Ocean Math: A Comparative Evaluation of Augmented Reality and Mixed Reality in a Gamified Educational Game for Learning Mathematical Functions

Juliel Bronzati Dourado[1], Marcos Américo[1]

[1]Programa de Pós-Graduação em Mídia e Tecnologia – Universidade Estadual Paulista "Júlio de Mesquista Filho" – Bauru – SP – Brasil

juliel.dourado@unesp.br, marcos.americo@unesp.br

Abstract. Introduction: *Teaching mathematical functions often involves challenges related to abstraction and student engagement. Immersive technologies such as Augmented Reality (AR) and Mixed Reality (MR) have emerged as promising approaches for educational games, although comparative studies between both technologies using equivalent pedagogical content remain limited.* ***Objective:*** *This paper evaluates Ocean Math, a gamified educational game designed to teach mathematical functions through immersive interaction.* ***Methodology or Steps:*** *An exploratory study was conducted with 102 students (aged 15-20) with similar educational backgrounds, comparing user experiences between AR (n=53) and smartphone-based MR (n=49) versions. Evaluation focused on usability, engagement, and immersion through Likert-scale questionnaires and observational analysis.* ***Results:*** *Findings reveal a consistent trade-off: the MR version promoted significantly higher emotional engagement (4.2 vs 3.7) and immersion, while the AR version provided superior usability (3.2 vs 2.5) and lower adaptation barriers. Despite higher interaction complexity in MR, both approaches were perceived as equally effective for learning mathematical concepts.* ***Conclusion:*** *The study highlights that while MR excels in presence and curiosity, AR remains more accessible for standard classroom settings, contributing to the design of immersive serious games.*

Keywords: *Mixed Reality, Augmented Reality, Serious Games, Gamification, Mathematics Education.*

1. Introduction

Teaching mathematical functions is often challenging due to their abstract nature, which can hinder students' conceptual understanding and reduce engagement during the learning process. In recent years, immersive technologies have emerged as promising educational tools by enabling more interactive, contextualized, and engaging learning experiences that encourage active student participation [Lampropoulos et al. 2022].

Among these technologies, Augmented Reality (AR) and Mixed Reality (MR) have received increasing attention for their potential to enrich educational environments through the integration of virtual content with the physical world [Maas and Hughes 2020]. While AR superimposes virtual elements onto the user's view of the real environment, MR extends this concept by providing a more immersive interaction in which users continuously perceive and interact with virtual objects while remaining aware of their physical surroundings. According to the mixed reality continuum proposed by Milgram and Kishino (1994), the immersive version of *Ocean Math* employs a smartphone mounted on a head-mounted display (HMD). The smartphone camera continuously captures the surrounding environment, allowing students to perceive the real world through a live video feed while virtual educational objects are spatially superimposed onto the physical environment. This configuration enables hands-free immersive interaction while preserving users' awareness of the real world.

Although immersive technologies have been increasingly adopted in education, comparative studies evaluating AR and MR under equivalent pedagogical conditions remain limited, particularly in mathematics education. Most previous studies investigate a single immersive technology or compare applications that differ simultaneously in educational content, interaction design, or instructional strategies, making it difficult to isolate the effects of the immersive technology itself on students' learning experiences [Merino et al. 2020]. Consequently, there is still limited evidence regarding the advantages and trade-offs associated with handheld AR and immersive MR configurations when the same educational objectives, mechanics, and learning activities are preserved.

To address this gap, this paper presents *Ocean Math*, a gamified educational application designed to support the teaching and learning of mathematical functions through immersive interaction. The application was initially developed as a handheld AR experience and subsequently adapted to a smartphone-based MR configuration while maintaining the same pedagogical objectives, educational content, game mechanics, and learning activities. This design enabled a comparative exploratory evaluation of both immersive conditions, focusing on usability, engagement, immersion, and interaction characteristics while minimizing the influence of pedagogical differences between the two versions.

