Creation of Materials from Tabular BRDFs

Mislene da Silva Nunes[1], Gastão Florêncio Miranda Jr. and Beatriz Trinchão Andrade
Graduate Program in Computer Science
Federal University of Sergipe
mislenesn@dcomp.ufs.br, gastao@mat.ufs.br, beatriz@dcomp.ufs.br

1. This work relates to a M.Sc. dissertation defended in February 22th, 2021.

***Abstract*—Rendering materials with a realistic appearance requires considering how they interact with the light. Bidirectional Reflectance Distribution Functions (BRDFs) are often used to achieve this goal. There are different ways to represent materials from BRDFs, which include tabular BRDFs, analytical models, and linear combinations of a BRDF database. In the last decade, the search for more realism in rendering increased the interest in using tabular BRDFs. However, this approach requires a long acquisition process and high storage space. This master dissertation proposes a pipeline to create new materials from a tabular BRDF database. During this process, we also explored two related topics: we compiled and proposed techniques to evaluate BRDFs, and developed an approach to preprocess and cluster a BRDF database. These researches presented insights and contributions that are useful for contexts other than ours and provided analysis that reinforced our choice of techniques to reach our goal. As a final result, our method creates new materials with realism and consistency.**

I. Introduction

Representing materials from the real world in computer graphics requires considering how they interact with light. A way to model the appearance and behavior of a material is through Bidirectional Reflectance Distribution Functions (BRDFs). These functions describe the reflectance of a point p on the surface through the quotient between reflected radiance and incoming irradiance in this point. For this purpose, BRDFs often use incoming ($\boldsymbol{\omega}_\mathbf{i}$) and outgoing ($\boldsymbol{\omega}_\mathbf{o}$) light directions as parameters, which, respectively, mean the radiance and irradiance directions (see Figure 1).

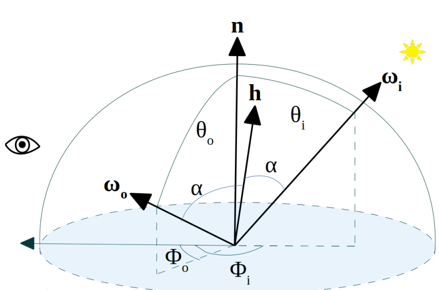

Fig. 1. Parameters of a BRDF: $\boldsymbol{\omega}_\mathbf{i} = (\theta_\mathbf{i}, \phi_\mathbf{i})$ and $\boldsymbol{\omega}_\mathbf{o} = (\theta_\mathbf{o}, \phi_\mathbf{o})$ represent, respectively, the incoming and outgoing light directions. Vector $\mathbf{n}$ means the surface normal vector and $\mathbf{h}$ the halfway vector between $\boldsymbol{\omega}_\mathbf{i}$ and $\boldsymbol{\omega}_\mathbf{o}$. Figure extracted from Nunes et al. [1]

A BRDF can be directly estimated from a real-world material through device and image-based methods, which sample reflectance data and store them in a tabular format. This approach is precise but requires high storage space and a long time of measurement acquisition [2]–[5]. An alternative is using analytical reflectance models, which are functions that aim to approximate tabular data [6]–[11]. Another approach is to estimate a linear combination of preexisting tabular BRDFs to provide materials based on data captured from real-world materials [12], [13]. A similar technique uses analytical BRDFs as components of the linear combination [14], [15].

Sampled tabular data enable rendering materials with the closest appearance to reality. However, this approach requires a large number of samples to reproduce the reality with fidelity. Thus, there are few tabular BRDF databases estimated from real-world materials, and they usually contain a relatively small amount of materials (e.g., 100 [5], 150 [16], and 62 [17] materials). To avoid long acquisition processes, approaches that take advantage of existing databases to create new materials based on tabular data are emerging [5], [18]–[20]. Besides, it is possible to personalize the new materials by choosing desired features about the appearance of materials or editing them [5], [19], [21].

