Trajectory Planning of an Aerial-Underwater Hybrid Vehicle Based on Heuristics for Energy Efficiency

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Abstract. *This work studies the trajectory planning of an unmanned hybrid aerial-underwater vehicle (HUAUV) called* Hydrone*, which is being developed by the Intelligent Robotics and Automation Group (NAUTEC) of the Federal University of Rio Grande (FURG). This study presents a new trajectory planning algorithm, based on closed-loop rapidly exploring random trees (CL-RRT). This algorithm is developed for an HUAUV and introduces two heuristics to improve its energy efficiency in hybrid tasks. Simulated experiments were carried out in 135 virtual scenarios, comparing three approaches: one without heuristics and two with the proposed heuristics. Simulated results demonstrate that using the heuristics can significantly reduce energy consumption and even improve the vehicle's average speed during missions. In particular, in* 95% *of the scenarios, the lowest energy consumption was achieved by one of the two heuristic-based algorithms. This article concludes by summarizing the findings and identifying potential future research opportunities.*

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1. Introduction

1.1. Hybrid Unmanned Aerial Underwater Vehicles

Hybrid Unmanned Aerial Underwater Vehicles (HUAUVs) are a class of mobile robots that combine the characteristics of unmanned aerial vehicles (UAVs) and unmanned underwater vehicles (UUVs). And the idea of combining these vehicle classes is not new [Drews et al. 2009].

By having the ability to navigate in two different media, HUAUVs are capable of obtaining a complete perception of environments close to water, such as riverbanks, ports, oil platforms, and even flooded caverns. This enables activities such as complete inspection of partially submerged structures, environmental monitoring and assessment, and search and rescue without the need for human assistance to switch between environments.

[1]`https://argo.furg.br/?BDTD13671`

However, navigating in both media demands a platform that can adapt to two densities that are almost three orders of magnitude apart. It must also be lightweight and waterproof. These challenges are not trivial and are being studied by research groups at many universities around the world [Alzu'bi et al. 2018, Ma et al. 2018, Lu et al. 2019, Chen et al. 2020].

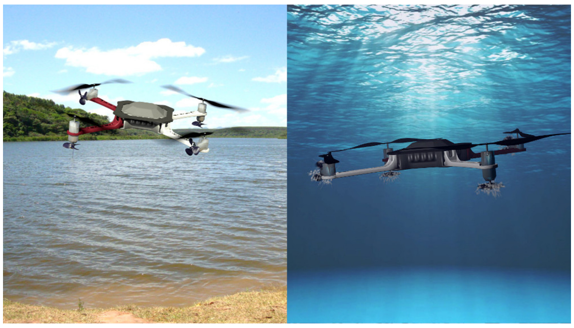

Figura 1. Hydrone concept [Horn et al. 2020]

1.2. Problem

Efficient and safe navigation in hybrid scenarios involves planning trajectories that minimize resource consumption and ensure physical integrity. These trajectories must account for dynamic changes due to differences in fluid density when searching for the best path to follow (Problem 1).

Problem 1 *Let a HUAUV be modeled with different dynamics for aerial and aquatic operations, with their respective control laws. This vehicle navigates in an environment $\mathcal{X}$, with static obstacles. Let the existence of a transition zone around the water surface be assumed at $z = 0$, in the range $-\mu \leq z \leq \mu$, for $\mu \geq 0$. Finally, assume a cost function to be provided such that it relates the vehicle's actions to the energy consumption along a trajectory. Then, the main goal is to find a trajectory from the vehicle's starting point to a given goal ($\boldsymbol{r}_{goal}$), such that $\boldsymbol{x}(t) \in \mathcal{X}_{free}$ for all $t > 0$. The planner algorithm must also minimize energy consumption and allow for a smooth transition between environments.*

1.3. Contributions

As contributions, this work presents a new trajectory planning algorithm for a multirotor Hybrid Unmanned Aerial Underwater Vehicle; it also presents two heuristics for energy-saving trajectory planning.

Regarding scientific publications, the standard nonheuristic algorithm of this work, called Hybrid CL-RRT, was presented in [Pinheiro et al. 2022] (JINT, Qualis A2). And the HUAUV survey of this work is also available in a pre-print version [Pinheiro et al. 2023] (submitted to RAS, Qualis A1).