The main contributions of this work are threefold:

- the design and implementation of an immersive serious game for teaching mathematical functions;
- a comparative evaluation of handheld Augmented Reality and smartphone-based Mixed Reality experiences under equivalent pedagogical conditions; and
- the identification of trade-offs related to usability, accessibility, engagement, immersion, and user interaction that may support the design of future immersive educational applications.

The remainder of this paper is organized as follows. Section 2 reviews related work on immersive technologies for mathematics education. Section 3 describes the design and development of *Ocean Math*. Section 4 presents the evaluation methodology. Section 5 reports the experimental results, followed by their discussion in Section 6. Finally, Section 7 concludes the paper and outlines directions for future work.

2. Related Work

Immersive technologies have become increasingly relevant in educational research due to their ability to promote interactive, contextualized, and student-centered learning experiences. Among these technologies, Augmented Reality (AR) and Mixed Reality (MR) have attracted considerable attention because they enable the integration of digital content with the physical environment, facilitating the visualization of abstract concepts and encouraging active learning [Maas and Hughes 2020]. In mathematics education, these characteristics are particularly valuable, as many mathematical concepts require students to mentally represent abstract objects, relationships, and functions that are often difficult to visualize using traditional instructional resources [Bower et al. 2014].

Several studies have reported positive educational outcomes associated with the use of AR in STEM education. According to the systematic review conducted by Akçayır and Akçayır (2017), AR applications frequently improve students' motivation, engagement, and conceptual understanding by enriching learning activities with interactive visual representations. Nevertheless, the authors also emphasize challenges related to usability, technical limitations, and the need for appropriate instructional design to maximize educational benefits.

In parallel, game-based learning and gamification have been widely adopted as strategies to increase motivation and participation in educational environments. By incorporating elements such as challenges, immediate feedback, rewards, and progression systems, educational games can stimulate active learning while maintaining students' interest throughout the instructional process [Deterding et al. 2011]. Romero (2020) argues that combining immersive technologies with gamification further enhances learner engagement by encouraging exploration, experimentation, and experiential learning.

More recently, Mixed Reality environments have expanded the possibilities of immersive educational applications by allowing users to interact with virtual objects while maintaining continuous awareness of the physical environment [Speicher et al. 2019]. Unlike conventional handheld AR experiences, smartphone-based MR configurations employing head-mounted displays enable hands-free interaction and a higher level of immersion while preserving the visualization of the surrounding real environment through the smartphone camera. These characteristics may influence users' perceptions of immersion, usability, and engagement during learning activities.

Despite the growing body of research on immersive educational technologies, direct comparisons between AR and MR remain relatively scarce, particularly in mathematics education. Most existing studies evaluate a single immersive technology or compare systems that differ simultaneously in educational content, interaction design, gameplay mechanics, or instructional strategies. Such methodological differences make it difficult to determine whether the observed outcomes are associated with the immersive technology itself or with differences in the learning experience.

To address this limitation, the present study adopts a controlled comparative approach. Both versions of *Ocean Math* share the same pedagogical objectives, mathematical content, gameplay mechanics, and learning activities, differing only in the immersive visualization and interaction modality. This design enables a more consistent exploratory comparison of handheld AR and smartphone-based MR with respect to usability, engagement, immersion, and user interaction.

Table 1 summarizes representative studies involving immersive educational technologies and highlights the comparative contribution of the proposed approach.

Table 1. Comparison of representative studies involving immersive educational technologies.

Study	Technology	Educational Focus	Comparative Analysis
Akçayır and Akçayır (2017)	AR	STEM Education	No
Maas and Hughes (2020)	AR/MR/VR	K-12 Education	Literature Review
Lampropoulos et al. (2022)	AR + Gamification	Education	No
This work (*Ocean Math*)	AR and MR	Mathematical Functions	Yes

As shown in Table 1, although immersive technologies have been extensively investigated in educational contexts, few studies directly compare AR and MR using equivalent pedagogical structures within the same educational application. This gap is even more evident in mathematics education, where controlled comparative investigations remain limited. By maintaining identical educational content and gameplay mechanics across both immersive conditions, *Ocean Math* contributes evidence regarding the influence of different immersive interaction modalities on students' learning experiences.

