Binary Image Denoising via Incremental Neighborhood-Based Energy Optimization: A Markov Random Field Approach

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***Abstract*—A digital image may exhibit undesired noise variations, making it necessary to apply preprocessing methods to reduce noise while preserving the original structure. In this context, this paper proposed a Markov random field-inspired energy approach for optimizing binary noise reduction, considering a variable neighborhood set. Our algorithm combines Iterated Conditional Modes with Markovian prior information about the image to compute the energy associated with each pixel based on its local neighborhood, which is progressively expanded at each iteration. Experimental results on binary images corrupted with 10% random noise demonstrate that the proposed approach achieves an agreement of up to 99.85% with the original noise-free images. Future research will focus on incorporating more prior knowledge about the image to enhance the energy model, as well as its extension to grayscale images, along with the exploration of new metrics and configuration strategies.**

I. Introduction

Digital images are a visual representation obtained by sampling a continuous scene from the real world. However, the acquisition process can be affected by factors such as sensor limitations, transmission interference, or adverse environmental conditions, introducing undesired variations into the image. These variations, known as noise, significantly compromise the results of image processing tasks such as segmentation, edge detection, pattern recognition, or feature extraction, complicating analysis and interpretability [1].

Binary images are no exception; despite their simpler nature, the presence of noise can significantly degrade image quality, leading to poor performance in applications such as object recognition, medical imaging, and document analysis [2], [3], highlighting the need for effective noise reduction techniques. Thus, preprocessing methods for noise reduction become essential to restore degraded images while preserving their original structure and effectively eliminating noise.

Traditional noise reduction methods typically involve mathematical operations, filtering-based approaches, and statistical analyses focused on identifying and suppressing noise [1]. Among statistical approaches, more robust techniques, such as Markov Random Field (MRF) methods, combine a set of prior assumptions with the observed noisy image to produce statistical estimates of the original image.

These MRF-based methods remain relevant not only for tasks such as image restoration, but also for texture modeling and image segmentation [4]. Moreover, recent academic studies continue to build upon and enhance these noise reduction techniques by proposing novel variations and improvements. For instance, Cherukuri (2024) [5] introduced an energy function grounded in the Markov property, emphasizing that adjacent pixels in a 4-neighborhood are emphatically related.

Despite positive results, achieving recoveries of up to 99.19% of the original binary image, Cherukuri's energy function's reliance on a 4-adjacency neighborhood restricts optimization algorithms from capturing more complex attributes, such as edges and finer details [5]. Additionally, the inherent randomness of certain metaheuristics used for energy optimization, such as simulated annealing, can lead to pixel misclassifications. Furthermore, in regions with elevated noise density, the noise tended to form clusters, resulting in residual artifacts.

In this context, this paper aims to address these challenges by extending Cherukuri's energy function to consider variable neighborhood sets and by proposing an incremental multi-metric neighborhood algorithm based on iterative constrained modes to overcome the noise clustering tendencies observed in 4-adjacency neighborhood dependency. Furthermore, modifications to the energy formulation are introduced, and more extensive experiments are conducted on a real-world dataset. This work contributes to the field of image processing by demonstrating the effectiveness of variable neighborhood structures in MRF-based approaches for improving the quality of binary images affected by noise.

The remainder of this paper is organized as follows. Section II describes the binary image restoration problem. Section III reviews the relevant literature. Section IV describes the methodology employed in this study, formalizing the proposed energy optimization approach. Finally, Section V presents the results and discussion, while Section VI concludes the paper by summarizing key findings and highlighting potential directions for future research.

II. Objective and Problem Formulation

Consider a binary image B as a one-dimensional array of pixels with values b_i, where i represents the pixel index. One noise addition technique consists of simply inverting the values of a percentage n of randomly selected pixels in a clean source image. This stochastic process leads to image degradation, complicating subsequent analysis and interpretation. Figure 1 illustrates the effects of the random noise inversion process applied to an example image at varying percentages.

(a) Source (b) 10% Noise (c) 30% Noise

Fig. 1. Random Noise Addition on Binary Images.

In this way, MRF-based techniques can be used for image restoration by assuming the image is a realization of an MRF with a distribution that captures the spatial context of the scene. Thus, given the prior distribution of the actual image and the observed noisy one, this problem can be formulated as an optimization problem [6]. Therefore, the primary objective of this paper is to denoise binary images corrupted by random noise using MRF-based optimization strategies.

