An Application for Medical Diagnosis Using Correlation Coefficient With Modal Operators and Operator Identifying and Unary

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Abstract. *This paper aims to study the Atanassov's correlation coefficient (A-CC) between two of Atanassov's intuitionistic fuzzy sets (A-IFS), obtained as images of intuitionistic fuzzy modal operators. The composition of modal operators are investigated, verifying under which conditions an A-CC preserves the main properties related to conjugate and complement operations performed on A-IFS. In addition, a simulation based on the proposal methodology using operators described above is applied to a medical diagnosis analysis.*

1. Introduction

The expression of uncertainty and imprecision has been widely discussed over the years, generating several extensions to the theory introduced by Zadeh [Zadeh 1965]. Atanassov's intuitionistic fuzzy sets (A-IFS) [Atanassov 1986] are used to explain the situations where there is hesitancy in the information describing the membership and non-membership degrees of elements in fuzzy sets (FS). An A-IFS considers the informations of these degrees and also providing the hesitation (uncertainty) margin related to the Atanassov's intuitionistic fuzzy index (A-IFIx), as reported by [Szmidt et al. 2012] approach. This approach leads to a great numbers of studies, all of them are closely connected with the correlation coefficient (A-CC) [Reiser et al. 2013, Bertei et al. 2016, Bertei and Reiser 2018, Bertei et al. 2021] between two intuitionistic fuzzy sets, mainly those applied to processes of decision-making, such as clustering analysis [Meng et al. 2016], digital image processing and medical diagnosis [Bertei and Reiser 2019].

The A-CC was conceived as a strict association between two variables and defined, in the Pearse correlation coefficient case, as a linear relationship of them, assigning +1 in the case of a positive linear relationship increasing their variable values and, -1 in the case of a negative (decreasing) linear relationship. In general, the correlation coefficient describes how one variable moves in relation to another. A positive correlation indicates that the two are moving in the same direction, achieving a correlation of +1 when they are moving together. A negative correlation coefficient indicates that they move in opposite directions instead. So, the closer an A-CC is to either -1 or 1, the stronger correlation between these two A-IFSs. Therefore, when the correlation between two A-IFS is 0, there is no linear relationship between them.

This article mainly focuses on intuitionistic fuzzy modal (A-IFM) operators [Atanassov 1986] which have been systematically studied by different authors [Atanassov 2012, Dencheva 2004]. In the A-IFS theory, many interpretations for the modality can be achieved, based on the corresponding modal operator expressions.

Some algebraic and characteristic properties of these operators were already examined and discussed in [Bertei and Reiser 2018, Bertei and Reiser 2019]. This article extend the results presented in [Bertei et al. 2021], studying A-CC relating to modal operators as necessity, possibility together with the A-model operator. Based on their analytical expressions, fuzzy data analysis for a problem in medical diagnosis was considered.

This paper is organized as follows: preliminaries is described in Section 2 and 3 presents the foundations on A-CC. In Section 4, this study includes the main results based on A-CC obtained by modal operators. In the Section 5 the study includes the main results based on A-CC obtained by modal operators and Identifying and Unary operator. In the Section 6 is presents an application for the medical diagnosis. Finally, conclusions and further work are discussed in Section 7.

2. Preliminary

Firstly, a brief account on A-IFS is stated. Consider a non-empty and finite universe $\mathscr{U} = \{x_1,\ldots,x_n\}$ and the unitary interval $[0,1] = U$. An A-IFS A_I based on $\mathscr{U}$ is expressed as

$$A_I = \{(x,\mu_{A_I}(x),\nu_{A_I}(x))\colon x \in \mathscr{U}\} \tag{1}$$

whenever the membership and non-membership functions $\mu_{A_I},\nu_{A_I} : \mathscr{U} \to U$ are related by the inequality $\mu_{A_I}(x_i)+\nu_{A_I}(x_i) \leq 1$, for all $i \in \mathbb{N}_n = \{1,2,\ldots,n\}$. An intuitionistic fuzzy index (IFIx) or hesitance degree of an A-IFS A_I is given as

