Uncertainty Quantification in Reservoir History Matching Using the Ensemble Smoother

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***Abstract*—Ensemble-based methods have been widely used in uncertainty quantification, particularly, in reservoir history matching. The search for a more robust method which holds high nonlinear problems is the focus for this area. The Ensemble Kalman Filter (EnKF) is a popular tool for these problems, but studies have noticed uncertainty in the results of the final ensemble, high dependent on the initial ensemble. The Ensemble Smoother (ES) is an alternative, with an easier impletation and low computational cost. However, it presents the same problem as the EnKF. The Ensemble Smoother with Multiple Data Assimilation (ES-MDA) seems to be a good alternative to these ensemble-based methods, once it assimilates tha same data multiple times. In this work, we analyze the efficiency of the Ensemble Smoother and the Ensemble Smoother with multiple data assimilation in a reservoir histoy matching of a turbidite model with 3 layers, considering permeability estimation and data mismatch.**

I. Introduction

One of the main challenges for oil and gas industry is to diminish uncertainty referred to a possible extraction of hydrocarbon. The search for a geometric model that represents the reservoir starts from the first data obtained before the production, such as seismic and environmental geology, and keeps being updated as long as new data is available. The final model is expected to predict, through flow simulators, the sensitive of the reservoir when different methods of oil recovery are applied.

Reservoir characterization has been in focus for a several years. One of this techniques is the automatic history matching, where production has already been done for some years, collaboratively with data observation, and the focus is to improve the reservoir model, through its parameters, intending to obtain similar simulated data with the ones collected during the production period. To perform history matching, besides the reservoir model, we need an optimization algorithm to estimate such parameters. Among all methods applied in reservoir engineering and history matching, we can cite the gradient-based Gauss-Newton [11], Levenberg-Marquardt [11] and BFGS [5]. However, computing the gradient of the objective function may not be easy. Therefore, recent researches have focused in method based on the Kalman Filter (KF) [7], a recursively filter for state estimation of a linear system.

The Ensemble-Kalman Filter (EnKF) is a Monte Carlo implementention of the KF, where the mean of an ensemble of states gives a good estimation of the model. The EnKF avoids some limitations of the KF. One example is that EnKF holds nonlinear systems. It was introduced by Evensen [4] and it has a large application in the science industry, such as economic science, geophysics, statistics and, more famous, in meteorology. Its first application in reservoir history matching was in 2001, by Lorentzen [8]. Later, in 2002, Nævdal [6] published a study using EnKF to estimate permeabilities of oil reservoirs. EnKF assimilates the data sequentially in time, which can be inconvenient when the objective is to incorporate the history matching in a flow simulator. Considering this limitation of EnKF, the Ensemble Smoother (ES), proposed by van Leeuwen and Evensen [12] does not assimilate data sequentially in time. Instead, it computes a global update as a result of assimilation of all data available. The major advantage of the ES is that it avoids restarts the simulator at each time step, which can be easier to implement and computational cost reducing. Nevertheless, Reynolds *et al.* [10] showed that every assimilation step of EnKF is similar to applying a Gauss-Newton iteration with full step. It means that the ES is similar to one iteration of Gauss-Newton. Hence, it may not provide acceptable data matches when applied to reservoir history matching. Based on this observation, Emerick and Reynolds [2] proposed the Ensemble Smoother with Multiple Data Assimilation (ES-MDA), which assimilates the same data multiple times, expecting that data match is improved at each data assimilation.

The first section of this work we formulate the problem from a Bayesian point of view. The second section gives a brief review of ensemble-based methods, focusing on EnKF, ES and ES-MDA. In the third section is presented the reservoir model used in this work and, in section five, we present and compare the results obtained by checking the results of ES and ES-MDA.

