

# **cusp<sup>~</sup> — energy-based synthesis through cusp catastrophe in Max/MSP**

Leon Steidle<sup>1</sup>

<sup>1</sup>Laboratório de Música – Universidade de São Paulo  
Av. Prof. Lúcio Martins Rodrigues, 443 - Butantã, São Paulo - SP

steidleleon@gmail.com

**Abstract.** This article presents *cusp<sup>~</sup>*, an audio-rate external for Max/MSP that implements René Thom’s cusp catastrophe in real time as a nonlinear operator of energy transformation. Grounded in Gilbert Simondon’s theory of metastable ontogenesis and Ilya Prigogine’s irreversible thermodynamics, the object solves, sample by sample, the cubic equation  $x^3 + \alpha x + \beta = 0$  via an internal cubic solver, exposing the multivalent region ( $\Delta < 0$ ) where hysteresis and bifurcation become compositional parameters. We describe the mathematical modeling of the quartic potential, the performative mapping of the parameters  $\alpha$ ,  $\beta$ , and *width*, and the branch memory logic that ensures sonic coherence across the hysteresis cycle. Three test presets demonstrate the correspondence between theoretical predictions, spectrograms, and  $\alpha$ –state cycles, validating the device as an operator of sonic individuation. As a contribution, we provide (i) cross-platform open-source code, (ii) exemplary patches for real-time control of bifurcations, and (iii) a conceptual framework articulating catastrophe theory, musical energy, and contemporary sound design—offering the SBCM community a rigorous and reproducible tool for composing with energy gradients.

## **1. Introduction**

The conceptual shift mapped by Grabócz [1]—from the organicist metaphysics of the 19<sup>th</sup> century to contemporary matrices of morphodynamics, chaos, and fractal geometry—repositions musical energy from a merely expressive attribute to an ontogenic principle of sonic individuation. Form is no longer a homeostatic totality but operates in a metastable regime, where critical instabilities catalyze new configurations. The musical work thus becomes an energetic diagram.

This turn converges with Simondon’s critique of hylomorphism [2]: form and matter are poles of a transductive process in which pre-individual energy discharges as a virtual differential—and with Prigogine’s irreversible thermodynamics [3], in which energy functions as an operator of irreversible differentiation in systems far from equilibrium. Thom [4], by formalizing catastrophe theory, provides a topological lexicon (fold, cusp, swallowtail, etc.) for mapping such metamorphoses, converting them into a compositional grammar capable of dramatizing sudden transitions in perceptual states, crystallizing the passage from potential to actual. Andreatta [5] reinforces the musical applicability of this phenomenological pathway in Thom’s theory, granting epistemological rigor to catastrophe theory as an effective compositional tool. According to the author, Thom’s diagrams cease to be metaphors and

become operative germs capable of guiding compositional design.

However, the musical reception of this grammar has remained fragmentary, lacking a tool that, by combining the topological precision of bifurcation models with their real-time phenomenological intelligibility, renders them effectively performable. This article addresses this gap by introducing *cusp<sup>~</sup>*, an audio-rate external for Max/MSP that materializes, in digital signal, the cusp catastrophe operator. Ported from the Pure Data code by Gaffney and Smyth [6], the object solves, sample by sample, the state equation  $x^3 + \alpha x + \beta = 0$  via an internal cubic solver, exposing the multivalent region in which the discriminant becomes negative ( $\Delta < 0$ )—a condition that yields three real roots and thus controllable hysteresis. The full source code and example patches are available at: <https://github.com/leonsteidle/cusp-max-external>.

## **2. Theoretical Foundations**

The theoretical framework woven throughout this article brings together three complementary lines of inquiry. First, recent reconceptualizations of energy as a generative, metastable principle recast sonic form as a continuous process of individuation (Section 2.1). Second, catastrophe theory—exemplified by the cusp potential—offers a precise topological grammar for modeling critical transitions and hysteresis in dynamical systems (Section 2.2). Finally, a phenomenological–hermeneutic perspective on topology furnishes an epistemic bridge between mathematical models and the listening experience, positioning bifurcations as audible dramaturgical events (Section 2.3). Together, these strands supply the conceptual bedrock on which the *cusp<sup>~</sup>* operator is built, aligning real-time signal processing with a rigorous ontology of energetic form.