III. Related Works

The use of statistical methods for noise reduction and image restoration is well established. Classic works, such as those by Geman [7] (1984), were fundamental in introducing techniques from statistical mechanics into image processing. This work specifically introduced an early MRF model for images, emphasizing the energy function as a principled approach to modeling image features, and serving as a foundational reference for subsequent research in the field.

For example, Besag [8] (1986) emphasizes the relationship between the characteristics of an actual scene and its representation by a non-degenerate MRF. This formulation, combined with a prior model of the actual scene, enables the estimation of the underlying clean image. Furthermore, this paper employs the Iterated Conditional Modes (ICM) method, which facilitates efficient estimation with rapid convergence. However, more general optimization algorithms, such as Simulated Annealing (SA), can also be applied to minimize the energy function, as demonstrated by Dubes et al. [9].

More recently, Cherukuri [5] (2024) proposed an approach for denoising binary images based on an MRF with a novel energy function optimized via SA. This energy function quantifies the relationship between the noisy and the desired clean image. The study subsequently compares the performance of this energy function when subject to optimization through ICM and SA methods. The latter achieves agreement results of up to 99.21%, compared to 96.21% obtained by ICM.

However, the reliance of Cherukuri's energy function on a 4-adjacency neighborhood restricts optimization algorithms from capturing more complex attributes, such as edges and finer details. Additionally, the inherent randomness of specific metaheuristics used for energy optimization, such as SA, can lead to pixel misclassifications. Furthermore, in regions with elevated noise density, the noise tends to form clusters, resulting in residual artifacts. To address these issues, this paper proposes an extension of Cherukuri's energy function to achieve a more effective noise reduction approach.

IV. Methodology

To overcome the challenges identified in Cherukuri's work, this paper extends the energy function by considering variable neighborhood sets. It proposes an incremental multi-metric neighborhood algorithm based on ICMs to address the noise clustering tendencies observed with 4-adjacency neighborhood dependencies. Furthermore, modifications to the energy formulation are introduced to incorporate normalized terms and to adjust the energy function to a non-negative formulation, as detailed in the following subsections.

A. Reference Energy Model

Cherukuri's energy function, when utilizing the Markov property to model this optimization problem, is primarily based on two properties closely related to the nature of images:

$P1$) There is a strong pixel-to-pixel correlation between the target (restored) image and the source (noisy) image;

$P2$) There is a strong relationship between neighboring pixels within each image;

To represent property $P1$, an associated energy function is used to express the pixel-to-pixel relationship $\{x_i, y_i\}$, reflecting the tendency for the restoring image X and the original noisy image Y to be overall similar. For this purpose, a simple energy term $-x_i y_i$ is employed to favor configurations in which x_i, y_i share the same sign. This equal signal configuration results in a lower energy, while configurations with opposite signs yield higher energy.

Conversely, to capture the correlation between neighboring pixels in the restoring image X, as illustrated by property $P2$, an additional energy term is introduced: $-x_i x_j$, where i, j denote indices of neighboring pixels in X. This term encourages local smoothness by assigning lower energy when neighboring pixels share the same sign, and higher energy otherwise.

Additionally, Cherukuri introduces an extra term hx_i, which is not related to the previously mentioned Markovian properties. This term introduces a bias into the energy function by favoring pixel values with a specific sign. Finally, Cherukuri sums the corresponding energy terms, resulting in the energy model illustrated in Equation 1:

$$E(X,Y) = h\sum_{i=0}^{X} x_i - \beta \sum_{j=0}^{\{i,j\}} x_i x_j - \eta \sum_{i=0}^{X} x_i y_i \qquad (1)$$

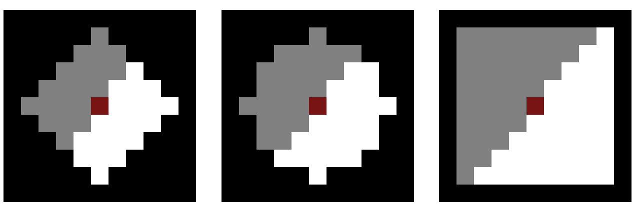
Where η, β are positive constants and $x_i, y_i \in \{-1, +1\}$.