II. Bayesian Formulation of the Problem

Let m be a N_m-dimensional random vector containing all parameters of a model. The theoretical data related to m can be described as

$$d = g(m) \tag{1}$$

where d is a N_d-dimensional vector and the function g is the function that relates m with d. In reservoir simulation, given a vector d, the uncertainty between d and a certain model vector m can be described as

$$d = g(m) + \epsilon \tag{2}$$

where $\epsilon = \mathcal{N}(0, C_D)$, C_D is the covariance matrix of d. In this case, we say that $d \sim \mathcal{N}(g(m), C_D)$. Thus, the probability density function (PDF) of d is

$$f(d|m) = \alpha_1 \; exp\left\{ -\frac{1}{2}(d - g(m))^T C_D^{-1}(d - g(m) \right\} \tag{3}$$

The Likelihood function of m, $\mathcal{L}(m)$, is defined by Equation 3, where $d = d_{obs}$ a vector of observations, $\mathcal{L}(m) = f(d_{obs}|m)$. Using Bayes' theorem to write $f(m|d_{obs})$:

$$\begin{aligned} \pi(m) = f(m|d_{obs}) &= \frac{f(d_{obs}|m)f(m)}{f(d_{obs}} \\ &= \frac{f(d_{obs}|m)f(m)}{\int_D f(d_{obs}|m)f(m)dm} \\ &= \alpha_2 \; \mathcal{L}(m)f(m) \end{aligned} \tag{4}$$

In Equation 4 α_2 is a normalizing constant that guarantees that $\int_D \pi(m)dm = 1$. Assuming that m is Gaussian with mean m_{prior} and covariance C_M, we can write $\pi(m)$ as

$$\begin{aligned} \pi(m) &= \alpha_2 \; \mathcal{L}(m)f(m) \\ &= \alpha \; exp\left\{ -\frac{1}{2}(m - m_{prior})^T C_M^{-1}(m - m_{prior}) \right\} \\ &\times exp\left\{ -\frac{1}{2}(g(m) - d_{obs})^T C_D^{-1}(g(m) - d_{obs}) \right\} \\ &= \alpha \; exp\{-O(m)\} \end{aligned} \tag{5}$$

where

$$O(m) = O_m(m) + O_d(m) \tag{6}$$

with

$$O_m(m) = \frac{1}{2}(m - m_{prior})^T C_M^{-1}(m - m_{prior}) \tag{7}$$

and

$$O_d(m) = \frac{1}{2}(g(m) - d_{obs})^T C_D^{-1}(g(m) - d_{obs}) \tag{8}$$

Finding a vector m that minimizes $O(m)$, also gives us a maximum value of $\pi(m)$. Thus, we say that $O(m)$ is the objective function that we want to minimize.

A. Maximum a Posteriori Estimate

Although the complete solution of the inverse problem be the posterior probability distribution for the model parameters, in reservoir simulation, the complete characterization of this PDF is impracticable. However, there is an exception for the linear model, with Gaussian distributon, as stated in the previous subsection. Assuming that $d = Gm$, where G is the sensitivity matrix that linearly relates d with m, it is known that if m is a minimum of $O(m)$, then $\nabla O(m) = 0$.

$$\begin{aligned} \nabla O(m) = C_M^{-1}&(m - m_{prior}) \\ &+ G^T C_D^{-1}(Gm - d_{obs}) \end{aligned} \tag{9}$$

Adding and subtracting Gm_{prior} to $(Gm - d_{obs})$ and setting $\nabla O(m) = 0$:

$$\begin{aligned} 0 = C_M^{-1}&(m - m_{prior}) \\ &+ G^T C_D^{-1}(Gm - d_{obs} + Gm_{prior} - Gm_{prior}) \end{aligned}$$

$$\begin{aligned} 0 = C_M^{-1}&(m - m_{prior}) \\ &+ G^T C_D^{-1} G(m - m_{prior}) + G^T C_D^{-1}(Gm_{prior} - d_{obs}) \end{aligned}$$

$$\begin{aligned} (C_M^{-1} + G^T C_D^{-1} G)&(m - m_{prior}) = \\ &G^T C_D^{-1}(Gm_{prior} - d_{obs}) \end{aligned}$$

$$\begin{aligned} m - m_{prior} = (&C_M^{-1} \\ &+ G^T C_D^{-1} G)^{-1} G^T C_D^{-1}(Gm_{prior} - d_{obs}) \end{aligned}$$

$$\begin{aligned} m &= m_{prior} \\ &+ (C_M^{-1} + G^T C_D^{-1} G)^{-1} G^T C_D^{-1}(Gm_{prior} - d_{obs}) \end{aligned} \tag{10}$$

We say that a vector m obtained from Equation 10 is the maximum a posteriori estimate of m given a vector of observed data d_{obs}, and we denote by m_{map}.