### **2.1. Energy and Individuation**

For Simondon (2005), potential energy is contained in a metastable system or medium, which is a pre-individual field characterized by being loaded with potentials and tensions. Individuation occurs as a resolution of this metastable state, through an operation that propagates this potential energy, allowing the individual to take shape and emerge: namely, through *transduction* [1]. In consonance, Ilya Prigogine (1996) demonstrates that, in systems far from equilibrium, energy functions as an irreversible operator of differentiation, sustaining dissipative structures [2]. These perspectives shift the focus from a substantial ontology to an allagmatic ontology of operations, in which

both Simondonian transduction and Prigoginian dissipation emerge as convergent modes of energetic individuation. Within this framework, the musical work appears as a genuine field of energetic gradients in constant reconfiguration, where each sonic gesture simultaneously embodies the resolution of pre-individual metastabilities and the constitution of new poles of order and difference.

## 2.2. Form, Singularity, and the Cusp

René Thom formalized topological bifurcations based on his work in cobordism. The transition from cobordism to catastrophe theory occurred when Thom reinterpreted geometric “borders” as loci of topological rupture, responsible for critical transitions in physical and biological systems. For Thom, borders represent boundary zones where qualitative changes—or singularities—occur in the evolution of a system. These singularities are precisely the points where different “sheets” of the potential meet and diverge, constituting true thresholds of topological individuation. In this sense, the border is an active agent that determines the emergence of new forms and regulates the passage from one state to another. Thom’s topological mediation—by transforming discontinuities into auditory or physical forms—mirrors Simondon’s insistence on the inseparability of matter, form, and information.

By formalizing topological bifurcations, René Thom (1972) redefines the notion of form as the structural stability of a potential [4]. In the case of the cusp, the potential  $V(x)$  presents a topology governed by the discriminant  $\Delta$ , which delineates two qualitatively distinct regimes: when  $\Delta > 0$ , the system is monovalent, with a single real root and no hysteresis; when  $\Delta < 0$ , multivalence arises, with three coexisting real roots—revealing a maximum degree of energetic multiplicity and enabling abrupt jumps between stable “sheets.” Energetically, each sheet corresponds to a local minimum of  $V$ , and the collapse of the discriminant (the cusp proper,  $\Delta = 0$ ) marks the threshold at which pre-individual energy “chooses” its trajectory of actualization, embodying the passage from the virtual to the actual. At this unstable critical point, the degeneration of the potential makes it possible for the system to decide which sheet to occupy, producing perceptible jumps and spectral hysteresis—granting the cusp its paradigmatic character as a site of critical transition.

This dialectic—where 1) cusp instability ( $\Delta = 0$ ) gives rise to morphological leaps, 2) multivalence ( $\Delta < 0$ ) opens a field of sonic possibilities, and 3) irreversibility guides the actualization of singularities—grounds the  $\text{cusp}^\sim$  operator, a device that materializes in real time the tension between potential energy and emergent form in computational music.

## 2.3. The “Second Path” of Thom’s Theory as Observed by Andreatta

Moreno Andreatta, taking up Jean Petitot’s distinction between a first path (mathematical-physical bifurcation theory) and a second path (hermeneutic-phenomenological) in the approach to Thomian theory, states that music offers the privileged laboratory of this second strand: from