Furthermore, throughout the development of this work, studies were carried out in areas complementary to trajectory planning, such as vehicle design, modeling, control, simulation, visual-based navigation, and prototyping [Grando et al. 2020, Horn et al. 2020, Aoki et al. 2021, Aoki et al. 2022a, Aoki et al. 2022b, Pedroso et al. 2022] (all six of them published in LARS, QualisCC B1). Other publications also explore the use of deep reinforcement learning for mapless navigation and obstacle avoidance in aerial and underwater environments [Grando et al. 2021, Grando et al. 2022a, Grando et al. 2022b] (one of them published in ICRA, QualisCC A1; and two of them published in LARS, QualisCC B1).

2. Literature review

Previous studies on Trajectory Planning for multirotor-based HUAUVs are limited. However, recent works have addressed similar problems.

In [Su et al. 2021], trajectory optimization for an HUAUV navigating between air and water is addressed. An improved teaching- and learning-based optimization (ITLBO) algorithm is proposed to minimize position and velocity errors. The simulation results demonstrate the effectiveness of the algorithm.

In [Liang et al. 2021], a heuristic generalized extensive neighborhood search (GLNS)–k-means algorithm is proposed for a multirotor, underwater-glider, and tail-sitter HUAUV. The algorithm combines k-means clustering and the GLNS algorithm to find optimal paths. An online replanning strategy is introduced and MATLAB simulations evaluate the approach.

[Wu et al. 2020] presents an improved teach-and-learn-based optimization (ITLBO) algorithm for trajectory optimization of a coaxial eight-rotor HUAUV. The approach considers navigation errors and collision probability. Simulation results demonstrate its effectiveness.

The three-dimensional multi-domain trajectory planning for multirotor HUAUVs is underexplored. Challenges like high-dimensional state space and abrupt environment transitions require customized approaches and knowledge of the robot's dynamic model. Section 3 presents the trajectory planning techniques and robot model used in this work.

3. Methodology

3.1. Hydrone vehicle model

The Hydrone vehicle is an HUAUV that has four vertical aerial actuators, two horizontal forward facing underwater actuators, and two vertical underwater actuators (Figure 2). Therefore, during flight, the vehicle model is the same as a UAV model. However, underwater, the Hydrone vehicle has an underactuated ROV-like model. Hence, it relies on the passive stability of the roll motion, achieved by carefully placing the vehicle's center of buoyancy above the center of mass. In addition, it can produce forward, upward, and downward forces, and pitch and yaw rotations [Horn et al. 2020].

3.2. Vehicle constraints

As presented, the HUAUV in question has some limitations. The main one is with regard to its underwater actuation. The model proposed by [Horn et al. 2020], has a pair of propellers facing forward as a way to act on the x-axis on the body frame. Still, it does not have actuators facing backwards. Thus, as this vehicle model was developed considering standard off-the-shelf products, its motor-propeller sets do not consider reversing motor rotation. Therefore, the vehicle depends on the drag force to reduce the forward speed underwater.

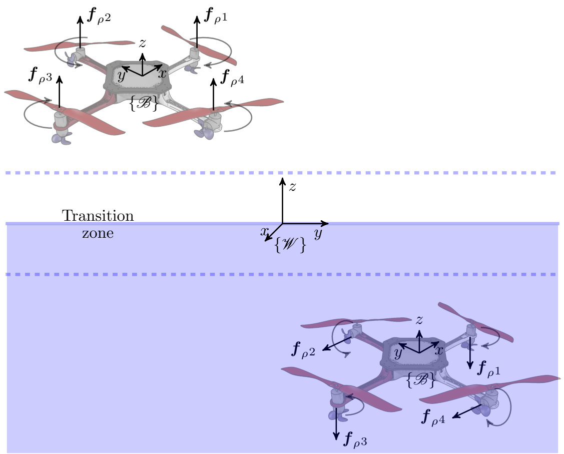

Figura 2. Hydrone forces and moments. Above, the vehicle operates in the aerial mode, while below it acts in the aquatic mode. Around the surface level ($z = 0$) there is a transition zone.

This significantly impacts its underwater maneuverability. First, the vehicle cannot rotate around the z axis in $\{\mathscr{B}\}$. Second, when it describes curves underwater, the radius of curvature of these curves depends on the vehicle momentum.