Another possibility of computing m_{map} comes from the matrix inversion property

$$\begin{aligned} (G^T C_D^{-1} G + C_M^{-1})^{-1} G^T C_D^{-1} \\ = C_M G^T (C_D + G C_M G^T)^{-1} \end{aligned} \tag{11}$$

Thus, Equation 10 can be rewritten as

$$\begin{aligned} m &= m_{prior} \\ &+ C_M G^T (C_D + G C_M G^T)^{-1}(Gm_{prior} - d_{obs}) \end{aligned} \tag{12}$$

Note that computing m_{map} using Equation 10 needs to solve a $N_M \times N_M$ matrix problem

$$(C_M^{-1} + G^T C_D^{-1} G)x = G^T C_D^{-1}(Gm_{prior} - d_{obs}) \tag{13}$$

while Equation 12 requires the solution of the $N_D \times N_D$ matrix problem

$$(C_D + G C_M G^T)y = (Gm_{prior} - d_{obs}) \tag{14}$$

Since computational efficiency is closely related to the size of the matrix problem, solvin Equation 14 seems to be more efficienty when $N_D < N_M$, i.e., the number of observations less than the number of model parameters, which is plausible assuming reservoir history matching problems.

III. Ensemble-Based Methods

In this section it is explained the concepts of the Ensemble Smoother (ES). We first present the Ensemble Kalman Filter (EnKF) and derive ES as a Kalman-Filter-Based method.

A. Ensemble Kalman Filter (EnKF)

The EnKF was proposed by Evensen [4] as a Monte Carlo Method in which the mean and covariance are estimated from an ensemble of states, which are updated sequentially in time. Typically, the number of ensemble members is much smaller than the number of the number of model parameters, whcich are the unknowns of the problem. Hence, the covariance estimate of the problem is most of the time a low-rank approximation. This problem leads EnKF to search for a solution of the problem in the space spanned by the initial ensemble [3].

Let N_e be number of member of the ensemble and m_j, $j = 1, \cdots, Ne$ model parameters members of the ensemble. The approximation of the covariance matrix C_M is given by:

$$C_M \approx \frac{1}{N_e - 1} \sum_{j=1}^{N_e} (m_j - \bar{m})(m_j - \bar{m})^T \tag{15}$$

where $\bar{m} = \frac{1}{N_e} \sum_{j=1}^{N_e} m_j$. Based on Equation 12, we want to formulate the EnKF update formula assuming the approximation of the covariance matrix C_M.

$$\begin{aligned} C_M G^T &\approx \frac{1}{N_e - 1} \sum_{j=1}^{N_e} (m_j - \bar{m})(m_j - \bar{m})^T G^T \\ &\approx \frac{1}{N_e - 1} \sum_{j=1}^{N_e} (m_j - \bar{m})[G(m_j - \bar{m})]^T \\ &\approx \frac{1}{N_e - 1} \sum_{j=1}^{N_e} (m_j - \bar{m})(d_j - \bar{d})^T \\ &\approx C_{MD} \end{aligned} \tag{16}$$

where $d_j \sim \mathcal{N}(g(m_j), C_D)$. We say that the matrix C_{MD} is the approximation from the ensemble of the cross covariance matrix between the model parameters and the theoretical data of the ensemble members. Another approximation using the ensemble of the Equation 12 is

$$\begin{aligned} G C_M G^T &\approx \frac{1}{N_e - 1} \sum_{j=1}^{N_e} G(m_j - \bar{m})(d_j - \bar{d})^T \\ &\approx \frac{1}{N_e - 1} \sum_{j=1}^{N_e} (d_j - \bar{d})(d_j - \bar{d})^T \\ &\approx C_{DD} \end{aligned} \tag{17}$$

Substituting Equations 16 and 17 in 12, we get that the update formula of the EnKF is

$$m_j^a = m_j^f + C_{MD}(C_D + C_{DD})^{-1}(d_j - d_{u,j}) \tag{18}$$

The superscript a reffers to the analysis step and f to the forecast step of EnKF. Because we want to analyse the quantification of uncertainty of the model ensemble, we assume some sampling errors by adding a perturbation in the vector of observed data d_{obs} for each ensemble member. Thus, we define $d_{u,j} = \mathcal{N}(g(m_j), C_D)$.