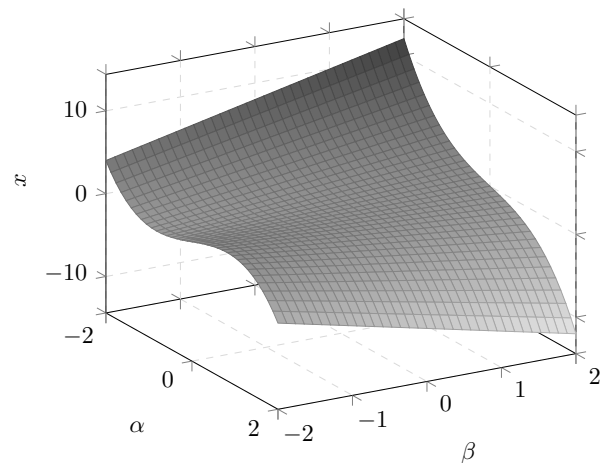


Figure 1: Surface of the cusp catastrophe

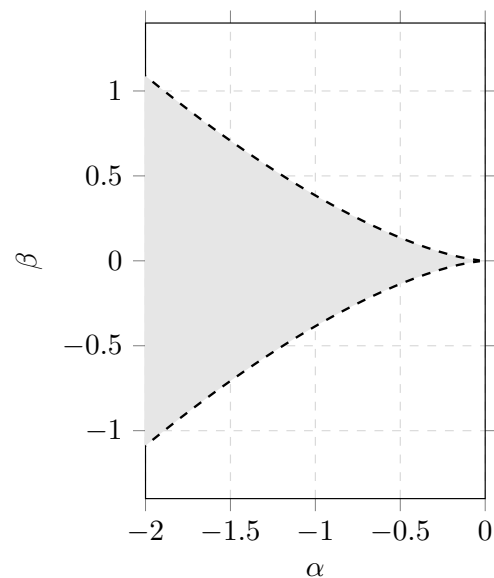


Figure 2: Hysteresis region ( $\Delta < 0$ )

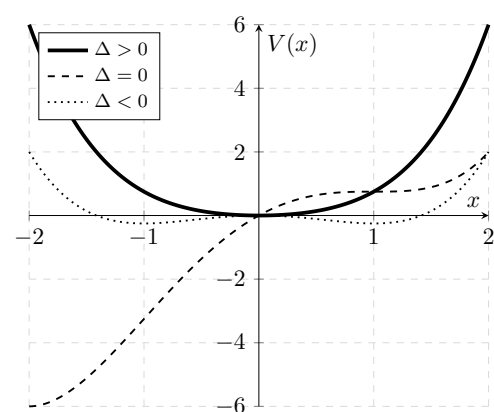


Figure 3: Quartic potential  $V(x)$  for the three discriminant regimes: monovalent ( $\Delta > 0$ ), cuspidal ( $\Delta = 0$ ), and multivalent ( $\Delta < 0$ ).

the audible contour of sonic forms, it is possible to deduce the topological germ that governs their underlying discontinuities [5]. It is the most systematic reinscription of

Thom's theory within contemporary mathematical-musical research.

The author begins with a diagnosis of the split that hindered the reception of Thom in the field of musicology: on one side, the axiomatic-structural vector (Forte, Lewin, the "French formal school"), whose combinatorial emphasis aligned with symbolic algebra; on the other, Thom's stance, critical of modern formalism and anchored in the primacy of emergent form. This fissure helps to understand why, between the 1970s and 1990s, *catastrophe* remained a peripheral concept in the face of the predominance of set-theory.

Andreatta's critical operation reveals the false dilemma underlying this opposition: alongside the "first path"—centered on mathematical bifurcations and critical transitions in physics—Thom sketches a "second path," of phenomenological-hermeneutic nature, which departs from morphological appearance to deduce latent internal dynamics. It is precisely along this path that music assumes a structural role: it offers audible fields in which discontinuities and emergences can be thought in their own dimension, without being reduced to numerical codification. By proposing this articulation, he shifts the debate beyond illustrative applications—which, since Risset, had proven punctual and scattered—conceiving an epistemological program in which differential topology, sonic morphology, and phenomenological critique are organically inter-implicated. Andreatta demonstrates that the phenomenology of sound—in which discontinuities and morphological leaps are directly perceived—constitutes a legitimate platform for reinscribing catastrophe theory at the core of compositional practice.

Three conceptual axes highlighted by Andreatta ground the approach developed here: topological mediation, which understands catastrophe as a translator of virtual energy into audible form; bifurcation as dramaturgy, in which folds and cusps are mobilized as instruments of narrative structuring in sonic space; and critical phenomenology, which affirms the inseparability between morphology and listening, shifting the analytical focus from absolute parameters to the dynamics of energetic force fields.

These three dimensions materialize paradigmatically in some compositional constellations that gravitate around the same morphogenetic sensibility. In French spectralism and its analytical-technological extensions, Hugues Dufourt [7]—as summarized by Andreatta—conceives sound as "an almost static form animated by infinitesimal vibrations": minimal perturbations are enough to trigger phase changes, while timbre, converted into a locus of symmetry rupture, causes new acoustic attractors to emerge—an operation entirely consonant with Thom's definition of form as disjunction inscribed in a continuous medium.