3.3. Power estimation

In the work by [Horn et al. 2019] the authors carried out a survey of energy consumption for some motor-propeller combinations for HUAUVs. Through these data, it is possible to derive polynomial functions of energy consumption that associate it with the forces generated by the sets of motors and propellers.

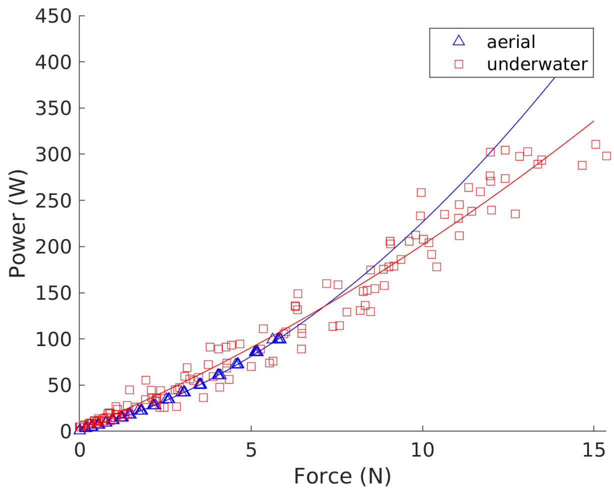

Figura 3. This figure shows the power consumption and polynomial regression curves for aerial and underwater actuation systems. Both curves follow a second-degree polynomial trend. The underwater system presents higher consumption than the aerial system from null force until 7,5N, while the aerial system assumes higher values for higher forces, widening the difference between the two systems.

These functions are assumed by this work to be significant enough to represent the overall energy consumption of a vehicle in a hybrid environment, although other systems would also consume battery energy.

4. Hybrid Closed-loop RRT

The CL-RRT algorithm [Kuwata et al. 2009] can be used in several situations. Among these, for planning the trajectory of unmanned aerial vehicles [Arslan et al. 2017, Zhu et al. 2018]. To this end, the algorithm uses the vehicle model to generate trajectories in the $\mathcal{X}$ space and verify possible collisions. Then, these trajectories are associated with nodes of the exploration tree.

To develop a planner that did not need to solve the vehicle dynamics equations for each node of the expanded tree, a simulated survey of the navigation costs associated with each environment and at transition was carried out. Significant variance in the data was verified, however, it was possible to observe that the water navigation cost is much lower than the air navigation cost. Thus, two techniques were proposed for optimizing the energy efficiency of HUAUVs:

1. the use of constant estimated costs for each medium, and for the transition;
2. and the use of a skewed tree expansion.

Another significant aspect of Hybrid CL-RRT is that it should be able to handle domain transition. As the vehicle model and several others in the literature focus on vertical medium transition, this planner maintains this approach to avoid possible disturbances and instabilities during vehicle navigation. In this way, this proposed planner also seeks to approximate the real application of this class of vehicles.

The main loop of the hybrid planner is presented in Algorithm 1. Essentially, this is the same as that adopted in the work by [Kuwata et al. 2009]. It has an initial phase of updating both the vehicle state and the map. Then, the propagation of the vehicle trajectory is made for the next instants, predicting its state after the planning. Then, the hybrid tree is expanded and the best path is selected. Finally, the repropagation procedure is performed, where the references associated with the best path are passed to the predictive model and the approximate trajectory to be performed by the vehicle is identified. If this trajectory is valid, then the references are passed to the navigation system; otherwise, the invalid nodes are marked before starting a new loop.

The expansion of this hybrid tree (Algorithm 2) is done similarly to that of a traditional tree: random points are sampled from the map; the tree is expanded in the direction of this point; if there is a collision in the edge between these two points, the new point is discarded, otherwise it is added to the tree; and so on. However, there is one important difference in this expansion: the medium transition.

In order to implement a hybrid trajectory planning approach, it is essential to devise effective strategies for transitioning between different domains. In our study, we adopted a specific strategy that involves conditioning the transition between nodes on a vertical edge. Essentially, when a new point is sampled and the closest existing point is either in the transition region or in another medium, we adjust the x and y coordinates of the position of the new point to ensure that it satisfies the vertical condition. In this way, we can facilitate a smoother and more efficient transition between different domains during the robot's trajectory planning.