For the problems with time dependent variables, such as pressure and saturation, the theoretical data is represented by $d_j = g(m_j, p_j)$. Thus, an augmented vector of state is defined as

$$y = [m^T p^T]^T \tag{19}$$

In this case, it is possible to use all equations derived before to assimilate data sequentially in time for each time step for the N_y-dimensional vector y

$$y_j^{n,a} = y_j^{n,f} + C_{YD}(C_D + C_{DD})^{-1}(d_j^n - d_{u,j}^n) \tag{20}$$

where the superscript n indicates the time step assimilation and

$$C_{YD} = \frac{1}{N_e - 1} \sum_{j=1}^{N_e} (y_j - \bar{y})(d_j - \bar(d))^T \tag{21}$$

where $\bar{y} = \sum_{j=1}^{N_e} \frac{1}{N_e} y_j$

B. Ensemble Smoother

Van Leeuwen and Evensen [12] proposed the Ensemble Smoother (ES) based on EnKF, but assimilating all data only once, avoiding restart reservoir simulation in each time step, which can improve time consuption and ease implementation. Because all data are assimilated simultaneously, we only need to consider the model parameters vector m, intead of the augmented vector y, as in EnKF. It makes ES as a parameter estimate method [2]. Its formulation is similar to the EnKF, and we write the analyzed vector of model parameters as in Equation 18.

C. Ensemble Smoother with Multiple Data Assimilation

The Ensemble Smoother with Multiple Data Assimilation (ES-MDA) was introduced by Emerick and Reynolds [2] aiming to improve the quality of history matching of data. It uses the ES as basis, but assimilating all data N_a times, expecting that quality of matching becomes better in each iteration. The only difference between ES and ES-MDA in the analysis update is that we need to premultiply the covariance matrix C_D by an inflant factor α_j, $j = 1, \cdots, N_e$.

$$\tilde{m}_j^a = \tilde{m}_j^f + C_{MD}(\alpha_j C_D + C_{DD})^{-1}(d_j - d_{u,j}) \tag{22}$$

In this formulation, $d_{u,j} \sim \mathcal{N}(d_{obs}, \sqrt{\alpha_j} C_D^{1/2})$ and the unique assumption under α is that

$$\sum_{j=1}^{N_e} \frac{1}{\alpha_j} = 1 \tag{23}$$

If α_j is chosen satisfying Equation 23, it is ensured that

$$E[\tilde{m}_j^a] = m_{map} \tag{24}$$

m_{map} defined in Equation 10, and

$$cov(\tilde{m}_j^a) = C_{map} \tag{25}$$

where C_{map}, defined in Equation 10, is

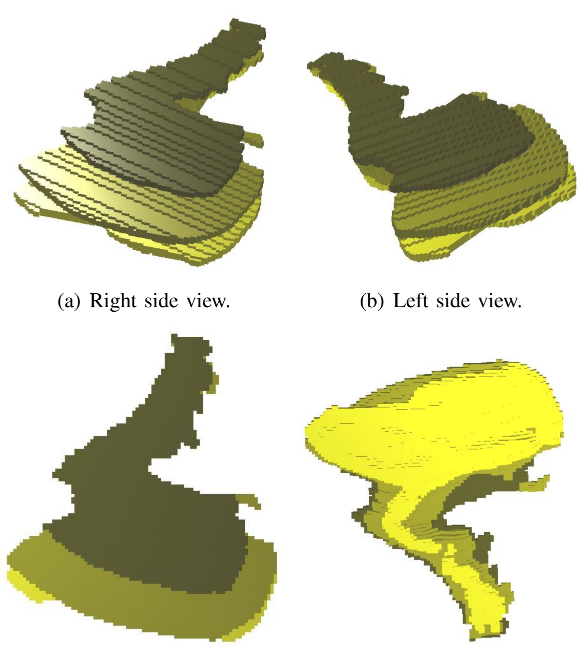
$$C_{map} = (C_D + GC_M G^T)^{-1} \tag{26}$$

