2018] used data from the United States during 2006-2011, while our study only focuses on a single city within a single year.

A patient trajectory network can be mathematically described as a directed graph $G = (V, E)$, where V and E are, respectively, a set of nodes and a set of edges that connect them. Since our data doesn't provide any unique identifier for physicians, we treated each unique `(specialty, unit)` tuple as a separate physician as a workaround. Consequently, each node $v_i \in V$ represents a "unique" physician identified by the combination of their specialty and the health unit where they practice, and each edge $(v_i, v_j) \in E$ represents a referral from physicians v_i to v_j. Since the same pair of physicians may have multiple patient referrals between them, we simplify the representation by weighting each edge with the total number of referrals observed from v_i to v_j.

We used a 60-day window to consider two consecutive occurrences of the same patient as a referral. Additionally, the edges in our referral network also contain two binary attributes: 'AMI' and 'inter-unit', which indicate, respectively, whether the referral was made directly after an AMI diagnosis and whether the patient was forwarded to a physician from a different health unit or not. As expected, we found that the same patient could have multiple distinct referral paths over the year. Since we are only interested in the analysis of AMI paths in this study, we removed any referral paths containing zero AMI occurrences. We also removed all instances of single-node paths. After that filtering, our final referral network consists of 847 "physicians" (nodes) and 2497 referrals (edges).

4. Results

This section presents preliminary results that already allow us to make important observations related to the use of network-based models and methods in the investigation of Acute Myocardial Infarction (AMI) care.

A core property of our dataset is that every patient included in the network construction was diagnosed with AMI during the course of 2014. For that reason, our referral paths cannot be used to relate network topologies to the occurrence of an AMI. However, we understand that a faster response time can often be crucial in a medical context [McNamara et al. 2006]. That prompted us to conduct a separate analysis of the patient referral paths, comparing paths with and without inter-unit referrals to assess their impact on overall statistics, which is presented in Table 1. Of the 410 patient referral paths, 333 (81.2%) have at least one inter-unit edge (we will refer to these paths as inter-unit paths from now on). Moreover, our findings show that inter-unit paths tend to last considerably longer, with an average duration (in days) nearly twice that of intra-unit paths.

In an initial analysis of the trajectory network, Figure 1(a) illustrates the degrees of the network. These distributions follow a pattern very similar to those observed in other social networks, frequently reported in the literature and used as benchmarks in social networks studies [Newman 2003]. The presence of a long tail indicates scale-free behavior, where a large number of connections are concentrated in a few nodes, while the majority of nodes exhibit a relatively low degree, suggesting that a small number of specialists act as hubs in AMI care. This finding serves as a validation of the social nature of the modeled network, serving as a baseline for subsequent experiments. As shown in Table 1, given that the average number of physicians involved in a referral path is smaller than its average length, one can argue that a patient's clinical trajectory is often characterized by repeated referrals to specialists who have previously treated them.

	Total	**Inter-unit**	**Intra-unit**
# paths	410	333 (81.2%)	77 (18.8%)
Avg. length	9.9	11.3	4
Median length	6	7	3
Avg. # physicians (nodes)	4.4	4.9	1.9
Median # physicians (nodes)	4	4	2
Avg gap for a referral (# days)	24.2	24	25
Avg time range (# days)	140.6	157.1	69.3
Median time range (# days)	129	149	62
Avg patient age	59.8	59.6	60.5
Median patient age	62	62	61
[male : female] proportion	52.1 : 47.9	50.8 : 49.2	65.2 : 34.8

Table 1. Main properties observed in the dataset

Rank Position	**Degree centrality**	**Eigenvector centrality**
1	Cardiologist Hosp. Dia Boi Mirim II	Nursing Tecnician Hosp. Dia Lapa
2	Cardiologist AMA Esp. Sta. Cecília	Nurse Hosp. Dia Lapa
3	Family Health Strategy UBS Vila Curuca	Nursing Assitant Hosp. Dia Lapa
4	Nursing Tecnician Hosp. Dia Lapa	Vascular surgeon Hosp. Dia Lapa
5	Cardiologist Hosp. Dia Mooca	Cardiologist AMA Esp. Sta. Cecília

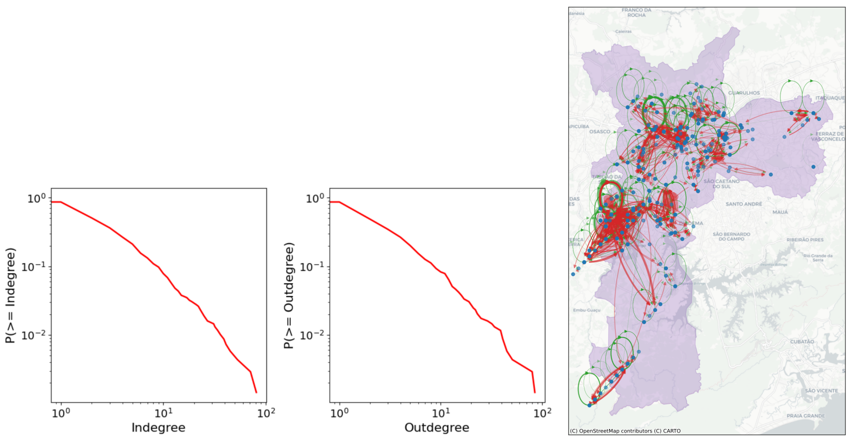
Table 2. Top-ranked nodes in the trajectory network.

