Exploring the influence of wind, vegetation and water sources on the spread of forest fires in the Brazilian Cerrado Biome using Cellular Automata

Heitor C. Brasiel[1], **Danielli A. Lima**[1]

[1]Laboratório de Inteligência Computacional e Robótica (LICRO)
Instituto Federal do Triângulo Mineiro (IFTM) Campus Patrocínio, MG - Brasil

heitor.brasiel@estudante.iftm.edu.br, danielli@iftm.edu.br

Abstract. *The fires in different Brazilian biomes have been increasingly frequent, leaving more and more consequences for the inhabitants, fauna, and flora of the affected region. Therefore, a computational model based on two-dimensional cellular automata was developed to simulate a real environment through software that predicts the behavior of the fire that starts at a predetermined location and spreads over a period of time. For the simulation, a mathematical-computational model based on cellular automata was developed in the Standard C Language, in which mathematical rules are given to order its evolution over time, generating more accurate results. Different scenarios were considered for the experiments, taking into account parameters such as combustion intensity, presence of wind, rivers, and homogeneous and heterogeneous vegetation. The results showed that these parameters can drastically affect the spread of the fire, with emphasis on the intensity of combustion in drier vegetation and the presence of water sources as a physical barrier.*

1. Introduction

Wildfires are a serious and recurrent problem in many regions of the world. In Brazil, the practice is quite common for land preparation for agricultural activities, since it is considered a low-cost alternative. Moreover, wildfires can occur naturally or by human actions, as a result of high temperatures, dry weather, electric discharges, spontaneous combustion, low relative humidity, friction between rocks, and even friction of some animals' fur with dry vegetation. However, when wildfires are caused by humans, they can be considered criminal, as they have a significant negative impact on the flora and fauna of the affected region [Alvarado et al. 2019].

The Cerrado biome, on the other hand, is considered a resilient ecosystem, with an impressive ability to recover quickly after a fire. The vegetation is able to sprout again and attract herbivorous animals in search of new forage, many of which are able to follow the wildfires and feed on insects and reptiles affected by the fire. However, during the dry season, the Cerrado is extremely affected by wildfires, which makes modeling the fire propagation in this biome a matter of great relevance [Ferreira et al. 2022].

In this context, the objective of this work is to model the propagation of wildfires in the Cerrado biome through the technique of cellular automata (CA) [Lima and Lima 2014]. The experiments carried out consider several types of vegetation, such as homogeneous and heterogeneous, in addition to the fire parameter and the presence of a river in the middle of the vegetation. The results of the experiments are presented and discussed in the following sections, with the purpose of contributing to the understanding of wildfire propagation in the Cerrado. By modeling wildfire propagation, it is possible to better understand the behavior of fire in the Cerrado biome and develop more effective strategies for fire prevention and control. In addition, the use of CA can provide a more detailed view of fire propagation patterns, allowing more precise predictions to be made about the impact of wildfires on the flora and fauna of the region.

Modeling wildfire propagation in the Cerrado can be an important tool for environmental planning and management. It is important to emphasize that protecting the Cerrado biome, one of the most biodiverse plant formations, is essential for the preservation of Brazilian biodiversity and for the maintenance of the ecosystem services that this ecosystem offers [Alvarado et al. 2019]. Therefore, it is crucial to adopt measures aimed at reducing the number of Cerrado wildfires, including raising awareness among the population about the risks and impacts of wildfires, promoting more sustainable agricultural practices, and implementing public policies that encourage the biome's conservation.

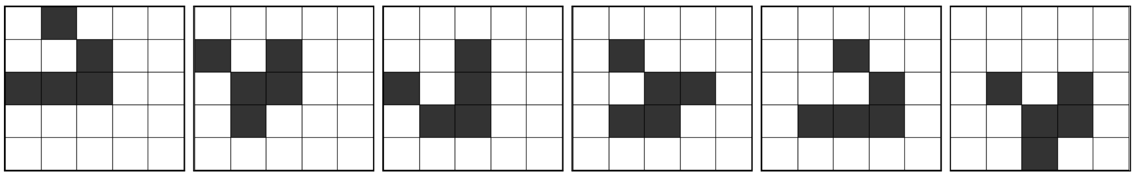
2. Theoretical foundation

In this section, we will present the concepts and definitions of Cellular Automata, which is a class of mathematical and computational models that rely on simple local rules to generate complex and emergent behaviors. Then, we will present some related works that have used this approach to simulate the propagation of forest fires in different biomes, including the Brazilian Cerrado.