The ES-MDA algorithm follows:

1. Choose the number of data assimilations N_a and the coefficients α_i, $i \in \{1, \cdots, N_a\}$;
2. For $i = 1$ to N_a:
   a) Run the ensemble from time zero.
   b) Perturb the observation vector d_{obs} for each ensemble member using $d_{u,j} = d_{obs} + \sqrt{(\alpha_i)} C_D^{1/2} z_d$, $j = 1, \cdots, Ne$ and $z_d \sim \mathcal{N}(0, I_d)$.
   c) Update the ensemble using Equation 22.

IV. Turbidite Reservoir Model

The turbidite reservoir model used in this work is presented in [1]. It is built using four single-valued B-Spline curves to delimit the width and thickness boundaries. The curves are connected with semi ellipses and, as long as turbidite lobes are commonly found in deep water, we use a bottom surface to simulate the submarine ground. The model is depicted in Figure 1.



(a) Right side view. (b) Left side view.

(c) Upper view. (d) Bottom view.

Fig. 1. Turbidite lobe reservoir model.

To perform well-tests in the reservoir model, it is assumed some simplifying hypothesis listed below:

- At time $t = 0$ the reservoir is in equilibrium, i.e., pressure is the same in all layers;
- Homogeneous in each layer and isotropic reservoir;
- Single-phase and isothermal flow with constant viscosity;
- Rock formation with low and constant compressibility;
- Constant production rate q.

V. Uncertainty Analysis

In this section we analyze the performance of two ensemble-based methods: the Ensemble Smoother (ES) and Ensemble Smoother with Multipe Data Assimilation (ES-MDA). For both methods we used an ensemble with $N_e = 50$ members. For the ES-MDA, we used $N_a = 10$. All members are vectors containing the permeability in each layer of the turbiditide reservoir. We make a comparison of each method by analyzing the result data from each ensemble and the observed data and the value of the data mismatch for each ensemble member. This data mismatch is computed using such equation:

$$O_d(m_j) = \frac{(d_{obs} - g(m_j))^T C_D (d_{obs} - g(m_j))}{N_d} \tag{27}$$

where N_d is the number of data. More information about the choice of objective function, see [9].

Figure 2 shows the result of computing $O_d(m_j)$ for all ensemble members. The solid red lines with asterisks refers to the ensemble computed for the members obtained with ES-MDA. The solid black lines with small balls refers to the ensemble obtained with ES. The main focus of ES-MDA is to improve the data match of ES with more assimilation of data, considering that ES assimilates all data only once [2]. Thus, it is expected that data match is not good. For the ensemble obtained with ES-MDA, we can see that O_d achieved smaller values. As observed by [2], 4 assimilations is enough to attain good results in reservoir history matching.

The good data match produced by ES-MDA can be observed comparing Figures 5 and 6. For both figures, the solid black lines refers to all ensemble members and the solid red line with asterisks is referred to the observed data. Figure 5 shows the resulting ensemble computed using ES. The uncertainty between the initial and final ensemble is reduced, but it is still have a considerable uncertainty level. Conversely, ES-MDA obtained good results with low uncertainty level.

Another comparison we can make is the value of $O_d(m_j)$ for the initial and final ensemble members for each method. Figure 3 shows the values of $O_d(m_j)$ for the initial and the final ensemble members obtained with ES. One can notice that O_d for the final ensemble is almost the same for the initial one. Figure 4, on the other hand, shows that ES-MDA can reduce the uncertainty of any ensemble member, diminishing O_d and, thus, improving data match.