3. Game Design and Development

Ocean Math is a gamified educational application designed to support the learning of mathematical functions through immersive interaction. Set in an ocean-themed environment, the game engages students in activities involving graph interpretation, coordinate positioning, and function analysis while exploring virtual educational elements distributed throughout the scenario.

The application was developed using the Unity game engine and the C# programming language. The handheld AR version was implemented with the AR Foundation framework, whereas the smartphone-based MR version employs a smartphone mounted on a head-mounted display (HMD). In this configuration, the smartphone camera continuously captures the surrounding environment, allowing students to perceive the real world while virtual educational objects are superimposed onto the live image. Navigation is performed using an Android-compatible joystick.

Both versions preserve the same pedagogical objectives, mathematical content, gameplay mechanics, challenge sequence, and scoring system. Therefore, the comparison focuses exclusively on differences in visualization and interaction, minimizing the influence of instructional design on the evaluation results.

During gameplay, students explore the environment to locate interactive objects containing mathematical challenges. Each activity requires the interpretation of functions, graphs, or Cartesian coordinates before progressing to the next stage. Immediate visual feedback informs whether the selected answer is correct, encouraging exploration, problem-solving, and continuous engagement throughout the learning experience.

Figure 1 illustrates the two interaction modes evaluated: the handheld AR version using touchscreen interaction (left) and the smartphone-based MR version using a head-mounted display and joystick navigation (right).

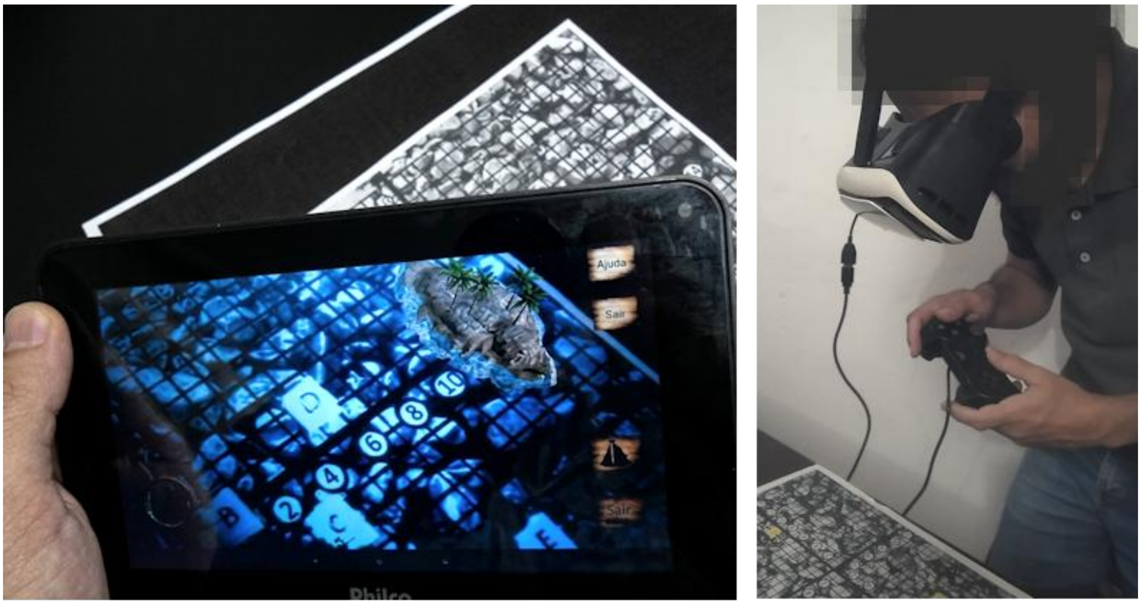

Figure 1. ***Ocean Math*** **interaction modes: (left) handheld AR version using touchscreen interaction; (right) smartphone-based MR version using a head-mounted display with continuous camera visualization and joystick navigation.**