$$\pi_{A_I}(x_i) = 1-\mu_{A_I}(x_i)-\nu_{A_I}(x_i). \tag{2}$$

And, the set of all above related A-IFS is denoted by $\mathscr{C}(A_I)$. Let $\tilde{U} = \{\tilde{x}_i = (x_{i1},x_{i2}) \in U^2 : x_{i1}+x_{i2} \leq 1\}$ be the set of all intuitionistic fuzzy values such that $\tilde{x}_i$ is a pair of membership and non-membership degrees of an element $x_i \in \mathscr{U}$, i.e. $(x_{i1},x_{i2}) = (\mu_{A_I}(x_i),\nu_{A_I}(x_i))$. And, the related IFIx is given as $\pi_{A_I}(x_i) = x_{i3} = 1-x_{i1}-x_{i2}$, $\forall i \in \mathbb{N}_n = \{1,2,\ldots,n\}$. The projections $l_{\tilde{U}^n}, r_{\tilde{U}^n} : \tilde{U}^n \to U^n$ are given by:

$$l_{\tilde{U}^n}(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n) = (x_{11},x_{21},\ldots,x_{n1}) \qquad r_{\tilde{U}^n}(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n) = (x_{12},x_{22},\ldots,x_{n2}). \tag{3}$$

The order relation $\leq_{\tilde{U}}$ on $\tilde{U}$ is defined as: $\tilde{x} \leq_{\tilde{U}} \tilde{y} \Leftrightarrow x_1 \leq y_1$ and $x_2 \geq y_2$. Moreover, $\tilde{0} = (0,1) \leq_{\tilde{U}} \tilde{x} \leq_{\tilde{U}} (1,0) = \tilde{1}$, for all $\tilde{x} \in \tilde{U}$.

Intuitionistic fuzzy negations and **intuitionistic automorphisms** are studied in the following. See more details in [Bustince et al. 2003]. An intuitionistic fuzzy negation (A-IFNs) $N_I : \tilde{U} \to \tilde{U}$ is a function verifying:

N_I**1** $N_I(\tilde{0}) = N_I(0,1) = \tilde{1}$ and $N_I(\tilde{1}) = N_I(1,0) = \tilde{0}$;
N_I**2** If $\tilde{x} \geq_{\tilde{U}} \tilde{y}$ then $N_I(\tilde{x}) \leq_{\tilde{U}} N_I(\tilde{y})$, $\forall \tilde{x},\tilde{y} \in \tilde{U}$.

In [Bustince et al. 2000], if an IFN N_I also satisfies the involutive property

N_I**3** $N_I(N_I(\tilde{x})) = \tilde{x}$, $\forall \tilde{x} \in \tilde{U}$,

N_I is called a strong A-IFN.

According with [Deschrijver et al. 2004, Theorem 3.6], N_I is a strong A-IFNs iff there exists a strong fuzzy negation N on U such that:

$$N_I(x_1,x_2) = (N(N_S(x_2)),N_S(N(x_1))). \tag{4}$$

Thus, N_I is an example of N-representable IFN. Moreover, if N is the standard fuzzy negation ($N(x) = N_S(x) = 1-x$) equation (4) can be given as

$$N_{SI}(\tilde{x}) = N_{S_I}(x_1, x_2) = (x_2, x_1). \tag{5}$$

By [Bustince et al. 2004], the complement of an IFS A w.r.t. N_I in (4) is given as

$$\overline{A} = \{(x, N_I(\mu_A(x), \nu_A(x))) : x \in \mathscr{U}\}. \tag{6}$$

And, when $N = N_S$ in Eq. (4), then the complement of an IFS A w.r.t. N_{SI} is expressed as

$$\overline{A} = \{(x, \nu_A(x), \mu_A(x)) : x \in \mathscr{U}\}. \tag{7}$$

The function $f_{N_I} : \tilde{U}^n \to \tilde{U}$ is the N_I-dual operator of $f : \tilde{U}^n \to \tilde{U}$ given as follows

$$f_{N_I}(\tilde{x}_1, \ldots, \tilde{x}_n) = N_I(f(N_I(x_1), \ldots, N_I(\tilde{x}_n))). \tag{8}$$

2.1. Intuitionistic Fuzzy Modal Operators

Following [Atanassov 1983], a pair of operators, the necessity operator given as $\Box : \tilde{U} \to U$, $\Box(x_1, x_2) = (x_1, 1 - x_1)$, and the possibility operator, defined as $\Diamond : \tilde{U} \to U$, $\Diamond(x_1, x_2) = (1 - x_2, x_2)$. These two operators and their properties resemble those of Modal Logic and both can be applied to an A-IFS A_I, transforming it into a fuzzy set (FS), $\Box A_I$ and $\Diamond A_I$.