VI. Conclusion

In this work we analyze the performance of the Ensemble Smoother and Ensemble Smoother with Multiple Data Assimilation. The reservoir model used is a turbidite lobe with

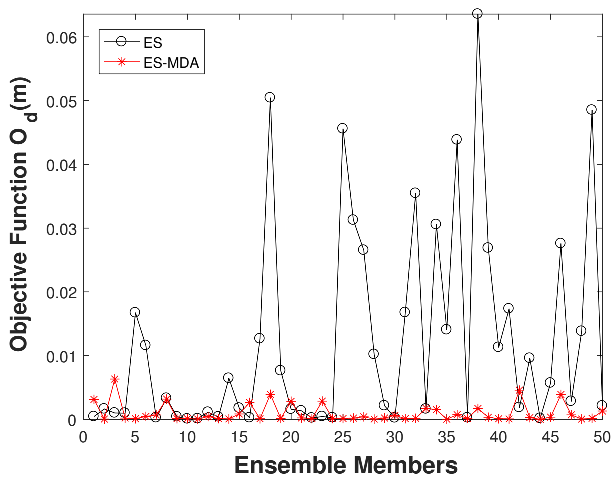


Fig. 2. Comparison between final ensemble members for ES and ES-MDA. The solid black line with small balls refers to ES and the solid red line with asterisks refers to ES-MDA

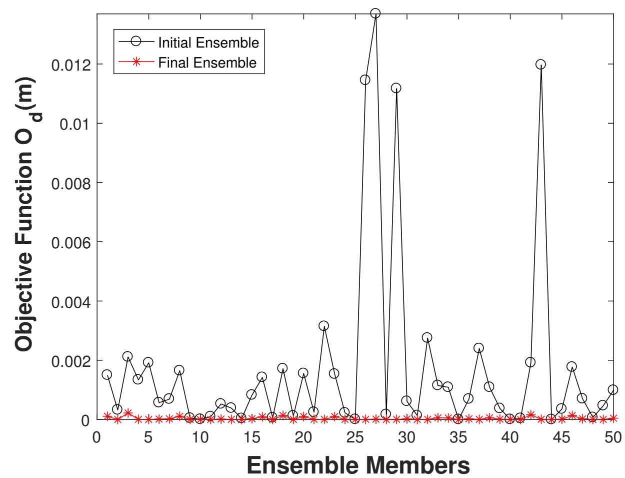


Fig. 4. Comparison between initial and final ensemble members fo ES-MDA. The black solid lines are ensemble members and the red line with asterisks is the observed data.The meaning of the lines is the same as in Figure 3.

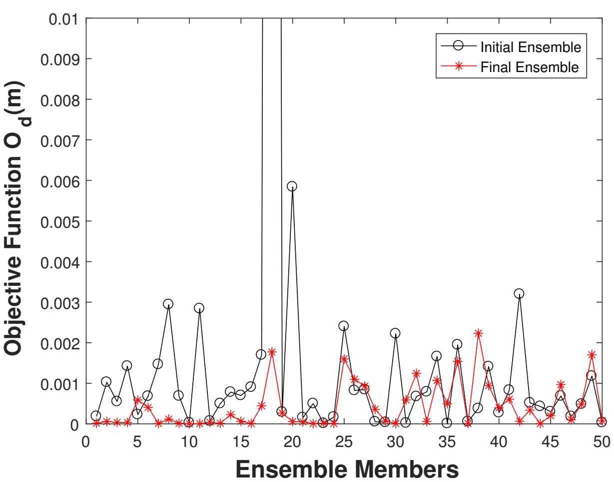


Fig. 3. Comparison between initial and final ensemble members fo ES. The solid black line with small balls refers to the initial ensemble and the solid red line with asterisks refers to the final ensemble. The value of O_d of the 18^{th} member of the initial ensemble reached 0.1.

three layers with homogeneous permeability in each layers. The observations leads us to conclude that the process of assimilating data multiple time of ES-MDA improves the data match substantially. The unique global update of ES seem to be not enough to provide a good update to the model parameters. The results shown here are in agreement with the ones presented in [3], where multiple ensemble-based methods is tested in a simple, but highly nonlinear model. There, ES-MDA also presented the best data match and uncertainty reduction among all methods.