3.1 Augmented Reality Version

The AR version was developed for Android devices using image-recognition markers positioned on a printed Cartesian board. Students interact with virtual educational objects superimposed onto the physical environment through touchscreen controls, performing activities involving coordinate interpretation, graph visualization, and function analysis. The familiarity of the mobile interaction model facilitates rapid adaptation during gameplay.

3.2 Smartphone-Based Mixed Reality Version

The MR version extends the immersive experience by mounting the smartphone on a head-mounted display while maintaining continuous visualization of the physical environment through the smartphone camera. Virtual educational elements remain superimposed onto the real-world view, enabling students to interact with both physical and digital elements simultaneously. Navigation is performed using a physical joystick, providing hands-free interaction and a first-person immersive experience.

Compared with the handheld AR version, the MR configuration encourages greater spatial exploration and environmental interaction, although it also introduces additional adaptation demands associated with joystick navigation and immersive visualization.

3.3 Transition from AR to MR

The transition from the handheld AR version to the smartphone-based MR configuration required modifications only to the interaction and visualization mechanisms. The educational objectives, mathematical content, gameplay mechanics, challenge sequence, and feedback remained unchanged. This design enabled a controlled exploratory comparison in which the observed differences can be primarily associated with the immersive interaction modality rather than with changes in the instructional proposal.

4. Methodology

This study adopted an exploratory comparative approach to investigate differences between AR and smartphone-based MR experiences in *Ocean Math* regarding usability, engagement, immersion, and interaction characteristics.

4.1 Participants

The study involved 102 participants aged between 15 and 20 years old, all students from technical and secondary education programs. The AR version was evaluated by 53 participants, while the MR version involved a similar cohort of 49 participants. This demographic homogeneity ensures that the comparative results focus on the technological interface rather than user profile variations.

4.2 Evaluation Procedure

Participants interacted individually with the game during supervised evaluation sessions. Initially, they received instructions regarding gameplay mechanics and interaction procedures. After the interaction sessions, participants answered a structured questionnaire specifically developed for this study, consisting of five-point Likert-scale items addressing usability, perceived learning, and engagement.

The evaluation investigated aspects including:

- ease of use;
- interaction difficulty;
- intuitive controls;
- comfort during use;
- clarity of explanations;
- entertainment;
- motivation to continue using the application.

4.3 Data Collection and Analysis

Data collection combined observational notes and questionnaire responses obtained after gameplay sessions. The analyses adopted a qualitative exploratory perspective supported by descriptive statistics including averages, medians, and standard deviations.

Rather than measuring long-term learning performance, the study focused on experiential aspects associated with immersive educational interaction, particularly differences involving usability, accessibility, immersion, and engagement between AR and MR conditions.

Considering the exploratory nature of the study and the participant sample size, no inferential statistical analyses were performed.

5. Results

5.1 Handheld Augmented Reality Experience

The AR version was evaluated with 53 students. Participants adapted quickly to touchscreen-based interaction and image-recognition markers, requiring minimal assistance. Higher scores were observed for clarity of explanations (4.1) and comfort during use (3.9), indicating good usability and accessibility. Entertainment (3.7) and motivation (3.5) showed slightly lower values.

5.2 Smartphone-Based Mixed Reality Experience

The MR version was evaluated with 49 students, most of whom had no prior experience with MR systems. Participants showed strong exploratory behavior and engagement during interaction. Higher scores were obtained for entertainment (4.2) and motivation

(4.1), while usability-related aspects were lower, including ease of use (2.5) and intuitive controls (2.9).