This master's dissertation's [22] primary goal is to develop a pipeline to create new custom materials from a tabular BRDF database [23]. We provide an appearance-driven space of materials according to the user's interest. In this space, navigating and retrieving novel, personalized materials is feasible. The navigation in this space provides a smooth appearance variation between neighboring points. We also performed two additional extensive studies. These studies provided analysis that reinforced our choice of techniques in the pipeline and presented insights and contributions that are useful for other contexts in the literature.

Our first study compiles and proposes techniques to evaluate BRDF representations [23]. Using such techniques help to verify if two BRDF representations are similar. Regarding our primary goal, we adopted a comparison function that stood out in this study to evaluate if the materials obtained from the navigation using our pipeline provided a smooth transition.

The second study develops an approach to preprocess and cluster a BRDF database [24]. To separate materials from a BRDF database into groups regarding their reflectance features helps to find materials with similar appearance and properties. This work developed an experimental study to investigate the use of image slices and dimensionality reduction as a preprocessing stage for clustering of BRDF database. Finding the best clusters of materials is an essential step in creating our appearance-driven space.

This way, we summarize our main contributions:

- A pipeline to create new and personalized materials [1];
- A compilation and proposal of techniques to evaluate BRDFs [23];
- An approach to preprocess and cluster a BRDF database [24].

Our proposed pipeline creates new materials according to the user's interest. It can be used in 3D artistic applications that range from the entertainment industry to computer-aided design and manufacture. In addition, techniques to evaluate BRDFs can be used to analyze if BRDFs are similar and as distance functions in cluster algorithms. The study about the clustering of the BRDF database can help researchers to group materials regarding their main reflectance features, selecting of basis BRDFs, reconstruction of BRDFs, and editing of material appearance.

This paper is organized as follows. Section II introduces related works on creating new materials from a tabular BRDF database, our primary goal. Sections III, IV, and V present an overview of the three main contributions of this master dissertation: an appearance-driven space to create new BRDFs [1], techniques for BRDF evaluation [23], and an approach to preprocess and cluster a BRDF database [24]. The reader is encouraged to refer to each referenced paper for more details. Finally, Section VI concludes this paper and presents our challenges, limitations, and future work perspectives.

II. Related Works

The search for more realism in rendering provided a greater interest in using tabular BRDFs to represent a material appearance. However, capturing the tabular BRDFs from real-world materials requires a long acquisition process and high storage space, bringing on few databases available. Thus, previous work developed approaches that use existing tabular BRDF databases to create new materials. The works presented in this section are related to our primary goal of creating new BRDFs. Please refer to Nunes et al. [23] and Nunes et al. [24] for related works regarding our other two main contributions (techniques for BRDF evaluation and an approach to preprocess and cluster a BRDF database).

Matusik et al. [5] captured the appearance of 100 isotropic materials from the real world and stored them in a tabular data structure. They proposed an approach that uses this tabular BRDF database to create new materials. To this end, it was used both linear and nonlinear dimensionality reduction methods, i.e., respectively, Principal components Analysis (PCA) and Charting [25], to find a low-dimensional manifold for their database. That approach contains a mapping between original and low-dimensional space, which makes possible the creation of new materials through interpolation or extrapolation. Also, it allows changing the material's properties through a set of parameters.

Wills et al. [26] developed a Multidimensional Scaling (MDS) variant to provide a low-dimensional perceptual embedding from a BRDF database. Through this variant of MDS, perceptual interpolation, and color integration techniques developed, they presented an approach to create new materials. This perceptual interpolation aims to represent the brightness of the material. The authors used Delaunay triangulation to obtain a convex hull from the embedding, and barycentric coordinates to interpolate the materials inside of triangles of this convex hull.

Representing a measured material requires a large number of samples to be captured from the real world. Nielsen et al. [18] showed that it is possible to reconstruct a measured BRDF from a limited number of samples. They used a log-relative mapping of the BRDF space, which consists of a linear approach combined with principal components from a tabular BRDF database. Furthermore, they find the best sampling directions and use these optimized and limited samples to reconstruct a tabular BRDF.