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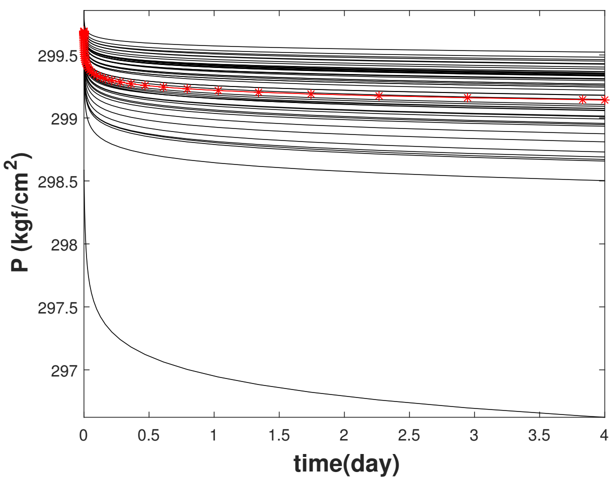
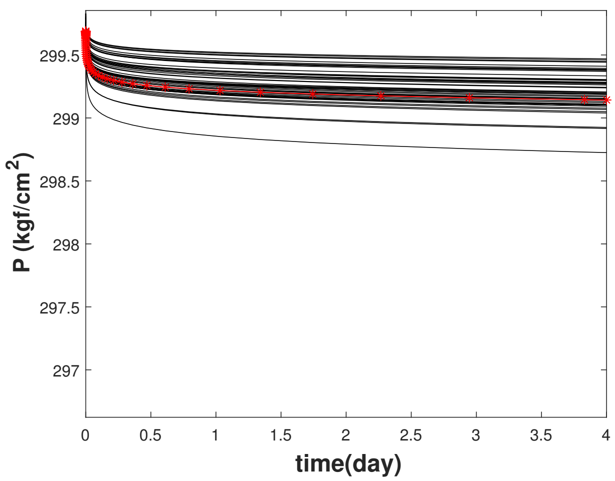
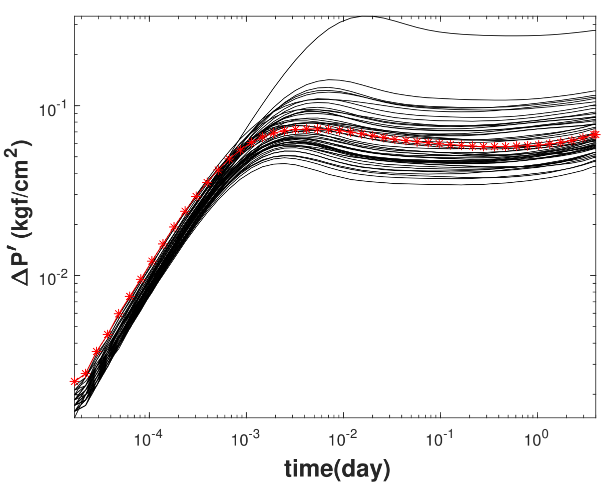


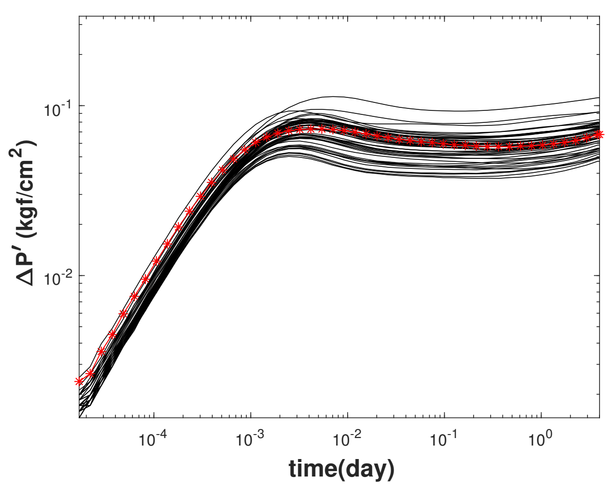
(a) Initial Ensemble.



(b) Final Ensemble.