2.1. Cellular automata

Cellular automata (CA) are computational systems based on sets of cells that interact with each other following predefined rules where each cell is represented by a state [Lima and Lima 2014]. They were proposed as mathematical models to simulate the complexity of natural systems like the one that will be treated in the article. The basic principle of CA consists of cells undergoing a state transformation (from the states of neighboring cells), where the evolution of the system is determined through transition rules: deterministic or probabilistic [Horibe et al. 2021].

The CA can be represented by a vector or matrix and are classified according to the number of dimensions in which the cells are arranged, and can be one-dimensional when the cells are arranged in a line, two-dimensional when the cells form a grid or three-dimensional when the cells are arranged in a cube. cells form a cube. The CA is composed of a set of cells (x_{ij}) in a lattice with dimension (d) that can be update during a certain ($t \in T$) time. Conway's Game of Life (GL) is a two-dimensional (2D) cellular automaton created by mathematician John Conway in 1970, as shown in Figure 1. The game is played on a grid of square cells, where each cell can be in one of two states: (1) alive, in black, or (0) dead, in white. The rules of Conway's GL are as follows [Lima and Lima 2014]: (i) any living cell with fewer than two living neighbors dies of loneliness; (ii) any live cell with two or three live neighbors continues to live in the next generation; (iii) any live cell with more than three live neighbors dies from overpopulation; (iv) any dead cell with exactly three live neighbors becomes a live cell. These rules are applied to each grid cell simultaneously to produce the next generation of cells. The game starts with an initial pattern of cells and then evolves from that pattern according to the game rules. The GL is an example of a complex dynamic system, with unpredictable



(a) Step $t = 0$. (b) Step $t = 1$. (c) Step $t = 2$. (d) Step $t = 3$. (e) Step $t = 4$. (f) Step $t = 5$.

Figure 1. Time evolution by ($t = 6$) steps of the GL, a 2D CA created by Conway.

emergent behavior based on simple rules.

Cellular automata are widely used in several areas of science to develop the spatial modeling of complex systems that have a large number of local interactions and that can exhibit unpredictable behavior, among them, we can mention robotics [Horibe et al. 2021], modeling of diseases [Monteiro et al. 2020], and even forest fire modeling [Lima and Lima 2014], which is the focus of our work. Different works have already been proposed with the objective of modeling fires and forest fires through CA, among them, we can mention the work of [Lima and Lima 2014] in which CA was used in a 2D form. In this article, their use to simulate fires will be explored, which is a complex process because it is influenced by several variables such as topography, climate, vegetation type, region, wind, among others. CA can simulate these factors on a smaller scale, allowing for more accurate modeling of the fire propagation process.

2.2. Related works

Fires have been occurring frequently throughout Brazilian territory, especially with the advancement of deforestation for the expansion of pasture areas and economic activities linked to agribusiness. In addition, hot and dry weather is also significantly influencing the increase in fire spread throughout the country, and in the Cerrado region it is no different, especially because it is a hot region [Miranda et al. 2009]. Thus, from climate change, which provides increasingly hot and dry weather, coupled with the natural action of winds, flames can increase and spread in a devastating way. Another factor that increases the probability of major damage in the Cerrado is the absence of rain during the dry season, causing large-scale fires to amplify [Alvarado et al. 2019].

Based on this context, CA are considered an important mathematical-computational tool capable of modeling dynamic and complex systems [Lima and Lima 2014], capable of modeling complex systems with a lot of realism, even replacing numerical simulation from differential equations. In addition, to simulate the dynamic action of CA, their cells undergo a state transformation (from the states of neighboring cells) through a transition rule: deterministic or probabilistic [Horibe et al. 2021]. CA can be applied in different contexts, including robotics [Horibe et al. 2021], disease modeling [Monteiro et al. 2020], and even modeling fires [Lima and Lima 2014]. This modeling can be considered an important predictor for environmental agencies to make decisions about fires. Different works have already been proposed with the aim of modeling fires and burns through CA, among them we can cite the work of [Lima and Lima 2014] using probabilistic 2D-CA for simulating different fire outbreaks in homogeneous forests using a preference matrix for simulating the wind. In [Lima et al. 2020] authors used a 2D-CA for modeling fire in closed environments, aiming at safe and panic-free evacuation of buildings. Moreover, in

[Ferreira et al. 2022], the authors improved the work of [Lima and Lima 2014] by adding a parameter improvement through genetic algorithms.

3. Proposal

In this section, we will present three models for the spread of fires. First, a basic model of CA rule updating will be introduced. Then, a model that considers the action of wind will be presented. Finally, a model that takes into account the different types of vegetation that affect fire propagation will be discussed.