The same logic governs François Bayle's acoustic listening [8], understood as a "region of sonic influence" where microtemporal variations function as auditory singularities capable of reorganizing the internal topology

of the *acousmôn* space, and the transformational architecture in object networks by Horacio Vaggione [9], in which parametric flows—grain, spectral density—guide the music-listener system through critical zones of singularity, activating new formal configurations.

Romanian spectralism, which includes Aurel Stroe, Horațiu Rădulescu, and Iancu Dumitrescu [10, 11, 12], shifts the focus to morphogenesis as a compositional principle: already in the 1970s, Stroe coined the term *muzică morfogenetică* to designate works governed by asymmetries, bifurcations, and metastable states, explicitly anchoring it in René Thom's catastrophe theory. In lectures and writings, he describes musical form as a nascent-growing-agonizing trajectory traversed by collapses (*prăbușiri*). Rădulescu develops the concept of *sound plasma*—sound as living matter subject to "critical ruptures" within the spectrum itself; Dumitrescu, in turn, describes sound as "a being that is born, develops, and dies while perpetually transforming all its components."

In all cases, the work is conceived as a process of modulation between catastrophic singularities, in which form reorganizes in real time, preserving the dialectic between continuity and rupture. In all these strands, the multiplicity of catastrophic singularities functions as a degree of compositional freedom, allowing the work to modulate between differentiated states, keeping the dialectic between form and content pulsating.

These principles justify the implementation of  $\text{cusp}^\sim$  as a device capable of rendering audible—and controllable in real time—the differential logic that governs energetic individuation.

### 3. Modeling and Implementation

The design of the  $\text{cusp}^\sim$  operator unfolds on two complementary fronts. First, a mathematical layer formalises the elementary cusp catastrophe: it defines the quartic potential, its cubic state equation, and the discriminant that separates monovalent from multivalent behavior. Second, a software layer translates this abstraction into an audio-rate DSP routine that solves the cubic equation sample by sample, maintaining the strict latency requirements of interactive computer-music practice. This split preserves theoretical transparency while ensuring real-time responsiveness.

#### 3.1. Mathematical Modeling of the Cusp

The mathematical backbone of the operator comprises three key elements. (i) The quartic potential produces the cubic state equation that determines the system's equilibrium points. (ii) The discriminant partitions the control-parameter plane, showing where a single root (stable behavior) or three roots (metastable, hysteretic behavior) will occur. (iii) A scaling parameter widens or narrows the multivalent band without altering the topology itself, giving composers fine control over the range of critical transitions. Together, these components map energetic gradients to audible phenomena—such as branch memory and abrupt spectral jumps—while remaining stable at audio sampling rates.

### 3.1.1. Potential and State Equation

The `cusptilde` operator is based on the quartic potential:

$$V(x) = \frac{1}{4}x^4 + \frac{\alpha}{2}x^2 + \beta x \quad (1)$$

where  $x$  is the system variable and  $(\alpha, \beta)$  are control parameters. From  $V(x)$ , the state equation is obtained by differentiation:

$$x^3 + \alpha x + \beta = 0 \quad (2)$$

which acts as the core of the cusp catastrophe by prescribing the states of the dynamic system that we control sonically or visually.

### 3.1.2. Discriminant and Bifurcation Regimes

The qualitative behavior of the solutions of (2) is governed by the discriminant:

$$\Delta = 4\alpha^3 + 27\beta^2 \quad (3)$$

When  $\Delta > 0$ , the system exhibits monovalence, that is, there is only one stable root and no branch memory phenomenon; whereas for  $\Delta < 0$ , three real roots coexist, characterizing the condition of multivalence and energetic metastability, allowing jumps between stable “sheets”. Finally, at the critical point of the cusp, defined by  $\Delta = 0$ , two of these roots coalesce and a bifurcation jump occurs—the hysteresis phenomenon—which marks the qualitative transition between the monovalent and multivalent regimes.