5.3 Comparative Analysis

Table 2 and Figure 2 summarize the descriptive results obtained for both interaction modalities.

Table 2. Descriptive statistics (mean, standard deviation, and median) for the AR and MR conditions.

Evaluation Aspect	AR Mean	AR SD	AR Median	MR Mean	MR SD	MR Median
Ease of Use	3.2	0.7	3	2.5	0.5	2
Interaction Difficulty	2.9	0.9	3	2.5	0.7	2
Intuitive Controls	3.5	0.7	3	2.9	0.6	3
Comfort During Use	3.9	0.8	4	3.4	0.6	3
Clarity of Explanations	4.1	0.8	4	4.0	0.6	4
Understanding of Functions	3.8	0.9	4	3.7	0.6	4
Interest in Learning	3.6	0.7	4	3.9	0.4	4
Entertainment	3.7	0.7	4	4.2	0.9	4
Motivation to Continue	3.5	0.7	4	4.1	0.6	4

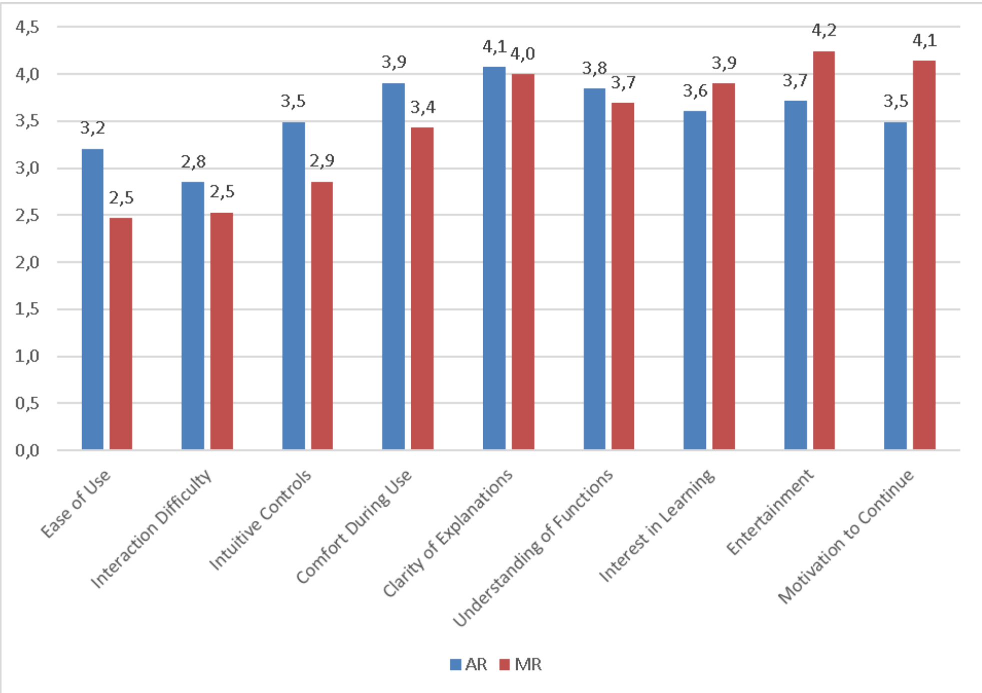

Figure 2. Comparative results between the handheld AR and smartphone-based MR versions of *Ocean Math*.

Overall, the descriptive results indicate different strengths for the two interaction modalities. The handheld AR version received higher ratings for usability-related aspects, including **Ease of Use**, **Comfort During Use**, and **Intuitive Controls**, suggesting that participants adapted more easily to the touchscreen-based interaction model.

In contrast, the smartphone-based MR version received higher ratings for **Entertainment**, **Motivation to Continue**, and **Interest in Learning**, indicating that the immersive interaction promoted a more engaging user experience. Both versions received similarly high evaluations for **Clarity of Explanations**, suggesting that the educational content remained understandable regardless of the interaction modality.