Definition 2.1. *[Atanassov 1999] Let A_I be an A-IFS. The sets $\Box A_I$-IFS and $\Diamond A_I$-IFS, related to the necessity and possibility modal operators are, respectively, given as*

$$\Box A_I = \{\langle x, \mu_{A_I}(x), 1 - \mu_{A_I}(x)\rangle | x \in \mathscr{U}\} \text{ and } \Diamond A_I = \{\langle x, 1 - \nu_{A_I}(x), \nu_{A_I}(x)\rangle | x \in \mathscr{U}\}. \tag{9}$$

When A_I is a fuzzy set ($\mu_{A_I}(x) = 1 - \nu_{A_I}(x)$) for each $x \in \mathscr{U}$, then $\Box A_I = A_I = \Diamond A_I$.

Proposition 2.1. *[Atanassov 1999, Prop. 1.42] For an A-IFS, the properties hold:*

$$\overline{\Box\overline{A_I}} = \Diamond A_I,\ \overline{\Diamond\overline{A_I}} = \Box A_I,\ \Diamond\Diamond A_I = \Diamond A_I,\ \Box\Box A_I = \Box A_I,\ \Box\Diamond A_I = \Diamond A_I,\ \Diamond\Box A_I = \Box A_I. \tag{10}$$

The operator $\triangle : \tilde{U} \to \tilde{U}$, given as $\triangle(x_1, x_2) = (x_1(1 - x_2), x_2(1 - x_1))$ can be used in Intuitionistic Fuzzy Expert Systems and Intuitionistic Fuzzy Estimations of Expert Knowledge [Atanassov 1999].

Definition 2.2. *[Atanassov 1999, Def. 1.93]The A-IFS $\triangle A_I$ is defined as*

$$\triangle A_I = \{\langle x, \mu_{A_I}(x).(1 - \nu_{A_I}(x)), \nu_{A_I}(x).(1 - \mu_{A_I}(x))\rangle | x \in \mathscr{U}\} \tag{11}$$

The A-IFS $\triangle A_I$ has the properties stated in the following theorem:

Theorem 2.1. *[Atanassov 1999, Theorem 1.94] For every A-IFS A, $\overline{\triangle\overline{A_I}} = \triangle A_I$.*

In the case of fuzzy sets, we have that $\triangle A_I = \{\langle x, \mu_{A_I}(x)^2, \nu_{A_I}(x)^2\rangle | x \in \mathscr{U}\}$.

Thus, the $\triangle$-operator is similar to Zadeh's idea expressing the *very*-qualifier [Zadeh 1975]. Moreover, if $\nu_{A_I}(x) = 1 - \mu_{A_I}(x)$, $\mu_{A_I}(x)^2 + \nu_{A_I}(x)^2 = 1 - 2\mu_{A_I}(x)(1 - \mu_{A_I}(x)) \leq 1$, i.e. when we have that $\mu_{A_I}(x) > 0$, then $\triangle A_I$ is a proper IFS with the non-determinacy degreee given as $\pi_{A_I}(x) = 2\mu_{A_I}(x)(1 - \mu_{A_I}(x))$.