Algorithm 1 Hybrid CL-RRT execution loop

Require: $\boldsymbol{x}(0)$, $\boldsymbol{r}_{\text{goal}}$
1: initialise $\mathcal{T}$ with node at $\boldsymbol{x}(0)$
2: **repeat**
3: update $\boldsymbol{x}(t)$ and $\mathcal{X}_{free}(t)$
4: propagate $\boldsymbol{x}(t)$ to $\boldsymbol{x}(t+T)$ with hybrid system model
5: **repeat**
6: expand $\mathcal{T}$
7: **until** time limit T is reached
8: choose the current best path $\mathcal{P} \in \mathcal{T}$
9: **if** $\mathcal{P} \in \emptyset$ **then**
10: apply safety action and **goto** line 18
11: **end if**
12: re-propagate from $\boldsymbol{x}(t+T)$ by using references associated with $\mathcal{P}$
13: **if** $\boldsymbol{x}(t) \in \mathcal{X}_{free}(t)\ \forall t \in [t, t+T]$ **then**
14: send $\mathcal{P}$ to the controller
15: **else**
16: mark infeasible parts of $\mathcal{T}$ and **goto** line 8
17: **end if**
18: $t \leftarrow t + T$
19: **until** reach $\boldsymbol{r}_{\text{goal}}$

Because this work is not yet concerned with strategies for the best medium transition, the strategy adopted provides only a form of transition. It is therefore assumed that this will be sufficient to satisfactorily generate paths that explore both domains. It is also assumed that the repropagation step (trajectory verification), before passing the trajectory to the navigation system, can predict collision situations well enough and that the HUAUV control can handle the references provided from these vertical transitions well enough.

4.1. Water-biased Hybrid CL-RRT

As verified in the energy consumption survey conducted by [Horn et al. 2019], the energy consumption per unit of force is higher for the aerial domain than for the aquatic domain. In addition, from the construction aspects of HUAUVs, it is known that these vehicles need to generate more force to keep the vehicle in the air rather than to keep it submerged. Thus, the first heuristic proposed to reduce energy consumption consisted in biasing the tree expansion to further explore the water environment. From this, it is expected that the vehicle will generate trajectories with lower amounts of force and, consequently, less energy consumption for the same displacement.

As described above, the expansion of an RRT takes place through the random sampling of points in space. Therefore, in order to bias this sampling, a technique was used in which pseudorandom numbers are generated in a certain interval. If the generated number is smaller than a constant percentage of the interval, then the point is sampled in the water domain; otherwise, it can be sampled anywhere in the hybrid space.

Algorithm 2 Hybrid CL-RRT expansion

Require: Map, $segment\_length$

$\boldsymbol{p}_{sample} \leftarrow sampleRandomPoint()$
$closest\_node \leftarrow getClosestNode(\boldsymbol{p}_{sample})$
$\boldsymbol{p}_{closest} \leftarrow closest\_node.pos$
$\boldsymbol{p}_{dir} \leftarrow \boldsymbol{p}_{sample} - \boldsymbol{p}_{closest}$
$\boldsymbol{p}_{new} \leftarrow \boldsymbol{p}_{closest} + \hat{\boldsymbol{p}}_{dir} \cdot segment\_length$
if $transition(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$ **then**
    $\boldsymbol{p}_{new} \leftarrow verticalTransition(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$
end if
if $collision(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$ **then**
    $return$
else
    $\mathcal{T}.addNode(\boldsymbol{p}_{new})$
end if

4.2. Estimated Costs Hybrid CL-RRT

The second proposed heuristic is based on the assumption that there are approximately constant average costs for traveling between two points in the same environment or the transition. Therefore, based on a survey of these average costs, it would be possible to estimate the cost associated with a path section (between two nodes). In this way, it would be possible to identify, in addition to the nodes closest to a certain position, those that have the lowest cost in a certain region. Thus, besides simply adding child nodes to the closest nodes, it would be possible to add the new nodes to the least costly ones in a given region in an attempt to minimize (at least locally) the cost of the paths.

The expansion function associated with this heuristic has a structure similar to the previous ones. However, as presented in Algorithm 3 on line 12, instead of simply adding the new node as a child of the closest node, the verification of which parent node (neighbor) in a certain radius of the new node (neighborhood) would generate a lower displacement cost is performed. After checking the best parent node, the new node is added to the tree as a child of that node. Then, since this node is now part of the tree, it is verified which nodes in the neighborhood can be reached with less cost. If a path is found that allows reaching any of the neighbors at a lower cost, rewiring is performed.