B. Energy Improvements

Although the proposed energy function yields positive results, achieving image agreement of up to 99.21%, its reliance on a 4-adjacency neighborhood limits the ability of optimization algorithms to capture more complex attributes, such as edges and finer details. Additionally, specific optimization methods, such as SA, can lead to pixel misclassifications in regions with high noise density, resulting in residual artifacts due to noise clustering.

To address these challenges, this paper extends the original function by considering the image as a metric space, allowing the selection of a variable neighborhood set based on a provided distance parameter. As a direct consequence, a degree of freedom is gained in choosing various metrics. This work adopts three classical metrics: Manhattan, Euclidean, and the Chebyshev Metric.

Moreover, a symmetric partitioning strategy of the neighborhood is also adopted, as shown in Figure 2, to ensure that the interaction between two neighboring pixels is not accounted for twice in the energy computation, as would occur with a full neighborhood. In particular, the lower-right fraction of the neighborhood is preferred, as it includes only pixels that have not yet been visited in the current iteration, and are therefore unaffected by recent changes. This avoids the need for a second image and aligns with the operational characteristics of the ICM algorithm.



(a) Partial Manhattan (b) Partial Euclidean (c) Partial Chebyshev

Fig. 2. Illustration of the neighborhood metrics for distance $d = 4$. The partial neighborhood is highlighted in white, while the full neighborhood comprises the union of the gray and white regions.

Furthermore, to ensure non-negativity in the energy model, additional modifications are introduced to both the pixel domain and the energy function itself. This non-negativity prevents term cancellation, promotes faster convergence, and yields a more intuitive interpretation. Starting with the pixel domain, a more standard definition is adopted, where each pixel is defined as $x_i \in \{0, 1\}$, in contrast to Cherukuri's formulation, which employs $x_i \in \{-1, 1\}$.

Next, modifications to the energy terms are also made to ensure non-negativity while preserving the representation of the respective Markovian properties. To this end, the logical equality operator $\overline{a \oplus b}$ (i.e., ¬XOR) is employed, which will hereafter be denoted as $Eq(a, b)$ for clarity. In addition to these changes, the biased term (hx_i) has been removed, as it was found to have no significant impact on the algorithm's results. Finally, the energy is also normalized for the total image size, yielding more interpretable values and enabling fair comparisons between images of different dimensions.

$$E(X, Y, m_d) = \frac{\beta \sum_{i=0}^{X} \sum_{j=0}^{M_d\{i,j\}} Eq(x_i, x_j) + \eta \sum_{i=0}^{X} Eq(x_i, y_i)}{\sqrt{X_{size}}} \tag{2}$$

The resulting energy function is presented in Equation 2, where M_d represents the metric M associated with a distance d, both to be provided by the user.

C. Proposed Algorithm

Once the energy formulation is established, it is crucial to address the optimization strategies employed to minimize energy and, consequently, remove noise from the image. Accordingly, this work proposes an extension to the ICM algorithm by expanding its strategy to an incremental neighborhood approach. The central hypothesis is that performing multiple ICM iterations with progressively larger neighborhood sets can effectively remove noise without leaving residual artifacts.

This strategy is fundamentally supported by the energy function's ability to accommodate multiple neighborhood systems. Moreover, it allows for the adoption of different weights and metrics at each iteration by parameterizing a configuration vector. Finally, the algorithm can be further optimized by computing only the variation in energy caused by local modifications, avoiding the need to recalculate the entire energy. This strategy is illustrated in the Algorithm 1.

Algorithm 1 Accumulative ICM

Require: Source Image Y; Configuration Vector C

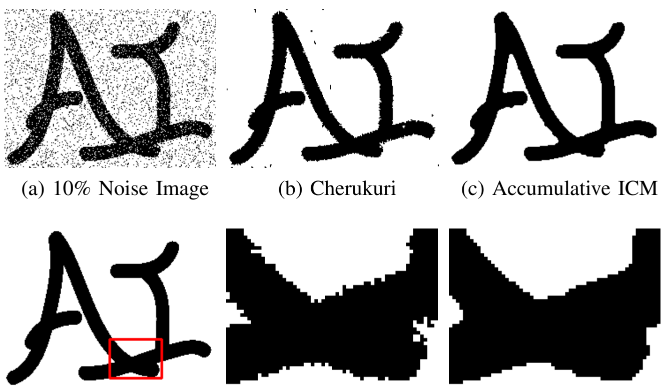
```
Image X ← Y
for each conf ∈ C do
  E ← energy(X, Y, conf)
  for r = 0 to X.rows do
    for c = 0 to X.cols do
      Pixel p ← ¬(X_rc)
      E' ← E + ΔE(X, Y, p, conf)
      if E' < E then
        X_rc ← p
        E ← E'
      end if
    end for
  end for
end for
return X
```

V. Results and Discussion

This section presents the results and analysis of computational experiments conducted to evaluate the proposed energy model. A comparative analysis with the original method is also provided under different configurations. The experiments include results on a single image as well as on a real dataset of numerical images across varying levels of noise.