3. Correlation from A-IFL

Using denotation related to equations (2), (3)a and (3)b:

$$(\mu_{A_I}(x_1), \mu_{A_I}(x_2), \ldots, \mu_{A_I}(x_n)) = (x_{11}, x_{21}, \ldots, x_{n1}) = \mathbf{x}_{i1};$$
$$(\nu_{A_I}(x_1), \nu_{A_I}(x_2), \ldots, \nu_{A_I}(x_n)) = (x_{12}, x_{22}, \ldots, x_{n2}) = \mathbf{x}_{i2};$$
$$(\pi_{A_I}(x_1), \pi_{A_I}(x_2), \ldots, \pi_{A_I}(x_n)) = (x_{13}, x_{23}, \ldots, x_{n3}) = \mathbf{x}_{i3}.$$

and the two corresponding classes of the quasi-arithmetic means are reported below:
(i) the arithmetic mean (AM) related to an A-IFS A_I; and
(ii) the quadratic mean (QM) is performed over the difference between each intuitionistic fuzzy value of an A-IFS A_I and the corresponding arithmetic mean

Thus, the quotient obtained from the product performed of such AM QM extends the A-CC definition to the A-IFS approach.

Definition 3.1. *[Szmidt and Kacprzyk 2012] The A-CC between A_I and B_I in $\mathscr{C}(A_I)$,*

$$\mathsf{C}_I(A_I,B_I) = \frac{1}{3}(C_1(A_I,B_I)+C_2(A_I,B_I)+C_3(A_I,B_I)) \tag{12}$$

is given when $k \in \{1,2,3\}$ based on the following equations:

$$C_k(A_I,B_I) = \frac{\sum_{i=1}^{n}\left(x_{ik}-\frac{1}{n}\sum_{j=1}^{n}x_{jk}\right)\left(y_{ik}-\frac{1}{n}\sum_{j=1}^{n}y_{jk}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{ik}-\frac{1}{n}\sum_{j=1}^{n}x_{jk}\right)^2\sum_{i=1}^{n}\left(y_{ik}-\frac{1}{n}\sum_{j=1}^{n}y_{jk}\right)^2}}$$

In [Szmidt and Kacprzyk 2012], the correlation coefficient $\mathsf{C}_I(A_I,B_I)$ in Eq. (12) considers both factors, the amount of reliability information expressed by: (i) the membership and non-membership degrees expressed by $C_1(A_I,B_I)$ and $C_2(A_I,B_I)$, respectively; and (ii) the hesitation margins in $C_3(A_I,B_I)$.

Additionally, these expressions just make sense for A-IFS variables whose values vary and avoid zero in the denominator. Moreover, $\mathsf{C}_I(A_I,B_I)$ fulfils the next properties: **(i)** $\mathsf{C}(A_I,B_I) = \mathsf{C}(A_I,B_I)$; **(ii)** If $A_I = B_I$ then $\mathsf{C}(A_I,B_I) = 1$; **(iii)** $-1 \leq \mathsf{C}(A_I,B_I) \leq 1$.

Proposition 3.1. *[Bertei et al. 2016, Prop.1] Let N be a strong A-IFNs, A_I and B_I be A-IFS and $\overline{A_I}$ and $\overline{B_I}$ be their corresponding complements. The following holds:*

$$C_1(A_I,\overline{B_I}) = C_2(\overline{A_I},B_I); \quad C_2(A_I,\overline{B_I}) = C_1(\overline{A_I},B_I); \quad C_3(A_I,\overline{B_I}) = C_3(\overline{A_I},B_I). \tag{13}$$

Corollary 3.1. *[Bertei et al. 2016, Corollary.1] Let N be a strong A-IFNs, A_I and B_I are A-IFS and $\overline{A_I}$ and $\overline{B_I}$ be their corresponding complements. The following holds:*

$$\mathsf{C}_I(A_I,\overline{B_I}) = \mathsf{C}_I(\overline{A_I},B_I). \tag{14}$$

4. A-CC Results on Modal Operators

Extending the results presented in [Bertei et al. 2016], the article [Bertei and Reiser 2018] studies the correlation between A-IFS obtained as the image of modal level operators as $\Diamond A_I$-IFS and $\Box A_I$-IFS that are obtained by action of dual and conjugate operators on $\tilde{U}$.