### 3.1.3. Scale Parameter `width`

To modulate the “opening” of the multivalent region without altering the fundamental shape of the potential, we chose to retain, as proposed by Gaffney & Smyth [6], the scale factor  $w$  (`width`) applied to the linear term, resulting in:

$$V_w(x) = \frac{1}{4}x^4 + \frac{\alpha}{2}x^2 + \beta wx \quad (4)$$

The corresponding derivative yields the modified cubic equation:

$$x^3 + \alpha x + \beta w = 0 \quad (5)$$

whose  $\Delta < 0$  region can be widened or narrowed by adjusting  $w$ . In this way, by varying  $w$ , one adjusts the size of the  $\Delta < 0$  region in the  $(\alpha, \beta)$  plane, allowing for fine control over the width of the metastable band.

### 3.1.4. Resolution Process

At each audio sample, `cusptilde` solves (5) via an internal cubic solver, calculates the discriminant  $\Delta$  to identify whether the system is in a monovalent or multivalent regime; then, the order of root calculation is determined according to this diagnosis, and the appropriate “sheet” of the potential to be output is selected using the hysteresis logic described in the following section. This procedure

ensures that the object translates energy gradients into precise morphological jumps in real time, maintaining coherence between the theoretical formulation and its DSP implementation.

## 3.2. Implementation of the `cusptilde` Object

The implementation of the `cusptilde` object derives directly from the original C code compiled in Pure Data by Gaffney & Smyth [6], having been reused in C for integration with *Max/MSP*. Two main elements are required: the Cycling ’74 Max SDK 8.2 (DSP integration and API) and a C toolchain compatible with the SDK. Cubic roots are computed by an internal solver; no external math libraries are required.

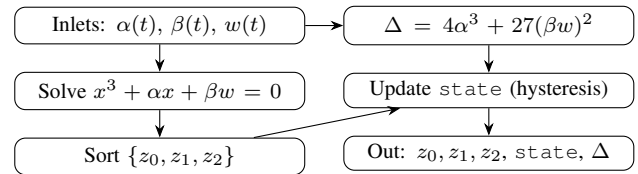
### 3.2.1. Data Structure and DSP Cycle

The fundamental data structure, defined as:

```
typedef struct _cusp {
    t_pxobject obj;
    double a, b;
    double width;
    double z[3];
    int state_var;
} t_cusp;
```

includes the field `obj` for the Max/MSP DSP object, the control parameters  $\alpha$  and  $\beta$ , the variable `width` which adjusts the bifurcation scale, the three-element vector  $z$  where the computed roots are stored, and the variable `state_var` which maintains the branch memory (value 0 or 1) to implement hysteresis.

In the audio processing flow, within the function `cusp_perform64`, the values of  $\alpha$ ,  $\beta$ , and  $w$  are read for each vector; next, the cubic is solved by the internal solver; then the three roots are ordered by the internal function `sort3`; the field `state_var` is updated according to the observed hysteresis behavior; finally, the routine outputs the three  $z$  values ( $z_0$ ,  $z_1$ , and  $z_2$ ), the current state, and the value of the discriminant.



**Figure 4: Compact DSP cycle of `cusptilde`: solve cubic, compute  $\Delta$ , hysteresis, outputs.**

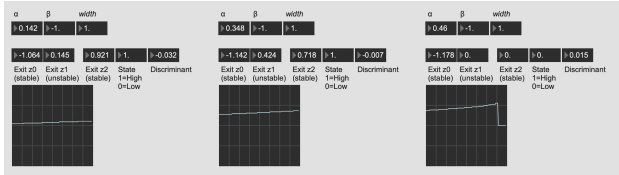
Regarding the input and output ports, the object provides three audio inlets: the first receives  $\alpha$  (quadratic coefficient), the second receives  $\beta$  (linear coefficient), and the third controls the bifurcation scale (`width`); it offers five outlets for  $z_0$ ,  $z_1$ ,  $z_2$ , `state`, and the discriminant.<sup>1</sup>

<sup>1</sup>Build (macOS): clone the Max SDK (`git clone https://github.com/Cycling74/max-sdk.git`); place `cusptilde.c` under `max-sdk-base/source/audio/cusptilde/`; build (`cmake -DCMAKE_OSX_ARCHITECTURES="arm64;x86_64" .. && make`); codesign (`codesign -s - cusptilde.mxo`); copy to `~/Documents/Max 8/Packages/cusp/externals/`. Full steps in the repo: [github.com/leonsteidle/cusp-max-external](https://github.com/leonsteidle/cusp-max-external).