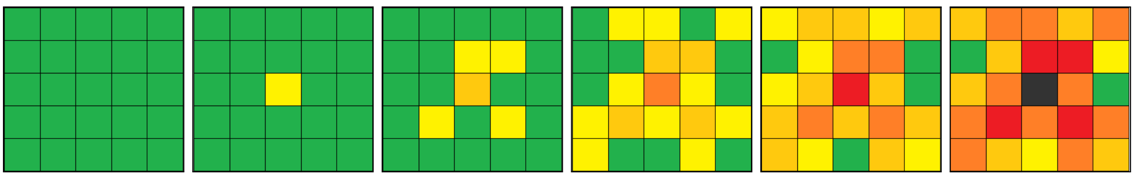
3.1. Initial base model

Initially, as shown in Figure 2, the CA states are set in T_A ($t = 0$), later, a fire focus is allocated to the grid $L_{5\times 5}$. At this moment, a fire is started in the lattice T_{B_i}, $1 \le i \le 4$. In

$$T_A \xrightarrow{p(x_{ij})} T_{B_1} \xrightarrow{t_{b_1}} T_{B_2} \xrightarrow{t_{b_2}} T_{B_3} \xrightarrow{t_{b_3}} T_{B_4} \xrightarrow{t_{b_4}} T_D$$

Figure 2. Transition of states from the general base model to the CA cells update.

other words, the fire intensity assumes 4 possible states (colors) depending on how long the vegetation has been burning. Thus, in this example $t_{b_i} = 2$ (burning time), the fire changes its intensity at each $t = 2$ steps. If cell x_{ij} has one or more neighboring cells on fire, then it has a non-zero probability ($p(x_{ij})$) of catching fire ($T_A \to T_B$) at a later time, in cases of homogeneous forests, $p(x_{ij}) = 0.6$. Finally, when the fire reaches a certain burning state, the tree is completely destroyed (dead) and goes to the state T_D. Figure 3



(a) Step $t = 0$. (b) Step $t = 2$. (c) Step $t = 4$. (d) Step $t = 6$. (e) Step $t = 8$. (f) Step $t = 10$.

Figure 3. Transitions of the CA to states between cells: (T_A) alive (green), (T_{B_i}) burning (light yellow, dark yellow, orange and red) and (T_D) dead (black).

shows the evolution of the CA using a lattice representation for times $t = 0$ (see Figure 3(a)), $t = 2$ (see Figure 3(b)), $t = 4$ (see Figure 3(c)), $t = 6$ (see Figure 3(d)), $t = 8$ (see Figure 3(e)), and $t = 10$ (see Figure 3(f)). Initially, the forest is intact, but as time passes, the flames begin to spread throughout the forest, and the fire states propagate for $t \in T$ steps (number of rule evolution).

3.2. Variation of wind speed

Wind is a crucial factor to consider when modeling wildfires using 2D CA. Wind ($\vec{w}$) can greatly influence the speed and direction of fire spread, as well as the shape and size of the fire front. Therefore, it is important to incorporate wind direction and speed into the CA model to create more accurate and realistic simulations of wildfires. In a CA model, wind can be represented by a matrix field that defines the direction and speed of wind at each cell. This matrix field can then be used to calculate the probability of a cell igniting and spreading fire based on the wind direction and speed, herein we used a wind preference matrix based on [Schadschneider et al. 2002, Lima and Lima 2014, Ferreira et al. 2022], as can be see in Figure 4. There are different directions and velocities that wind can

100	100	100
100	Xij	100
100	100	100

(a) Wind $\vec{w} = 0$.

100	95,5	93,5
100	Xij	93,5
100	95,5	93,5

(b) Wind $\vec{w} = 2$.

100	77,5	67,5
100	Xij	67,5
100	77,5	67,5

(c) Wind $\vec{w} = 10$.

100	55	35
100	Xij	35
100	55	35

(d) Wind $\vec{w} = 20$.

Figure 4. Burning probabilities per neighboring cell according to the value of the wind variable.

assume, for example, wind ($\vec{w} = 0$) in Figure 4(a), wind ($\vec{w} = 2$) in Figure 4(b), wind ($\vec{w} = 10$) in Figure 4(c) and wind ($\vec{w} = 20$) in Figure 4(d).

In landscapes with diverse vegetation, wind plays a crucial role in fire spread. It can either facilitate the spread through flammable areas or hinder it in less flammable regions. Therefore, integrating wind dynamics into CA models is essential for accurate simulation of wildfires. By considering wind, these models provide insights into fire behavior in different environments, enabling the development of effective strategies for wildfire prevention and control.

3.3. Variation in the type of vegetation

The occurrence and spread of forest fires depend heavily on the type of vegetation present in a given area. Some types of vegetation are more prone to fire, such as dry grasslands and forests with high levels of dead wood, while others are more resistant, such as swamps and moist tropical forests. In addition to vegetation type, the presence of fuel, temperature, humidity, and wind can also contribute to the ignition and spread of forest fires.