Proposition 4.1. *[Bertei and Reiser 2018, Proposition IV.3.] Let A_I-AIFS and $\Diamond A_I$-IFS given in Eqs. (1) and (9b). The A-CC between A_I-IFS and $\Diamond A_I$-IFS is given as*

$$\mathsf{C}_I(A_I,\Diamond A_I) = \frac{1}{3}\left(C_1(A_I,\Diamond A_I)+1\right), \tag{15}$$

whenever the following holds

$$C_1(A_I,\Diamond A_I) = \frac{\sum_{i=1}^{n}\left(x_{i1}-\frac{1}{n}\sum_{j=1}^{n}x_{j1}\right)\left(-x_{i2}+\frac{1}{n}\sum_{j=1}^{n}x_{j2}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i1}-\frac{1}{n}\sum_{j=1}^{n}x_{j1}\right)^2\sum_{i=1}^{n}\left(-x_{i2}+\frac{1}{n}\sum_{j=1}^{n}x_{j2}\right)^2}}.$$

Proposition 4.2. *[Bertei and Reiser 2018, Proposition IV.3.] The correlation coefficient between an A_I-IFS and a $\overline{\Diamond A_I}$-IFS is given as*

$$\mathsf{C}_I(A_I,\overline{\Diamond A_I}) = -\mathsf{C}_I\left(A_I,\Diamond A_I\right) \tag{16}$$

Proposition 4.3. *[Bertei and Reiser 2018, Proposition IV.5.] Let A_I-IFS, $\Box A_I$-IFS and $\Diamond A_I$-IFS given by Eqs. 1, 9(a) and (b). The following holds:*

$$\mathsf{C}_I(A_I,\Box A_I) = \mathsf{C}_I\left(A_I,\Diamond A_I\right) \tag{17}$$

Proposition 4.4. *[Bertei and Reiser 2018, Proposition IV.7.] Let A_I be an A-IFS. The correlation between A-IFS A_I and $\overline{\Box A_I}$ is given as*

$$\mathsf{C}_I(A_I, \overline{\Box A_I}) = -\mathsf{C}_I(A_I, \Box A_I) \tag{18}$$

Proposition 4.5. *[Bertei and Reiser 2018, Proposition IV.9.] Let A_I-IFS, $\Diamond A_I$-IFS and $\Box A_I$-IFS given as Eqs. 1, 9(a) and (b), respectively. The following holds:*

$$\mathsf{C}_I(\Diamond A_I, \Box A_I) = \frac{1}{3}\left(C_1(A_I, \Diamond A_I) + C_2(A_I, \Box A_I)\right) \tag{19}$$

Proposition 4.6. *[Bertei and Reiser 2018, Proposition IV.11.] Let $\Box A_I$-IFS and $\Diamond A_I$-IFS given by Eqs. 9(a) and (b), respectively. The following holds:*

$$\mathsf{C}_I(\overline{\Diamond A_I}, \Box A_I) = \frac{2}{3}\left(C_2(\overline{\Diamond A_I}, \Box A_I)\right) \tag{20}$$

whenever the following expression holds

$$C_2(\overline{\Diamond A_I}, \Box A_I) = \frac{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)^2 \sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)^2}}.$$

Proposition 4.7. *[Bertei and Reiser 2018, Prop. IV.13.] For an A-IFS A_I, we have that:*

$$\mathsf{C}_I(\overline{\Diamond A_I}, \overline{\Box A_I}) = \frac{2}{3}(C_1(A_I, \Diamond A_I)) \tag{21}$$

5. A-CC Results on Modal and $\triangle A_I$ Operators

This section reports main results of A-CC related to A-IFS, $\Box A_I$-IFS, $\Diamond A_I$-IFS, and $\triangle A_I$ obtained by the action of dual and conjugate operators on $\tilde{U}$.