A. Single Image Visual Comparison

To illustrate the performance of the Accumulative ICM algorithm, a qualitative analysis was conducted on a single image corrupted with 10% random noise. It can be observed that, while Cherukuri's model resulted in poor edge preservation and residual artifacts, the proposed approach achieved a more visually coherent reconstruction, exhibiting no residual artifacts and fewer visual distortions along the borders, as shown in Figure 3. In terms of agreement, Cherukuri's model achieved 99.17%, whereas the proposed method achieved 99.53%.



(a) 10% Noise Image (b) Cherukuri (c) Accumulative ICM

(d) Source Image (e) Close-up Cherukuri (f) Close-up Acc. ICM

Fig. 3. Denoising performance between Cherukuri's SA method based on 4-adjacency energy and the proposed Accumulative ICM algorithm with an incremental neighborhood approach applied to the new energy model.

Furthermore, while Cherukuri's strategy relied on Simulated Annealing with a total of 30 iterations, the proposed method achieved better results using only 6 iterations under the Manhattan metric. This efficiency is attributed to the inherently greedy nature of the base ICM algorithm, which, when combined with an incremental neighborhood strategy, enables rapid energy convergence, leading to better outcomes.

B. Configuration Comparison

To further evaluate the performance of the proposed algorithm, a series of tests was conducted on numerical images extracted from the MNIST dataset [10]. A total of 200 images (20 of each digit) were selected, resized, converted to binary via thresholding, and subjected to varying levels of random noise. Subsequently, a series of experiments was conducted to evaluate image restoration using various energy configurations. The results are summarized in Table I.

Among the tested configurations, the incremental neighborhood strategy demonstrated superior performance, consistently achieving the highest average agreement with the original images. Additionally, the use of a partial neighborhood configuration contributed to further improvements, especially when combined with the incremental strategy. Furthermore, the incorporation of new metrics had a positive impact, with the inclusion of the Chebyshev metric enabling the algorithm to produce more accurate results under high noise levels. The best results achieve agreement rates of up to 99.85% on images corrupted with 10% noise.

TABLE I
SUMMARY OF IMAGE RESTORATION PERFORMANCE

Metric	Distance	10% Noise		20% Noise		30% Noise		40% Noise	
		Full	Partial	Full	Partial	Full	Partial	Full	Partial
Manhattan	1	99.73	96.42	98.66	87.96	94.19	76.64	79.52	63.69
	2	99.76	**99.85**	99.19	99.58	97.73	98.88	92.87	93.58
	$\{1,2\}$	99.84	**99.85**	99.58	99.59	98.95	98.72	93.30	91.04
Euclidean	1	99.73	96.42	98.66	87.96	94.19	76.64	78.52	63.69
	2	99.76	**99.85**	99.19	99.58	97.73	98.88	92.87	93.58
	$\{1,2\}$	99.84	**99.85**	99.58	99.59	98.95	98.72	93.30	91.04
Chebyshev	1	99.81	99.76	99.42	98.77	98.41	94.31	93.61	79.54
	2	99.50	99.80	98.13	99.31	93.54	97.50	84.60	90.62
	$\{1,2\}$	99.74	99.84	99.41	**99.64**	98.53	**99.16**	95.46	**96.33**

VI. CONCLUSION

This work proposed an MRF-inspired energy model for optimizing binary noise reduction. The proposed method employs an ICM-based algorithm to minimize a novel energy function that incorporates multi-metric variable neighborhoods across iterations. Experimental results on images corrupted with 10% noise demonstrate that the proposed approach produces restored images with no residual artifacts and fewer visual distortions along the borders, achieving recovery rates of up to 99.85%. Future research will focus on enhancing the energy model by incorporating additional prior knowledge about the image, as well as extending it to grayscale images and exploring new configuration strategies.

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