## 4. Results

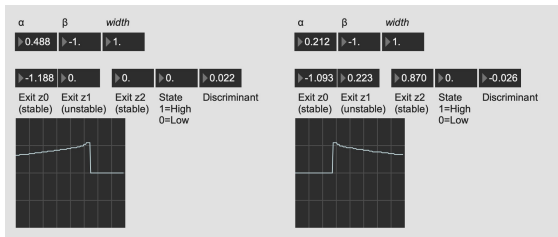
To verify the behavior of `cuspt`, the provided patch already includes three test presets selectable via buttons: preset 1, preset 2, or preset 3. Each configuration was executed at audio rate (48 kHz, 64-sample vector). Below, we present the theoretical expectation of each preset and the main empirical results extracted from the spectrograms, hysteresis cycles, and discriminant measurements.

In **preset 1 (vertical sweep)**,  $\beta$  is set to  $-1$  and  $\text{width}$  to  $1$ , while  $\alpha$  is slowly swept from  $-0.5$  to  $+0.5$  at  $0.1$  Hz; theoretically, the emergence of three real roots is expected for  $|\alpha| < 0.384$ , and a single root outside this range, which is confirmed in the spectrogram (Fig. 5). In the patch, this is clearly observed through the abrupt change in the object’s output behavior when entering and exiting the critical zone, revealing a more complex (metastable) dynamic in the tri-root region, in contrast with the stability and uniformity of the monostable behavior outside this critical interval.



**Figure 5: Preset 1 – vertical sweep ( $\beta = -1$ ,  $\text{width} = 1$ ,  $\alpha$ :  $-0.5 \rightarrow +0.5$ ; transition segment only).**

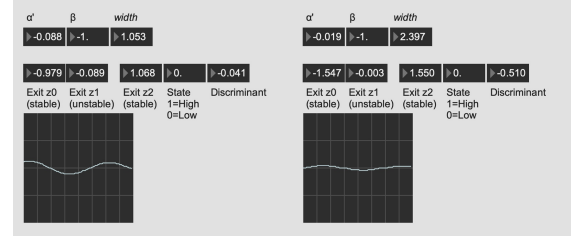
In **preset 2 (hysteresis cycle)**,  $\beta$  is kept at  $-1$ ,  $\text{width}$  at  $1$ , and a sweep is performed over the interval  $\alpha$ :  $-0.5 \rightarrow +0.5 \rightarrow -0.5$  at  $0.1$  Hz; the prediction of a complete hysteresis loop with switching at  $\alpha_h = \pm \frac{2}{3\sqrt{3}} \cdot \sqrt{|\beta w|}$  appears clearly in the graph of  $\alpha$  versus output state, with transitions at approximately  $\pm 0.385$  forming a loop whose area is about  $0.051 \alpha^2$  units (Fig. 6), validating the flip-flop mechanism of branch memory.



**Figure 6: Preset 2 – hysteresis cycle ( $\beta = -1$ ,  $\text{width} = 1$ ,  $\alpha$ :  $-0.5 \rightarrow +0.5 \rightarrow -0.5$ ; transition segment only).**

In **preset 3 (variation of width)**, a slow variation was applied around  $\alpha = 0$  (between  $-0.1$  and  $+0.1$  at  $0.1$  Hz) while the  $\text{width}$  parameter was gradually increased from  $0.5$  to  $4$ . This parameterization clearly reveals the effect of narrowing the metastable region as  $\text{width}$  progressively increases. The comparative spectrogram (Fig. 7) clearly shows this effect through the narrow-

ing of the transition zones and the intensification of granular textures associated with root multiplicity, while maintaining good stability and clarity in the sonic results.



**Figure 7: Preset 3 – width variation ( $\alpha \approx 0$ ,  $\text{width}$ :  $0.5 \rightarrow 4$ ; transition segment only).**

## 5. Discussion

Instrumentalizing the cusp in an MSP object amounts to equipping the composer with a true operator of individuation, as each sweep of the parameters  $\alpha$  and  $\beta$  reveals how the performative contingent brings forth actualities—or sonic jumps—from a virtual substrate embodied by the polynomial field. In energetic terms,  $z(t)$  are the instantaneous critical points of  $V$  (the real roots of  $V'(x) = x^3 + \alpha x + \beta$ ). By jumping between these “sheets,” the system produces perceptible entropy pulses that materialize the passage from virtual to actual; the unstable outlet  $z_1$ , in turn, symbolizes the multiplicity inherent to being—“more-than-one” [2], in close dialogue with Simondon’s notion of the pre-individual. In this way, `cuspt` functions as an organ of ontological mediation that connects the differential algebra of the cusp to sonic matter, unveiling the topological physics underlying aesthetic experience.