Serrano et al. [19] presented approaches to creating and editing materials. To create materials, the authors used a mapping strategy similar to Nielsen et al. [18]'s approach, which applied a log-relative linear mapping and used PCA to provide a low-dimensional space of BRDFs. Then, they built a convex hull for this low-dimensional space through a uniform distribution with Gibbs sampling [4]. To edit the appearance of materials, they built a set of attributes, a mapping between each attribute, and coefficients of the principal components that uses a radial basis function network. That mapping defines the control of space to edit the appearance of materials.

Aiming to create new materials based on tabular BRDFs, Nunes et al. [20] presented an approach that uses PCA to generate a low-dimensional space of BRDFs. Besides, they used Delaunay triangulation to provide a mesh of points, in which these points represent the materials of the low-dimensional BRDF space. Thus, any point inside this mesh can be used to create a new material in the original space through interpolating the materials within it. The authors proposed a navigation strategy in the low-dimensional space that provides a set of novel materials between two existing materials of the database.

Hu et al. [27] developed an approach to creating new materials, which uses deep learning to provide a low-dimensional manifold from the measured BRDF database. The authors represent each BRDF as a sequence of image slices. This low-dimensional manifold makes it possible to navigate and create new materials with a smooth transition. They also proposed BRDF editing using the set of attributes proposed by Guo et al. [28]. Furthermore, they perform BRDF recovery from a single image of material using another deep neural network.

In this work, we present our pipeline to create new materials considering the user's interest. Previous works such as Wills et al. [26] and Nielsen et al. [18] used the linear dimensionality reduction method to achieve the same goal. Here, it is feasible to choose between linear and nonlinear dimensionality reduction methods. Unlike Matusik et al. [5] and Serrano et al. [19], the user does not need to know about specific BRDF properties since our approach only requires the user to choose the desired appearance through the selection of existing materials. Our reduced space is based on the selected materials by the user. We differ from most works presented in this section by using image slices to preprocess tabular BRDF database before applying a dimensionality reduction method and clustering a BRDF database. Hu et al. [27] also used image slices in their approach to creating new materials. However, Hu et al. represent a BRDF as a sequence of image slices while we represent a BRDF with only one image slice. This choice enables us to keep the main reflectance features while maintaining a compact representation.

III. An Appearance-Driven Space to Create New BRDFs

This section presents our pipeline to create new materials from a tabular BRDF database [1]. In addition to that database, the pipeline receives as input a set of indexes of materials belonging to that database, selected by the user according to appearance of the desired material. This information will guide the generation of a BRDF space.

The pipeline first preprocesses a tabular BRDF database using image slices, which cosiders the main reflectance features of a material, such as diffuse reflectance, specular peak, Fresnel effect, retro-reflection, and retro-reflection in grazing angles. The clustering of that preprocessed database is performed (the k-means and k-medoids algorithms were compared in this stage). Then, the dimensionality of the database is reduced. Here, MDS and ISOMAP (Isometric Feature Mapping) were compared. From the reduced space obtained, clusters of materials, and indexes of the materials chosen by the user, we build an appearance-driven space of BRDFs. In addition, a mapping from that space to the original tabular BRDF space is defined. It is also possible to navigate in this appearance-driven space to create new materials. Figure 2 shows a summary of the pipeline stages.