Modeling forest fires using CA requires careful consideration of the vegetation type in the simulated environment. Two approaches can be employed: homogeneous and heterogeneous vegetation models. A homogeneous model assumes uniform vegetation coverage throughout the area, allowing for detailed analysis of fire behavior within a specific vegetation context. In contrast, a heterogeneous model incorporates different vegetation types across distinct areas, providing more realistic simulations that account for the complex interactions between diverse vegetation and landscape characteristics found in real-world scenarios.

For example, in a heterogeneous model, a river can act as a barrier, preventing the spread of fire to a neighboring patch of vegetation. Additionally, the presence of different types of vegetation with different levels of flammability can lead to the creation of firebreaks, which can slow down or stop the spread of fire. Thus, different types of vegetation are added in the heterogeneous' state (S_{HT}) approach. As a result, different burning probabilities are observed from different vegetation properties: **dry forest** (T_{DF}), we consider the probability $T_A \rightarrow T_{DF}$ is $p(x_{ij}) = 0.9$, **transitory forest** or **mixed vegetation forest** (T_{MF}), we consider the probability $T_A \rightarrow T_{MF}$ is $p(x_{ij}) = 0.6$, **humid forest** (T_{HF}), we consider the probability $T_A \to T_{HF}$ is $p(x_{ij}) = 0.3$ and **rivers** (presence of water) (T_R), we consider the probability $T_A \to T_R$ is $p(x_{ij}) = 0.0$ and $\forall\{T_{DF}, T_{MF}, T_{HF}\} \in S_{HT}$. For example, in our model, if the cell encounters the barrier of a river, the river does not catch fire, so the fire can be interrupted by the presence of water. The types of vegetation in a forest can impact how wildfires spread. Drier areas are more at risk than wetter areas, so it's crucial to consider vegetation diversity and other landscape features when modeling forest fires, improving simulations and helps with prevention and control strategies.

3.4. Implementation of the final model

The methodology used in this work consists of modeling the propagation of forest fires in the Cerrado biome using the cellular automaton (CA) technique, which is a mathematical-computational tool capable of modeling complex and dynamic systems. In addition, several experiments will be carried out, considering different types of vegetation: homogeneous and heterogeneous. The algorithm follows a step-by-step simulation that can be basically represented by a few steps:

1. Initializes a matrix for CA lattice $L_{m \times n}$, where each cell x_{ij} represents an area of vegetation and is initially filled with state 0 (live or intact area) for $t = 0$;
   (a) Initializes a preference matrix $\vec{W}$ representing the wind vector and sets the parameters for wind direction and velocity.
   (b) Initializes the vegetation states, that can include heterogeneous forest dry (T_{DF}), transitory forest or mixed vegetation forest (T_{MF}), humid (T_{HF}) and rivers (T_R).
2. Creates a fire focus in the forest and changes the state of the matrix cell to 1 (burning) for $t_{b_i} \in T_{B_i} = \{1, 2, 3, 4, 5\}$;
3. For each cell that is in state 0, calculate the probability of starting to burn in the current iteration from the state of neighboring cells in the previous iteration, considering factors such as wind and type of vegetation;
4. Updates the state of the cells for the next iteration, increasing the state by 1 for all cells that were burning in the previous iteration ($t = t + 1$) and changing the state of the cells to fully burned, then T_D;
5. Repeat step 3 and 4 for T defined iterations;
6. Evaluates fire propagation and affected area metrics as the percentage of total green area burned.

The state diagram for the forest burning model can be represented in Figure 5, including the initialization of the matrix and vegetation states, creation of fire focus, calculation of burning probability, updating cell states, and fire evaluation.

4. Experiments results

The experiments carried out here aim to make qualitative comparisons about the effect of altering some of the parameters of the forest fire propagation model in homogeneous forests, considering 2D-CA with lattices of $L_{100 \times 100}$ cells, with probabilistic transition rules and non-periodic boundary conditions.

The first experiment presented in Figure 6 demonstrates a homogeneous vegetation exposed to different burning times related to the cell's burning speed. In Figure 6(a), the fire has a shorter burning time t_{b_i}, so the cell that catches fire quickly consumes the fuel and becomes black. In the other Figures 6(b), 6(c), and 6(d), we have cases of