It is clear that this approach would increase the number of steps for the expansion of the tree, which would probably lead to a longer execution time for each new insertion. However, this work seeks to evaluate the possibility of reducing energy consumption in missions performed by HUAUVs, more specifically reducing the energy consumption of their motors, and not reducing execution time or consumption of computational resources.

To carry out the survey of cost estimates, several techniques could be used: from an experimental survey to consulting manufacturers' catalogs, or even using generic models for motors. However, for this work, this survey was considered to be possible using a significant number (in the order of thousands) of small trajectories in simulation. It is clear that for this the motor model used in the simulation should be consistent and close to the real one, as is the case.

Algorithm 3 Estimated Costs Hybrid CL-RRT expansion

Require: Map, $segment\_length$, $cost_{air}$, $cost_{uw}$, $cost_{trans}$, r

$\boldsymbol{p}_{sample} \leftarrow sampleRandomPoint()$
$closest\_node \leftarrow getClosestNode(\boldsymbol{p}_{sample})$
$\boldsymbol{p}_{closest} \leftarrow closest\_node.pos$
$\boldsymbol{p}_{dir} \leftarrow \boldsymbol{p}_{sample} - \boldsymbol{p}_{closest}$
$\boldsymbol{p}_{new} \leftarrow \boldsymbol{p}_{closest} + \hat{\boldsymbol{p}}_{dir} \times segment\_length$
if $transition(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$ **then**
    $\boldsymbol{p}_{new} \leftarrow verticalTransition(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$
end if
if $collision(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$ **then**
    $return$
else
    $min\_cost \leftarrow closest\_node.cost + getCost(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$
    $min\_cost\_node \leftarrow closest\_node$
    $neighbours \leftarrow getNodesInRadius(\boldsymbol{p}_{new}, r)$
    for $node \in neighbours$ **do**
        if $transition(\boldsymbol{p}_{node}, \boldsymbol{p}_{new})$ **then**
            $continue$
        else
            $\boldsymbol{p}_{node} \leftarrow node.pos$
            $this\_cost \leftarrow closest\_node.cost + getCost(\boldsymbol{p}_{node}, \boldsymbol{p}_{new})$
            if $this\_cost < min\_cost$ **then**
                $min\_cost \leftarrow this\_cost$
                $min\_cost\_node \leftarrow node$
            end if
        end if
    end for
    $\mathcal{T}.addNode(\boldsymbol{p}_{new})$
    $rewireTree(new_node, neighbours)$
end if

5. Simulated results

For testing the goal-reaching effectiveness and the energy-saving efficiency of the heuristics, experiments were conducted in 200 different scenarios. In each scenario, the following were randomly changed: the arrangement of obstacles, the starting point, and the goal region.

The scenario randomness allowed for obstacles to be significantly close to the vehicle since the beginning of its operation. And this, as well as the fact that the vehicle had no way to brake underwater, led to some unsuccessful missions, resulting in the vehicle not being able to reach its goal safely. Thus, the results for the number of successful missions for each planner were as presented in Table 1.

The water-biased approach allowed the vehicle to avoid some aerial obstacles that the other two planners struggled with. This led to more successful missions for the Water-biased CL-RRT, followed by the Estimated Costs Hybrid CL-RRT and the standard Hy-

Tabela 1. Successful missions by planning algorithm out of 200.

Planner algorithm	Successful missions
Hybrid CL-RRT	$152(76\%)$
Water-biased Hybrid CL-RRT	$158(79\%)$
Estimated Costs Hybrid CL-RRT	$154(77\%)$

brid CL-RRT. In total, 135 (67.5%) scenarios were completed by all three algorithms.

Therefore, the results for the use of the three planners in the 135 scenarios are presented below. Figure 4 presents the data for the average energy consumption of each of the algorithms. According to this graph, it can be observed that the algorithm without heuristics (called original in the graph legend) shows a larger dispersion of values. In contrast, the hybrid planning algorithms with heuristics present less-dispersed values, which are also apparently a little lower than those of the standard algorithm. Still, the estimated-cost heuristic seems to produce similarly grouped values but with a slightly smaller average than the water-bias heuristic.