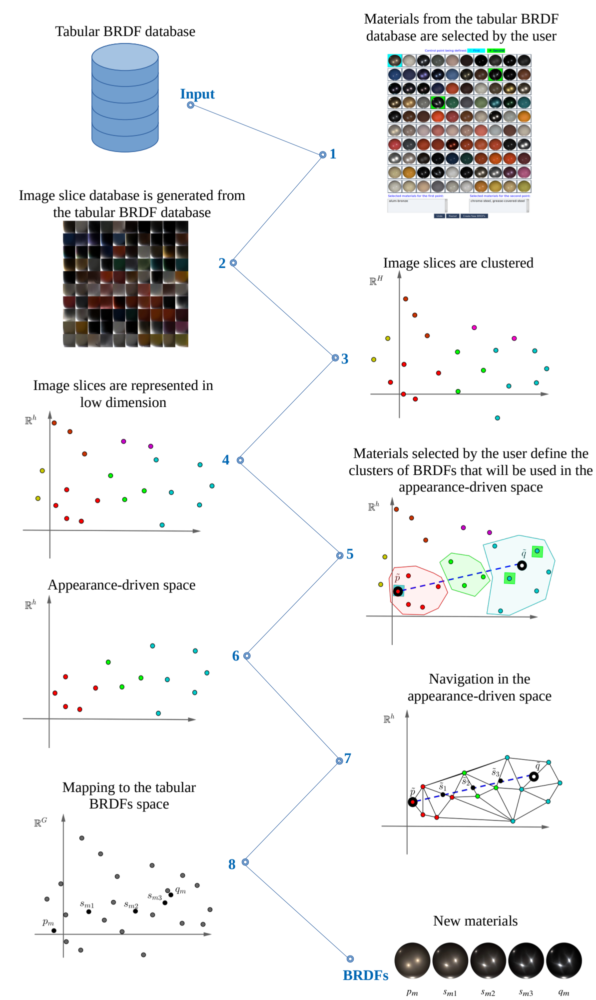

Fig. 2. Summary of the stages of the proposed pipeline. The space $\mathbb{R}^H$ represents the image slice database in its original dimension, the space $\mathbb{R}^h$ represents the image slice database in its low dimension, and the space $\mathbb{R}^G$ represents the tabular BRDF database, in which $G > H > h$. Figure extracted from Nunes et al. [1].

A. Results

New materials were created using the proposed pipeline. We used the Mitsubishi Electric Research Laboratories (MERL) BRDF database, which contains 100 isotropic materials captured from the real world. In addition, we also used image slices from BRDFs of that database.

The complexity of our pipeline is the sum of the complexities of the dimensionality reduction method (MDS or ISOMAP, which are, respectively, $O[N^3]$ [29] and $O[2N^3]$ [30]), Delaunay Triangulation ($O[N^{\lceil h/2 \rceil}]$ [31]), and clustering algorithm (k-means or k-medoids, which are, respectively, $O(kNi)$ and $O(k(N-k)^2)$ [32]). Here, N represents the number of image slices, h is the dimension of the low-dimensional space, k is the number of clusters, and i is the number of iterations. Thus, for instance, the cost of our method using ISOMAP and k-means is $O[2N^3] + O[N^{\lceil h/2 \rceil}] + O(kNi)$.

We selected the best clustering results according to the average silhouette index. K-means presented the greatest average silhouette for all elements of the database compared to k-medoids. The MDS and ISOMAP dimensionality reduction methods were compared in our pipeline. Please refer to Nunes et al. [1] for a comprehensive comparison of the results.

Figures 3 and 4 show examples of renderings of materials created using our pipeline. The first material is defined by the average of the blue-metallic-paint2 and green-metallic-paint2, and the second is pearl-paint. In this experiment, the results from the MDS and ISOMAP methods were compared.

Figures 5 and 6 show the Root Mean Square Error (RMSE) results for the sequence of all materials generated in this experiment. Comparing the renderings and RMSE results, notice that for the MDS method, the sequence started with a smooth transition in the appearance of the materials. As the materials approach of the pearl-paint, the new materials did not present an appearance next to this material. For the ISOMAP method, as the materials approach the pearl-paint, the new materials presented an appearance next to this material.

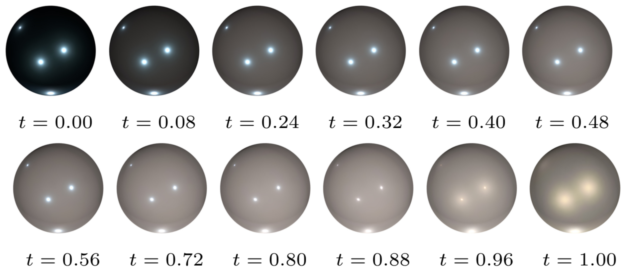

Fig. 3. Renderings of the materials obtained from the transition between two materials of interest. The first material ($t = 0$) is defined by the average of blue-metallic-paint2 and green-metallic-paint2, and the second ($t = 1$) is pearl-paint. MDS method was used in this experiment. Figure extracted from Nunes et al. [1], [22].