These findings reveal a trade-off between usability and engagement. While the AR implementation appears to reduce interaction barriers and facilitate task execution, the MR configuration seems to encourage greater user involvement and exploratory behavior, despite requiring additional adaptation during the initial interaction.

Because this study adopted an exploratory design based on descriptive statistics, these results should be interpreted as indicative trends rather than statistically significant differences. Future studies employing inferential statistical analyses and larger samples may further investigate the observed patterns.

6. Discussion

The comparative evaluation highlights distinct strengths associated with the two immersive interaction modalities. The handheld AR version received higher evaluations for usability-related aspects, including ease of use and comfort during gameplay, whereas the smartphone-based MR version obtained higher ratings for entertainment and motivation to continue using the application. These findings are consistent with previous studies indicating that increased immersion may enhance engagement while simultaneously increasing interaction demands [Maas and Hughes 2020].

Observations recorded during the evaluation sessions indicated different interaction patterns between the two conditions. Participants using the MR configuration frequently explored the virtual environment beyond the required activities, suggesting that the immersive experience encouraged curiosity and voluntary exploration. In contrast, the handheld AR version enabled faster adaptation and required less assistance during gameplay, indicating that its interaction model may be more suitable for educational contexts where rapid familiarization is desirable.

Regarding learning perception, both versions obtained similar evaluations for **Clarity of Explanations** and **Understanding of Functions**, suggesting that preserving the same pedagogical structure across both conditions contributed to comparable perceptions of the educational content. These results indicate that the differences observed between the two versions were primarily related to the interaction experience rather than to the instructional design.

Some aspects should be considered when interpreting these findings. The evaluations were conducted with different classes of students presenting similar educational profiles, and most participants assigned to the MR condition had no previous experience with immersive technologies. This lack of familiarity may have influenced the lower usability ratings observed for the MR version, particularly during the initial adaptation to joystick-based navigation.

Finally, because this study adopted an exploratory design based on descriptive analyses and a questionnaire specifically developed for this research, the results should be interpreted as evidence of participants' perceptions rather than definitive measures of learning effectiveness. Future studies involving validated assessment instruments, inferential statistical analyses, and objective learning measures may provide a broader understanding of the educational impact of different immersive interaction modalities.

7. Conclusion

This paper presented *Ocean Math*, a gamified educational application designed to support the teaching and learning of mathematical functions through immersive technologies. By preserving the same pedagogical objectives, mathematical content, and gameplay mechanics across both versions, the study enabled an exploratory comparison between handheld Augmented Reality (AR) and smartphone-based Mixed Reality (MR) under equivalent instructional conditions.

The descriptive results indicate that the two interaction modalities exhibit complementary strengths. The handheld AR version was associated with higher perceived usability and easier interaction, whereas the smartphone-based MR version received higher evaluations for entertainment, motivation, and exploratory interaction. At the same time, both versions obtained similar evaluations regarding the clarity of explanations and the perceived understanding of mathematical functions, suggesting that the educational content remained consistent regardless of the interaction modality.

These findings reinforce the importance of balancing immersion, usability, and accessibility when designing educational applications based on immersive technologies. Rather than identifying a superior interaction modality, the results suggest that the choice between AR and MR should consider the intended educational context, the characteristics of the target audience, and the desired learning experience.

This study has some limitations. The evaluation adopted an exploratory approach based on participants' perceptions, using a questionnaire specifically developed for this research, and did not include objective learning assessments or long-term follow-up. In addition, the two interaction modalities were evaluated with different classes, although they shared similar educational profiles.

Future work includes validating the evaluation instrument with larger samples, incorporating objective measures of learning performance through pre- and post-tests, and applying inferential statistical analyses to further investigate the differences between immersive interaction modalities. Additional studies involving students from different educational levels may also contribute to the generalization of the findings.

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