By embedding René Thom’s Cusp Catastrophe directly inside the MSP signal chain, `cuspt` translates, at the level of musical creation, what Ferraz [13] calls *tempo-música*: a metastable zone where energetic flows collide and, in doing so, give rise to form. Through the allagmatic lens of Simondon, `cuspt` becomes a transductive operator that installs, between silence and sound, a permanently unfinished in-between.

Future perspectives include migrating cubic resolution to GPU-based architectures, enabling applications in high-order ambisonics and massive rendering of energetic fields; integrating the operator into generative neural networks, acting as an explicit bifurcation module to enrich conditional creative processes; and incorporating informational metrics—such as Shannon entropy and Lempel–Ziv complexity—to objectively quantify the perceptual gain resulting from hysteresis. These directions strengthen the hypothesis that differential topology can function as a unified grammar for synthesis, control, and analysis at the frontier of contemporary computational music.

## Software Availability

The complete implementation of `cuspt` external, together with example patches and installation instructions, is

publicly available at: <https://github.com/leonsteidle/cusp-max-external>. A reproducible listening patch is included in the repository as `listening_demo.maxpat`. It implements the mapping used in the conclusion: `z0` drives the `svf~` cutoff (mapped to 750–3500 Hz, with short smoothing via `slide~`), and  $(1-|\Delta|)$  drives the `svf~` resonance (scaled to 0.20–0.60), while  $\beta = -1$ , `width=1`, and a slow sweep of  $\alpha$  generates the audible hysteretic jumps.

## 6. Conclusion

Integrating differential topology into DSP practices makes it possible to compose energy as a critical traversal of potential surfaces. The cusp forms a minimal—yet sufficient—lexicon to sculpt tensions within a two-parameter system. The multivalent region is the energetic fulcrum sustaining musical morphogenesis. The implementation in *Max/MSP* legitimizes the possibility of measuring, modulating, and composing energy as a critical crossing of topological surfaces—a gesture that reconciles catastrophe theory, spectral aesthetics, and DSP technology within a unified grammar of becoming. Compatibility with Max SDK 8.2 demonstrates the technical feasibility of integrating differential topology into DSP practices, creating an experimental space where algebra, aesthetics, and performance coexist.

As an operative close, we offer a minimal, musical demonstration that directly connects the cusp’s topology to listening: white noise  $\rightarrow$  `svf~`, with the `cutoff` driven by `z0` and the `resonance` (0–1) driven by  $1-|\Delta|$ , both provided by `cusp~`, while  $\alpha$  is swept slowly with  $\beta = -1$  and `width=1`. Because the object solves the cubic at audio rate and preserves branch memory (`state`), the system’s topological transitions become *audible*: as  $\Delta \rightarrow 0$  (mono-multivalency threshold), discrete shifts of `cutoff` with concurrent rises of `resonance` yield clear timbral ruptures; over an up–down sweep of  $\alpha$ , the commutations occur at distinct abscissas (hysteresis), typically near  $\alpha \approx \pm 0.385$  for  $\beta = -1$ . In this sense, the bifurcation grammar is not merely descriptive but acts as a direct musical parameterization that organizes gesture, timbre, and form in real time—fully realizable with the stated mapping and without recourse to additional figures. A minimal *Max* patch reproducing this setup is provided as `listening_demo.maxpat`. In it, `z0` from `cusp~` is smoothed (`slide~ 10 10.`) and mapped to the `svf~` cutoff (750–3500 Hz), while  $(1-|\Delta|)$ —taken from the discriminant outlet, rectified and clipped—is mapped to the `svf~` resonance (0.20–0.60). With  $\beta = -1$ , `width=1`, and a slow up–down sweep of  $\alpha$ , the same hysteretic timbral jumps reported here are reproducible by ear without additional figures.