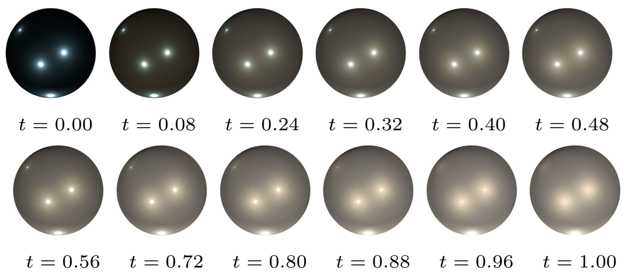

Fig. 4. Renderings of the materials obtained from the transition between two materials of interest. The first material ($t = 0$) is the average of the blue-metallic-paint2 and green-metallic-paint2, and the second ($t = 1$) is pearl-paint. ISOMAP method was used in this experiment. Figure extracted from Nunes et al. [1], [22].

These renderings illustrate the navigation performed in the appearance-driven space and show a smooth transition between the materials in the original space of BRDFs.

B. Conclusions

This study proposed a pipeline to create new custom materials from a tabular BRDF database. To illustrate our approach the MERL BRDF database [5] was used. Thus, new materials were created using the pipeline with MDS and ISOMAP dimensionality reduction methods. They presented interesting results with smooth transition through navigation in the appearance-driven space and fidelity to material proprieties.

For a sequence the new materials created, the ISOMAP method presented transition that changed more smoothly the appearance of the materials compared to MDS method (see examples in Figure 7). In addition, materials from pipeline using ISOMAP method provided transitions that also contain appearance of materials different from those selected by the user as reference, increasing the variety of materials.

IV. Techniques for BRDF Evaluation

BRDF evaluation techniques compare different BRDFs to know how similar they are. This is an essential topic during the development of a new BRDF representation since it enables the researcher to compare reflectance data from this new BRDF representation with reference tabular or analytical data. This way, different techniques can be used to evaluate BRDFs.

In Nunes et al. [23], we classified a set of 12 techniques from the literature into three categories: comparison functions, rendered images, and plots. These techniques were used to evaluate three classical reflectance models and one state-of-the-art BRDF representation. In addition, we proposed a new comparison function called Mean Absolute Error Peak Signal-to-Noise Radio that showed to be robust and stable.

A. Results

To illustrate the techniques studied, the MERL BRDF database [5] was used as ground truth. The reflectance models chosen were Blinn-Phong [6], Cook-Torrance [7], Ashikhmin-Shirley [10], and the start-of-art Bagher et al. model [11]. To represent the MERL BRDF database materials, we adopted, for the classical models, the parameters from Ngan et al. [33], and for the Bagher et al. model, the parameters provided by the model author [34]. Figure 8 shows the results obtained from this analysis for 86 materials, in which it is noticed that Bagher et al. model stood out.

Regarding rendered images and polar plots, during the evaluation of the 86 materials, the Bagher et al. model was the best model to represent the appearance of these materials. However, for reflectance features such as specular and Fresnel peaks, that model presented difficulty in reproducing them. An alternative would be using the Ashikhmin-Shirley model that represented these features well but sometimes demonstrated exceeded reflectance values.

B. Conclusions

This study classified 12 techniques to evaluate BRDFs into three categories: comparison functions, rendered images, and plots. Based on comparison functions, we presented a new one. These techniques were used to evaluate Blinn-Phong, Cook-Torrance, Ashikhmin-Shirley, and Bagher et al. models fitting materials from the MERL BRDF database. The results showed that the Bagher et al. model was the best among the reflectance models.