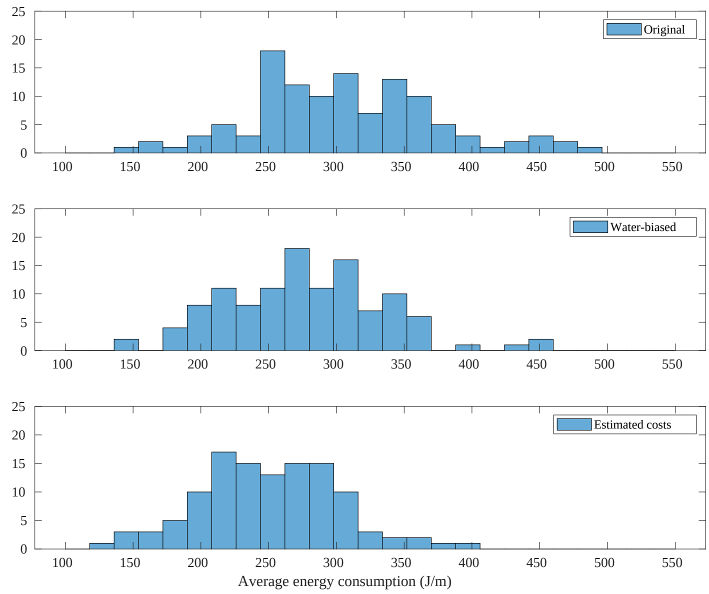

Figura 4. Average energy consumption by planning algorithm. The vertical axis shows the number of samples in the interval represented by the respective blue bar.

Through the graph in Figure 5 it is possible to confirm the statements about the previous graph, such as the statement that the algorithm without heuristics produced a larger sample interval for the average energy consumption in the scenarios considered. In addition to this, it can be seen that despite being similar, the distributions for the heuristics present significant differences, since the upper bound for the confidence interval for the median of the distribution of the estimated cost heuristic is $258{,}69\,\mathrm{J/m}$ and the lower bound for the confidence interval for the median of the distribution of the water bias heuristic is $266{,}31\ \mathrm{J/m}$.

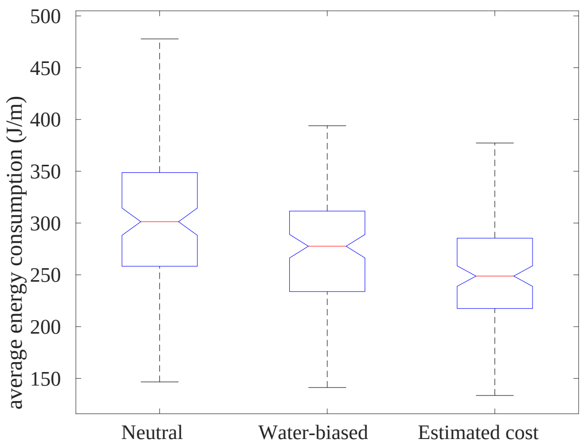

Figura 5. Average energy consumption by planning algorithm.

Finally, a last perspective is the comparison of the algorithms for each scenario. In this situation, the performance of each algorithm in relation to the others is separately assessed for each one of the cases. Such results are presented in Table 2.

Tabela 2. Comparison between the energy consumption data of each algorithm for each scenario. The number of times each algorithm was the best (lowest consumption) or the worst (highest consumption) among the three for the 135 scenarios considered.

Planner algorithm	Best	Worst
Hybrid CL-RRT	7 (5.18%)	82 (60.74%)
Water-biased Hybrid CL-RRT	39 (28.89%)	47 (34.81%)
Estimated Costs Hybrid CL-RRT	89 (65.93%)	6 (4.44%)

6. Conclusion

In conclusion, this work achieved its objectives successfully by formulating a base trajectory planning algorithm and introducing two heuristics. The comprehensive comparison conducted demonstrated the strengths of the proposed algorithms. Future work could involve studying the impact of atmospheric and underwater currents on the algorithms' performance, exploring the use of node rewiring in other algorithms, and conducting real-time experiments with the planning algorithms in controlled environments using HUAUV systems. These opportunities for further research were identified but not explored due to time and resource constraints.

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