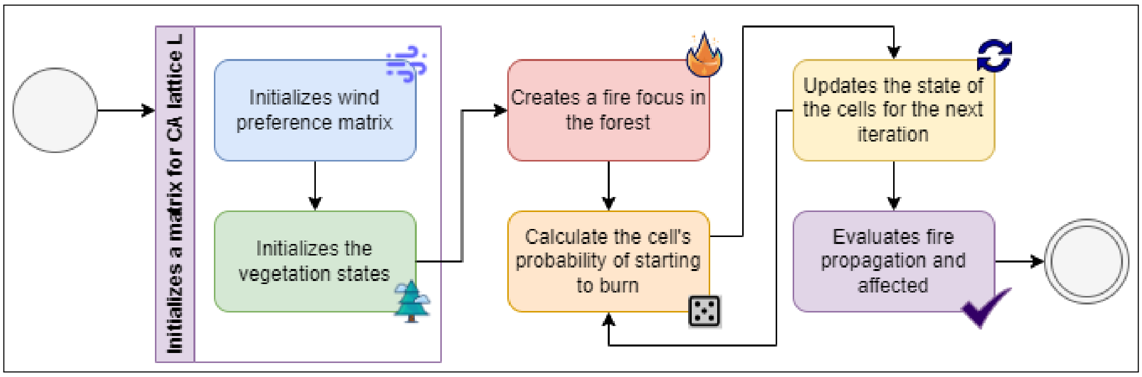
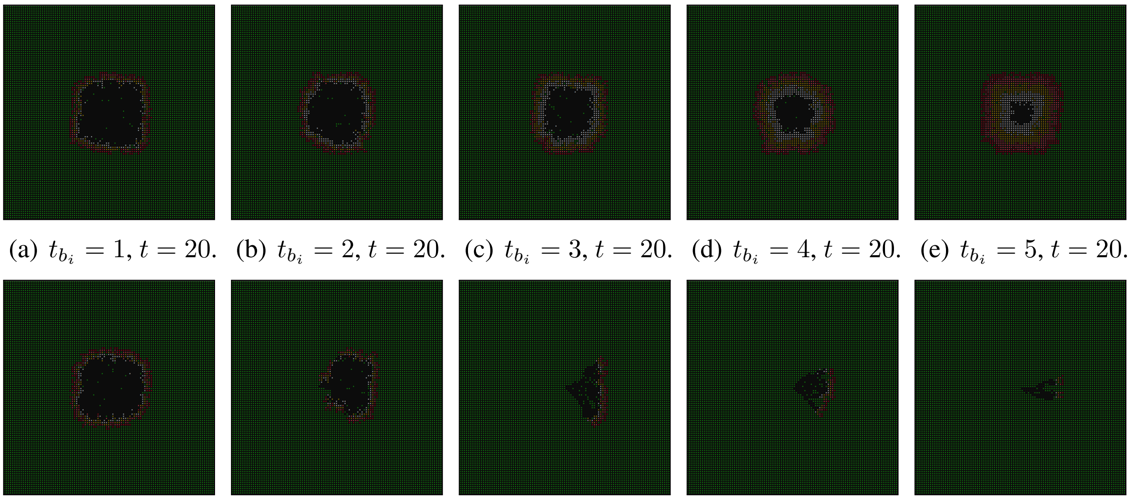


Figure 5. Steps of the CA model for simulating Cerrado biome forest fires.



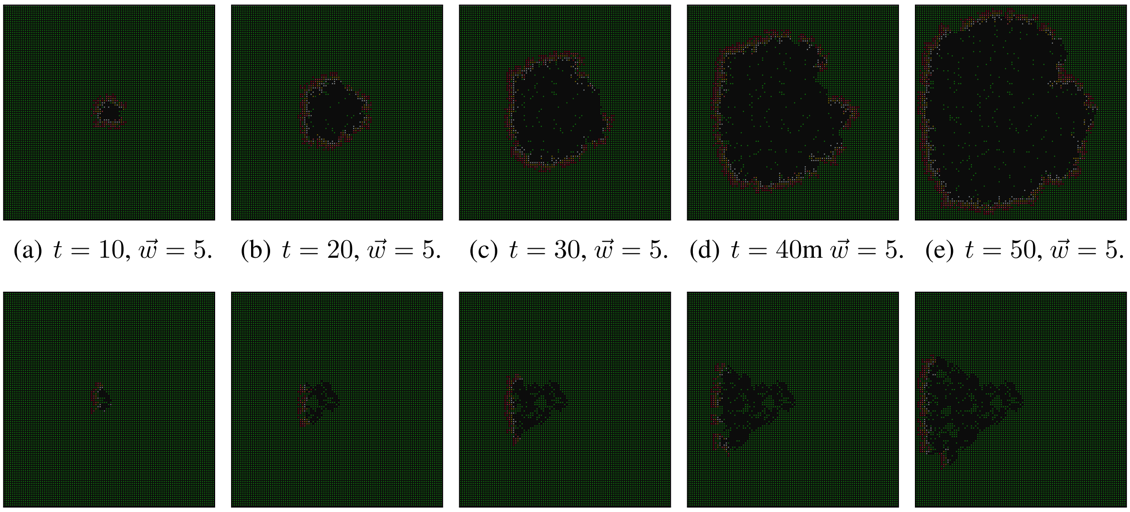
(a) $t_{b_i} = 1, t = 20.$ (b) $t_{b_i} = 2, t = 20.$ (c) $t_{b_i} = 3, t = 20.$ (d) $t_{b_i} = 4, t = 20.$ (e) $t_{b_i} = 5, t = 20.$