The main contributions of this work can be summarized in a single bifurcation operator applied to computational music: first, we developed an open-source external, called `cusp~`, capable of solving the cubic equation  $x^3 + \alpha x + \beta = 0$  at each sample. Second, we incorporated performative control through hysteresis and predictable morphological jumps, whose effects were validated both

in spectrograms and in specific hysteresis tests. Finally, we conceptually articulated differential topology, energetic ontogenesis, and DSP in a unified framework: `cusp~` offers the composer a true grammar of bifurcation, translating virtual energy gradients into emergent sonic pulses. By providing source code, cross-platform binaries, and audio examples, we establish an experimental space where bifurcation, metastability, and hysteresis become tangible compositional parameters, reconciling catastrophe theory, spectral aesthetics, and DSP technology into a single grammar of energetic becoming.

## 7. Bibliography

This work is situated at the crossroads of recent theoretical convergences between philosophy, mathematics, and contemporary music. Drawing on Simondon’s concept of energetic individuation [2], Prigogine’s thermodynamic irreversibility [3], and Thom’s topology of morphogenesis [4], the article explores how critical transitions and metastable configurations inform both the structure and perception of sound. The conceptual synthesis aligns with Grabócz’s analysis of scientific influences on musical form [1] and Andreatta’s application of catastrophe theory in musical morphogenesis [5]. These theoretical perspectives find practical articulation in the spectral aesthetics of Dufourt [7] and Bayle [8], and in the morphogenetic practices of composers like Vaggione [9], Stroe [10], Rădulescu [11], and Dumitrescu [12]. Finally, Ferraz’s concept of *tempo-música* [13] echoes the energetic collisions that this article seeks to model in computational sound synthesis through the `cusp~` operator.

## References

- [1] M. Grabócz. The influence of scientific theories on musical form in contemporary instrumental and electroacoustic works. *Studia Musicologica Academiae Scientiarum Hungaricae*, 60(1-4):129–146, 2020. (Engl. trans. by P. Clarke of the 2016 French original).
- [2] G. Simondon. *L’Individuation à la lumière des notions de forme et d’information*. Millon, Grenoble, France, 2005.
- [3] I. Prigogine. *O fim das certezas: tempo, caos e as leis da natureza*. Editora UNESP, São Paulo, Brazil, 1996.
- [4] R. Thom. *Stabilité structurelle et morphogénèse*. InterÉditions, Paris, France, 1972. Eng. trans.: *Structural Stability and Morphogenesis*, London, U.K.: CRC Press, 2018.
- [5] M. Andreatta. Structural stability and musical morphogenesis: catastrophe theory and mathematical-musical research. In *René Thom: Mathematical and Philosophical Portrait*. CNRS Éditions, Paris, France, 2018. Available: <https://hal.science/hal-02021953>.
- [6] B. B. Gaffney and T. Smyth. Acoustics-like dynamics in signal-based synthesis through parameter mapping. In *Proc. Sound and Music Computing Conf. (SMC)*, Stockholm, Sweden, 2013.
- [7] H. Dufourt. *Spectral Music: An Epistemological Revolution*. Delatour France, Sampzon, France, 2014. (Orig. French title *La musique spectrale*).
- [8] F. Bayle. Space and more. *Organised Sound*, 12(3):241–249, 2007.

- [9] H. Vaggione. Musical composition and computer resources: questions of approach. In M. Solomos, A. Soulez, and H. Vaggione, editors, *Formal / Informal: Music – Philosophy*, pages 91–117. L’Harmattan, Paris, France, 2003. Available: <https://hal.science/hal-02055239>.
- [10] O. Nemescu. Aurel stroe – the original. *Revista Muzica*, (6):27–38, 2017.
- [11] H. Rădulescu. *Sound Plasma: Music of the Future Sign*. Edition Modern, Munich, Germany, 1975.
- [12] I. Dumitrescu. The irradiant force of sound. In R. Reigle and P. Whitehead, editors, *Spectral World Musics: Proceedings of the Istanbul Spectral Music Conference*, pages 221–229. Pan Yayincilik, Istanbul, Turkey, 2008. (Transcript by A.-M. Avram of the author’s 2003 lecture).
- [13] S. Ferraz. *Tempo-música, música-tempo: as colisões de Chronos-Caos*. 7 Letras, Rio de Janeiro, 2025.