Using the three categories helps to better evaluate the BRDF fit, since they provide complementary information. We suggest using at least one technique from each of the three categories to get a comprehensive evaluation. From the ones we explored, the set of techniques that we suggest is RMSE, visual comparison and color-coded difference images, image slices and polar plot.

V. An Approach to Preprocess and Cluster a BRDF Database

Clustering a BRDF database provides materials separated into different groups. It can help in research that involves the selection of basis BRDFs, reconstruction of BRDFs, personalization of the appearance of materials, and finding material properties. Aiming to provide clusters of materials regarding their reflectance features, we propose a strategy to preprocess the BRDF database using image slices to provide an image slice database [24]. Thus, each material is represented by its main reflectance features, such as diffuse reflection, specular

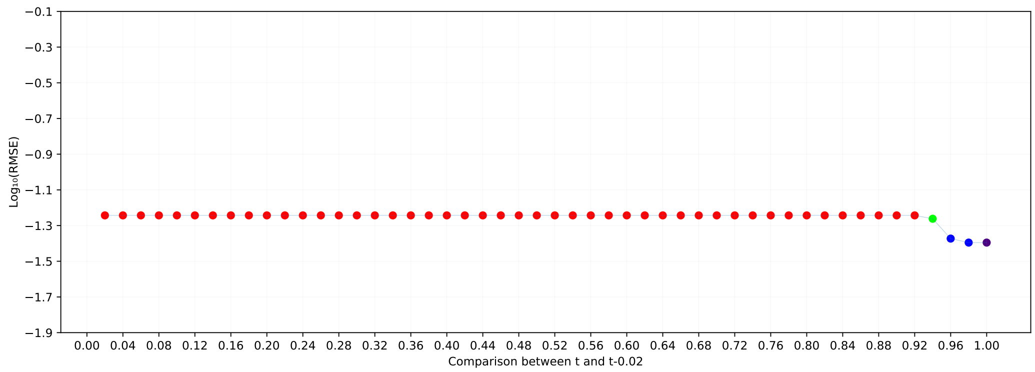

Fig. 5. RMSE of the comparison of materials obtained from the transition between two materials of interest. For this result the MDS method was used. The errors were mapped through a log transformation on base 10. Each different color represent a specific simplex in the appearance-driven space, in which the material belongs to. Figure extracted from Nunes et al. [1], [22].

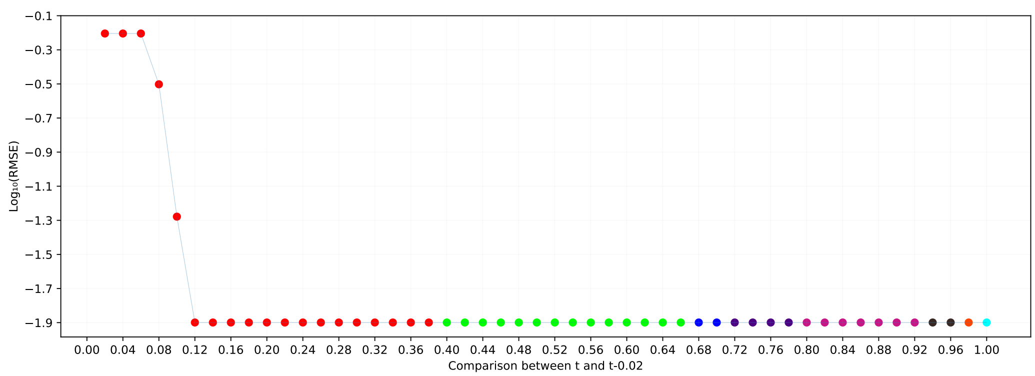

Fig. 6. RMSE of the comparison of materials obtained from the transition between two materials of interest. For this result the ISOMAP method was used. The errors were mapped through a log transformation on base 10. Each different color represent a specific simplex in the appearance-driven space, in which the material belongs to. Figure extracted from Nunes et al. [1], [22].

peaks, Fresnel effect, retro-reflection, and retro-reflection in grazing angles.