(f) $\vec{w} = 0, t = 20.$ (g) $\vec{w} = 5, t = 20.$ (h) $\vec{w} = 10, t = 20.$ (i) $\vec{w} = 15, t = 20.$ (j) $\vec{w} = 20, t = 20.$

Figure 6. Different simulation scenarios of 2D-CA considering the absence and presence of the wind factor.

intermediate probabilities. The experiments conducted here aim to perform qualitative comparisons regarding the effect of changing some of the parameters of the forest fire propagation model in homogeneous forests, considering 2D-CA with lattices of $L_{100\times100}$ squared x_{ij} cells, with probabilistic transition rules ($p(x_{ij})$) and non-periodic boundary conditions. The first experiment presented in Figure 6 demonstrates the same vegetation exposed to different burning times t_b, which are related to the burning speed of the cell. In Figure 6(a), the fire has a shorter burning time t_b, and therefore, the cell that catches fire quickly consumes the fuel and turns black. In Figures 6(b), 6(c), and 6(d), we have cases of intermediate probabilities. In Figure 6(e), a longer burning time was defined, so the fire takes longer to consume all the fuel until it goes out. Similarly, in Figure 6(f), the presence of wind $\vec{w}$ (wind direction) is almost insignificant, whereas for Figures 6(g), 6(h), 6(i), and 6(j), wind intensities gradually increased, with probabilities $\{0, 5, 10, 15, 20\}$, generating a visual effect of the wind blowing towards the east.

The experiments presented in Figure 7 show the evolution of fire in a forest as a function of wind direction and intensity. To do so, two situations with different wind intensities were simulated: (i) $\vec{w} = 5$ (see Figures 7(a), 7(b), 7(c), 7(d), and 7(e)), and (ii)



(a) $t = 10, \vec{w} = 5$. (b) $t = 20, \vec{w} = 5$. (c) $t = 30, \vec{w} = 5$. (d) $t = 40\mathrm{m}\ \vec{w} = 5$. (e) $t = 50, \vec{w} = 5$.

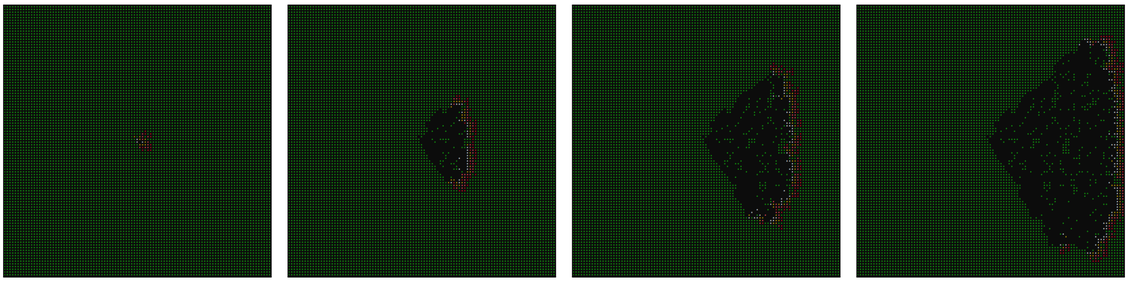
(f) $t = 10, \vec{w} = 15$. (g) $t = 20, \vec{w} = 15$. (h) $t = 30, \vec{w} = 15$. (i) $t = 40, \vec{w} = 15$. (j) $t = 50, \vec{w} = 15$.

Figure 7. Graphical representation of the evolution of forest fire propagation in an environment with different wind intensities modeled by CA.