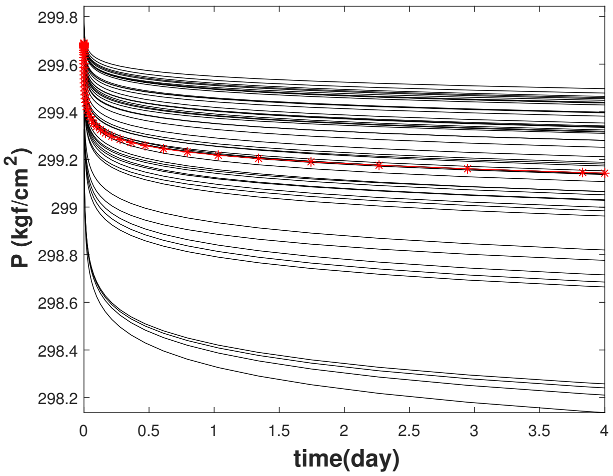


(c) Initial Ensemble.

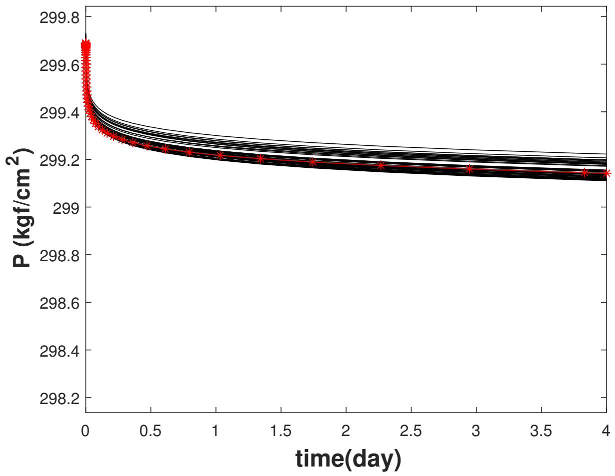


(d) Final Ensemble.

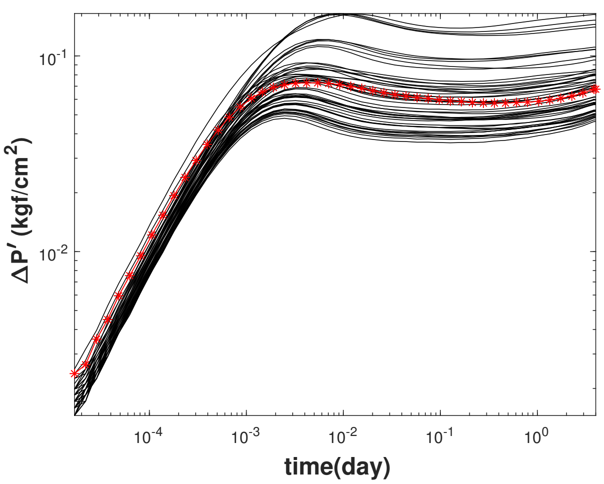
Fig. 5. Pressure data match for the turbidite reservoir model with ES. The top figures show the data match for pressure and the bottom ones show the data match for the logarithmic derivative of pressure. The solid black lines refers to the ensemble members and the solid red line with asterisks refers to the observed data.



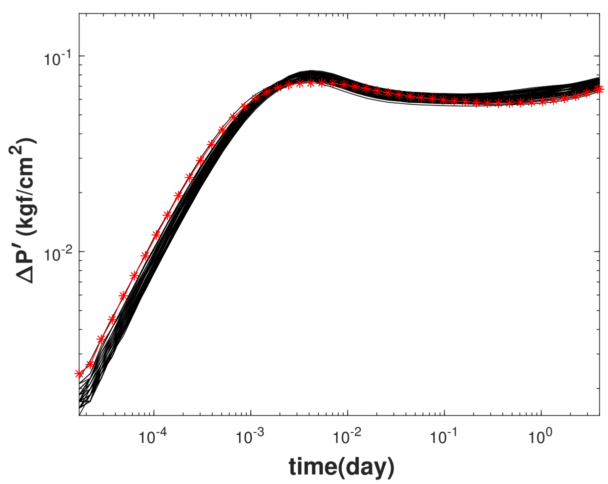
(a) Initial Ensemble.



(b) Final Ensemble.



(c) Initial Ensemble.



(d) Final Ensemble.

Fig. 6. Pressure data match for the turbidite reservoir model with ES-MDA with $N_a = 10$. The figures and lines shown here has the same meaning as in Figure 5.