We performed an experimental study about the use of a dimensionality reduction method applied to the image slice database before clustering it. This study compared three clustering algorithms using statistical tests and their image slices from cluster results. In addition, we compared the results obtained with and without applying a dimensionality reduction method to the image slice database before clustering it.

A. Results

We chose the Locally-Linear Embedding (LLE) method to reduce the image slice database dimension before clustering it. The k-means, k-medoids, and spectral clustering algorithms were selected to perform the clustering of that database and compared. RMSE was used as a distance measure for these algorithms. It was a promising measure since the resulting cluster of materials presented similar reflectance features found in image slices. LLE presents a complexity of $O(N \log N) + O(Nhd^3) + O(sN^2)$, where d and s are, respectively, the number of nearest neighbors and embedding dimension [35]. Compared to k-means and k-medoids complexities (see Section III-A), spectral clustering is $O(N^3)$ [36].

The best low-dimensional image slice databases were selected to be used as input to the k-means, k-medoids, and spectral clustering algorithms. Thus, statistical tests were applied to the results of the clustering of such databases using these algorithms. According to the Friedman test, there is a statistically significant difference among the medians of the silhouette index of the clustering algorithms. The best overall results according to the median silhouette index for k-means, k-medoids, and spectral clustering were 0.5808, 0.5716, 0.5233. Thus, k-means presented the best overall result compared to the other algorithms.

B. Conclusions

This study presents a strategy to preprocess and cluster a BRDF database that considers the main reflectance features of the BRDFs. To preprocess the BRDF database, the image slice representation and the LLE dimensionality reduction method were used, resulting in low-dimensional image slice databases. To this end, the MERL BRDF database [5] was adopted, and the best low-dimensional image slice databases were found.

K-means presented the best overall result compared to the other algorithms. However, for some low-dimensional image slice databases, k-medoids presented better results. Therefore, we suggest finding the best low-dimensional representation of

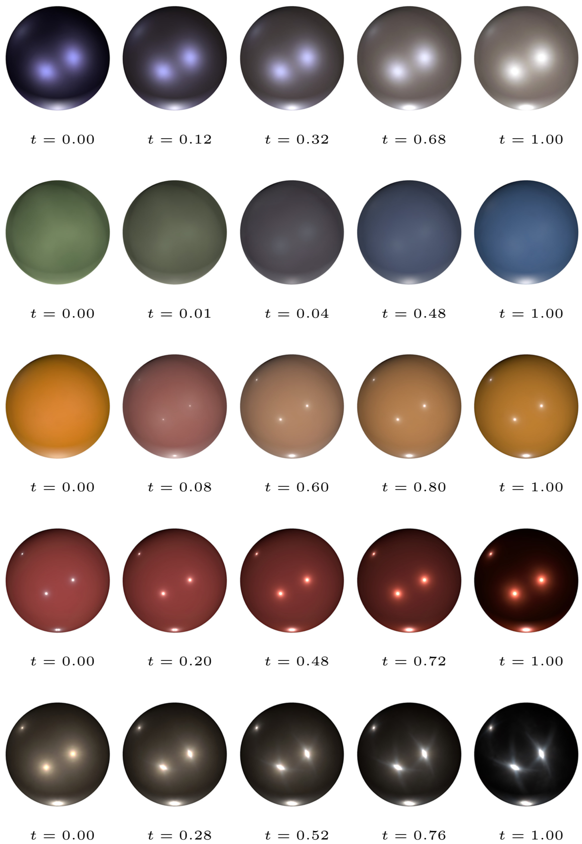

Fig. 7. Materials obtained using our approach with ISOMAP, illustrating transitions with: (first line) variation in the diffuse and specular colors (from blue-metallic-paint to silver-paint); (second line) variation in the material diffuse color (from green-latex to blue-rubber); (third line) variation in the material specularity (from orange-paint to specular-yellow-phenolic and yellow-matte-plastic); (forth line) variation in the material glossiness (from maroon-plastic to red-metallic-paint); (fifth line) different diffuse and specular reflectance properties (from alum-bronze to chrome-steel and grease-covered-steel); ISOMAP with 6 principal components and 29 neighbors and nearest-neighbor with 29 neighbors were used in this experiment. Figure extracted from Nunes et al. [1].

the database and using our evaluation approach to choose the clustering algorithm according to the best result of clustering of that database.