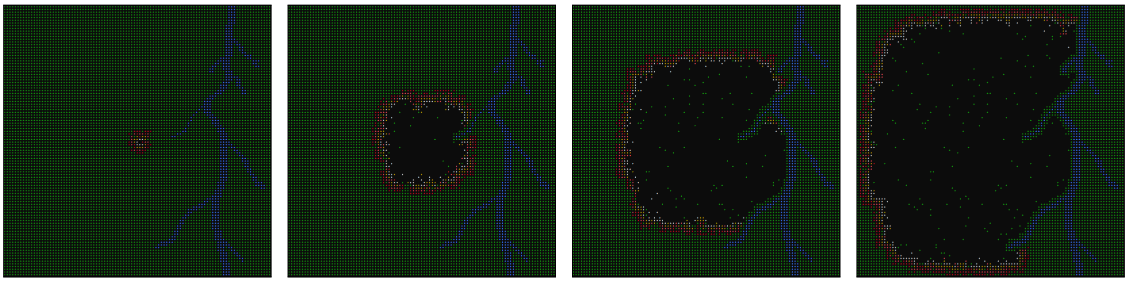
$\vec{w} = 15$ (see Figures 7(f), 7(g), 7(h), 7(i), and 7(j)), both with wind direction east-west and $t_{b_i} = 1, \forall i \in \{1, 2, 3, 4\}$. As we can observe, the fire spreads more rapidly in the direction of the wind, creating a more extended front line, since all cells are homogeneous, that is, with the same vegetation density. These experiments are important to understand how wind can affect the spread of fire in a forest and how it is crucial to consider wind direction and intensity when modeling fire propagation.

In Figure 8, the final experiment explores different scenarios involving rivers, wind, and variations in vegetation composition. The simulations were conducted at multiple time points ($t = 5, 20, 25, 50$). The first simulation (Figures 8(a), 8(b), 8(c), and 8(d)) focuses on wind effects in homogeneous forests, with a cell's probability of burning set at $P(x_{ij}) = 60\%$ when its neighboring cells are on fire. The wind direction vector indicates eastward movement, resulting in complete forest fire impact on the lattice's right portion by $t = 50$. In Figures 8(e), 8(f), 8(g), and 8(h), we have a scenario where a river is located on the eastern side of the lattice and in this case, the wind parameter was not included, and thus, the forest fire propagation is interrupted by the river. In Figures 8(i), 8(j), 8(k), and 8(l), we observe that the spread of fires occurs more rapidly in the eastern direction, however, the burning is interrupted by a physical water barrier (river). Later, in Figures 8(m), 8(n), 8(o), and 8(p), in the last experiment with heterogeneous vegetation, it is possible to notice that the more humid vegetation has greater difficulty in spreading the fire than the intermediate and dry vegetation, being a major fuel for the fire.

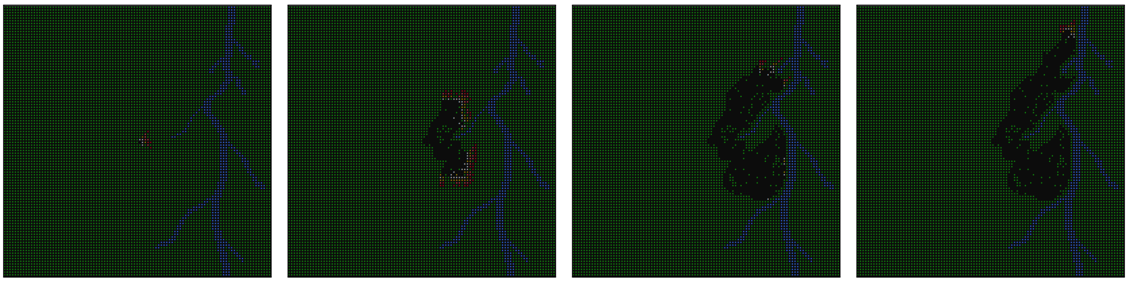
The results obtained in the experiments are very relevant for the wildfires in Cerrado, considered a mixed forest. Through the analysis of different parameters, variations in the speed of fire propagation and wind direction were identified. In the first experiment, it was observed that the burning time of cells influences the speed of fire propagation. Additionally, the presence of wind also showed a significant effect, where the higher the wind intensity, the faster the fire propagates in a triangular shape. In the last experiment, it was possible to verify that the rivers' (water) existence can interrupt the wildfires and limit its expansion. Furthermore, the analysis of heterogeneous vegetation showed that the pres-



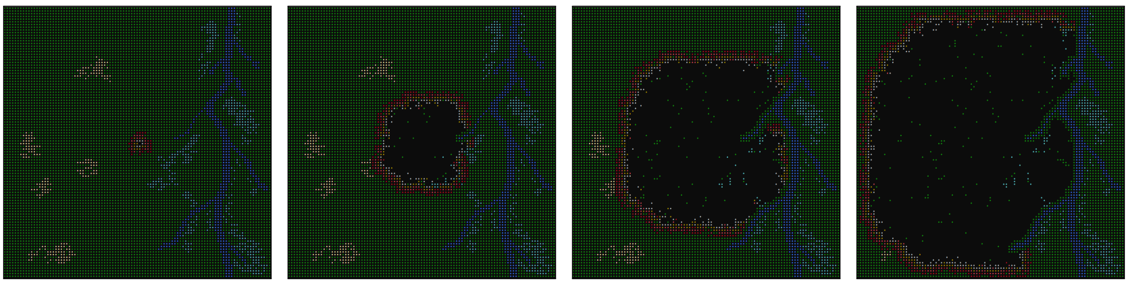
(a) $\vec{w} = 8, t = 5.$ (b) $\vec{w} = 8, t = 20.$ (c) $\vec{w} = 8, t = 35.$ (d) $\vec{w} = 8, t = 50.$