We investigated the structural nature of the connections, considering reciprocity, i.e., the occurrence of reciprocal edges between the same pair of nodes; and assortativity, i.e., the degree correlation observed in the nodes linked by an edge. The analysis of the directed relations between physicians reveal a reciprocity index $\rho = 0.469$, indicating that almost half of the relations are reciprocal. In a complementary way, the investigation of the general assortativity[1] shows an index $r = -0.02$, a neutral assortativity that reveals an absence of homophily: there is no correlation between the degrees of the connected nodes. Notably, the reciprocity index aligns with values reported for other social networks, suggesting a relevant number of referrals from specialists that result in a return to the originating specialist. However, unlike social networks frequently reported in the literature, which exhibit relevant positive assortativity, the trajectory network described here indicates that the topological importance of the specialists is not relevant for determining referral relationships. Moreover, the neutral assortativity suggests that referrals are likely driven by geographical proximity or immediate availability rather than a strictly hierarchical preference, critical for time-sensitive conditions like AMI.

The investigation of the intra/inter-unit nature of the referrals is based on the principle that connection patterns between specialists can play a fundamental role in the study of the treatment of conditions in a specific population, complemented here by the study of the trajectory network. In this sense, Table 2 shows the Top-5 ranked specialists by Degree and Eigenvector centralities (sorted in descending order). The first and second lines of each cell present, respectively, the specialty and the associated unit. Notably, the rankings differ considerably, with few nodes appearing in both top-five positions, highlighting the value of multiple approaches for a complementary assessment. While

[1] Assortativity index ($\rho \in [-1, +1]$) assesses if the nodes tend to connect to nodes with similar degrees. $\rho \to +1 (\rho \to -1)$ indicates a relevant assortativity(disassortativity). $\rho \sim 0$ indicates a neutral assortativity.

cardiologists hold prominent topological importance in terms of degree, other specialties are more central in terms of eigenvector centrality, indicating a potential overload of these professionals in the referral network. This distinction suggests that nursing and vascular surgery professionals, despite having fewer direct connections, may occupy strategically critical positions in the network. Their role as bridges connecting highly connected specialists could make them potential bottlenecks, where overload might trigger cascading effects throughout the entire AMI care pathway. These findings highlight the need for interventions that extend beyond the most visible specialists to include these structurally important. This can also be investigated by observing the spatial distribution of referrals, as shown in Figure 1(b), which allows a visual identification of not only the predominance of intra-unit referrals (green edges) but also the patterns of inter-unit referrals (red edges), suggesting a possible hierarchical structure of care. Thus, while a more in-depth assessment of referral characteristics within a spatial context is beyond the scope of this work, it suggests the potential for a trajectory study that accounts for not only dynamic aspects but also economic and geographical factors related to the urban environment. Consequently, socioeconomic properties linked to different city regions could be combined with trajectory network characteristics, offering significant potential to reveal patterns that support decision-making for the global improvement of the healthcare system.



(a) (b)

Figure 1. (a) Degree distribution: Indegree (left); Outdegree (right). (b) Spatial visualization of the referral network.

5. Final remarks and future works

This study presented a network-based investigation of AMI patient trajectories, drawing on concepts from social network science. Using ambulatory data from the city of São Paulo, we modeled referral patterns as a directed, weighted graph. An investigation into the trajectory network revealed the scale-free nature of the topological organization of specialists in patient treatments. Furthermore, the referral relationships indicate no clear preference for node degrees in the formation of edges, sugegsting that, although there are major centers oordinating AMI treatment, the referral dynamics are not segregated by a 'hierarchy of importance'. An investigation from geographical perspective reveals a high spatial concentration of referrals. These tend to be organized in a regionalized manner or, even more significantly, internally within the units where treatment occurs.

It is important to highlight that, due to its "ongoing" status, this work only introduces our investigation into AMI patient trajectories through a network-based lens, rather than drawing any definitive conclusions. Several promising directions have been outlined to advance the understanding of this phenomenon: (i) identifying communities of specialists and relating them to the characteristics of the trajectories they take part in; (ii) examining the relationship between such specialist communities and the socioeconomic attributes of their corresponding geographic regions; and (iii) disaggregating trajectories to explore their temporal dynamics in association with referral features, including medical specialties, intra- or inter-unit transfers, and geographic location.

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