As the k-medoids algorithm keeps an original element of the database as representative of the cluster, it is an alternative to k-means for applications that requires using existing materials. Examples of these applications are the reconstruction of BRDFs and editing the appearance of materials.

From an analysis of the resulting clusters, we realized that the image slices had an influence in their results since the clusters, in general, presented predominantly specific reflectance features found in image slices. This analysis showed that our proposed approach is promising.

VI. Conclusion

This work aims to create new custom materials from a tabular BRDF database. Our pipeline presented interesting results, creating new materials that are influenced by the appearance of the reference materials (selected by the user). Besides, the results showed new plausible materials with visual richness and fidelity to the material properties. Through navigation in the appearance-driven space of BRDFs, our work generated sequences of materials presenting smooth appearance transitions.

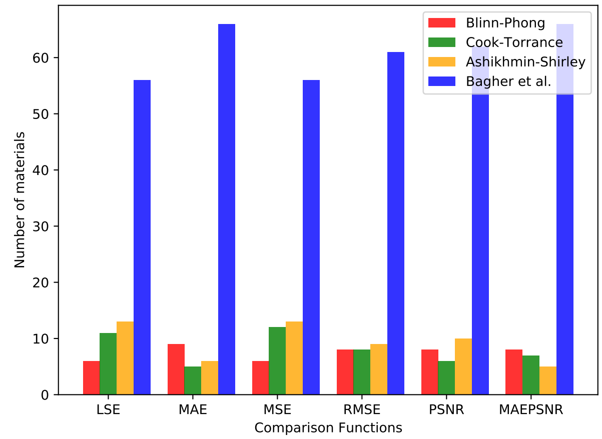

Fig. 8. Evaluation of 86 materials from the MERL BRDF database [5] fitted by reflectance models. It shows the number of materials that the reflectance model did the best fit according to the comparison function. Figure extracted from Nunes et al. [23].

Correlated to the main goal of this work, two studies were performed. The first one researched techniques to evaluate BRDFs. It presented promising results since these techniques presented a comprehensive evaluation that includes numeric and visual analysis. In addition, based on the results of this study, we adopted the RMSE comparison function to evaluate the transition of the new materials created. The second study refers to preprocessing and clustering a BRDF database. Its results showed that using image slices contributes to the clustering, providing materials separated into groups according to the main reflectance features. This improvement is a contribution to the development of the appearance-driven space.

A. *Challenges and limitations*

While developing the contributions presented in this master's dissertation, some challenges and limitations were found. In the first contribution (Section III), we needed to perform validation strategies with more subjective criteria to evaluate the appearance of the new materials. To this end, we adopted a comparison function and compared materials belonging to a sequence of materials. In the second contribution (Section IV), we had to compare the proposed comparison function with an existing one. Then, we developed a voting strategy regarding a set of comparison function results. In the third one (Section V), we needed to perform the proposed experiments in an extensive BRDF database. Thus, we divided the materials of the database into three parts, providing three databases.

B. *Future Work*

In future work, we propose to investigate the use of other nonlinear dimensionality reduction methods in clustering image slices and during the creation of the appearance-driven space. We also intend to develop a strategy to navigate the appearance-driven space that receives the contribution of the materials belonging to the minimum path. To evaluate the sequence of the new materials created, we would like to investigate strategies that use perceptual analysis and subjective criteria.

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PUBLICATIONS

From this master's dissertation, three papers were published in journals:

- Nunes et al. [1]. An Appearance-Driven Space to Create New BRDFs. Computer & Graphics. Elsevier, 2022;
- Nunes et al. [23]. Techniques for BRDF Evaluation. The Visual Computer. Springer, 2021;
- Nunes et al. [24]. An Approach to Preprocess and Cluster a BRDF Database. Graphical Models. Elsevier, 2022.

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