(e) $T_R, t = 5.$ (f) $T_R, t = 20.$ (g) $T_R, t = 35.$ (h) $T_R, t = 50.$



(i) $T_R, \vec{w} = 8, t = 5.$ (j) $T_R, \vec{w} = 8, t = 20.$ (k) $T_R, \vec{w} = 8, t = 35.$ (l) $T_R, \vec{w} = 8, t = 50.$



(m) $S_{HT}, t = 5.$ (n) $S_{HT}, t = 20.$ (o) $S_{HT}, t = 35.$ (p) $S_{HT}, t = 50.$

Figure 8. Different scenarios with two-dimensional CA considering homogeneous, heterogeneous vegetation, with the presence of river and wind.

ence of more humid areas can hinder fire propagation compared to dry and intermediate areas. Therefore, the obtained results significantly contribute to the development of more precise and efficient models for predicting and controlling fires.

5. Conclusions and future work

Through modeling and simulation of Cerrado fire spread through different scenarios, it was possible to perceive that parameters can drastically affect the outcome of a wildfire.

Thus, we can observe that wildfires are influenced by the intensity of combustion, especially in drier vegetation. Another observed parameter is that in homogeneous vegetation, fire spreads in a more standardized and radial way, except when there is a river. In the case of heterogeneous vegetation, different burning probabilities were programmed, and in this sense, the wetter it is, the less likely it is to burn and it is possible to stop a wildfire through the presence of water sources. As future work, experiments should be complemented with other analyses to obtain even more robust results. We can consider modeling different real biomes, analyzing other parameters that may affect the propagation of forest fires, such as soil moisture and altitude, as well as developing prevention and firefighting strategies based on the results found in the experiments. Additionally, it is important to continue investigating the effects of wind and cell burn rate on fire propagation, as well as studying the impacts of climate change on the occurrence and severity of forest fires.

Acknowledgments

Authors DAL and HCB would like to thank FAPEMIG for the scholarship and CNPq for funding the project Call/CNPq/MCTIC/FNDCT 18/2021 - 423105/2021-3.

References

Alvarado, S. T., de Carvalho, I. S., Ferraz, T. M., and Silva, T. S. (2019). Effects of fire suppression policies on fire regimes in protected areas in the cerrado. *Biodiversidade Brasileira-BioBrasil*, (1):200–200.

Ferreira, M. E. A., Quinta, A. L., Lima, D. A., Martins, L. G., and Oliveira, G. (2022). Automatic evolutionary adjustment of cellular automata model for forest fire propagation. In *International Conference on Cellular Automata for Research and Industry*, pages 235–245. Springer.

Horibe, K., Walker, K., and Risi, S. (2021). Regenerating soft robots through neural cellular automata. In *Genetic Programming: 24th European Conference, EuroGP 2021, Held as Part of EvoStar 2021, Virtual Event, April 7–9, 2021, Proceedings 24*, pages 36–50. Springer.

Lima, D. A., Cabral Jr, E., Almeida, I. T., Andrade, J. P., Fonseca, J. P., Santos, M. E., Nunes, N. T., and Bernardes, V. H. (2020). A fire elitist cellular automaton-based model to verify pedestrian flow simulated in real environments using arduino. *Proc. Series of the Brazilian Society of Computational and App. Mathematics*, 7(1).

Lima, H. A. and Lima, D. A. (2014). Autômatos celulares estocásticos bidimensionais aplicados a simulação de propagação de incêndios em florestas homogêneas. In *Workshop de Comp. Aplicada a Gestão do Meio Amb. e Rec. Naturais*, pages 15–24. SBC.

Miranda, H. S., Sato, M. N., Neto, W. N., and Aires, F. S. (2009). Fires in the cerrado, the brazilian savanna. In *Tropical fire ecology*, pages 427–450. Springer.

Monteiro, L., Fanti, V., and Tessaro, A. (2020). On the spread of sars-cov-2 under quarantine: A study based on probabilistic cellular automaton. *Ecological Complexity*, 44:100879.

Schadschneider, A., Kirchner, A., and Nishinari, K. (2002). Ca approach to collective phenomena in pedestrian dynamics. In *5th ACRI*, pages 239–248. Springer.