Proposition 5.1. *Let $\Diamond A_I - IFS$ and the operator $\triangle A_I$ given in Equations (9b) and (11). The correlation coefficient between $\triangle A_I$ and $\Diamond(\triangle A_I)$ is given as*

$$\mathsf{C}_I(\triangle A_I, \Diamond(\triangle A_I)) = \frac{1}{3}(C_1(\triangle A_I, \Diamond(\triangle A_I)) + 1) \tag{22}$$

whenever the following holds

$$C_1(\triangle A_I, \Diamond(\triangle A_I)) = (-1)\frac{\sum_{i=1}^{n}\left((x_{i1}(1-x_{i2})) - \frac{1}{n}\sum_{j=1}^{n}(x_{j1}(1-x_{j2}))\right)\left((x_{i2}(1-x_{i1})) - \frac{1}{n}\sum_{j=1}^{n}(x_{j2}(1-x_{j1}))\right)}{\sqrt{\sum_{i=1}^{n}\left((x_{i1}(1-x_{i2})) - \frac{1}{n}\sum_{j=1}^{n}(x_{j1}(1-x_{j2}))\right)^2 \sum_{i=1}^{n}\left((x_{i2}(1-x_{i1})) - \frac{1}{n}\sum_{j=1}^{n}(x_{j2}(1-x_{j1}))\right)^2}}$$

Proof. Straightforward from Proposition 4.1. □

Proposition 5.2. *The A-CC between the operator $\triangle A_I$ and $\overline{\Diamond(\triangle A_I)} - IFS$ is given as*

$$\mathsf{C}_I(\triangle A_I, \overline{\Diamond(\triangle A_I)}) = -\mathsf{C}_I(\triangle A_I, \Diamond(\triangle A_I)) \tag{23}$$

Proof. Straightforward from Proposition 4.2. □

Proposition 5.3. *Let $\Box A_I - IFS$ and $\triangle A_I$ be operators given in Equations (9a) and (11). The correlation coefficient between $\triangle A_I$ and $\Box(\triangle A_I)$ is given as*

$$\mathsf{C}_I(\triangle A_I, \Box(\triangle A_I)) = \mathsf{C}_I(\triangle A_I, \Diamond(\triangle A_I)) \tag{24}$$

Proof. Straightforward from Proposition 4.3. □

Proposition 5.4. *The A-CC analysis between $\triangle A_I$ and $\overline{\Box(\triangle A_I)}$ operators is expressed as*

$$\mathsf{C}_I(\triangle A_I, \overline{\Box(\triangle A_I)}) = -\mathsf{C}_I(\triangle A_I, \Box(\triangle A_I)) \tag{25}$$

Proof. Straightforward from Proposition 4.4. □

Proposition 5.5. *Let $\Box A_I - IFS$, $\Diamond A_I - IFS$ and $\triangle A_I$ be operators given in Equations (9a), (9b) and (11). The A-CC between $\Box(\triangle A_I)$ and $\Diamond(\triangle A_I)$ is given as*

$$\mathsf{C}_I(\Box(\triangle A_I), \Diamond(\triangle A_I)) = \frac{1}{3}(C_1(\triangle A_I, \Diamond(\triangle A_I)) + C_2(\triangle A_I, \Box(\triangle A_I))) \quad (26)$$

Proof. Straightforward from Proposition 4.5. □

Proposition 5.6. *Let $\Box A_I - IFS$, $\Diamond A_I - IFS$ and $\triangle A_I$ be operators given in Equations (9a), (9b) and (11), respectively. The next results are verified:*

$$\mathsf{C}_I(\Box(\triangle A_I), \overline{\Diamond(\triangle A_I)}) = \frac{2}{3}(C_2(\Box(\triangle A_I), \overline{\Diamond(\triangle A_I)}) \quad (27)$$

whenever the following expression holds

$$C_2(\Box(\triangle A_I), \overline{\Diamond(\triangle A_I)}) = \frac{\sum_{i=1}^{n}\left((x_{i2}(1-x_{i1})) - \frac{1}{n}\sum_{j=1}^{n}(x_{j2}(1-x_{j1}))\right)\left((x_{i1}(1-x_{i2})) - \frac{1}{n}\sum_{j=1}^{n}(x_{j1}(1-x_{j2}))\right)}{\sqrt{\sum_{i=1}^{n}\left((x_{i2}(1-x_{i1})) - \frac{1}{n}\sum_{j=1}^{n}(x_{j2}(1-x_{j1}))\right)^2 \sum_{i=1}^{n}\left((x_{i1}(1-x_{i2})) - \frac{1}{n}\sum_{j=1}^{n}(x_{j1}(1-x_{j2}))\right)^2}}$$

Proof. Straightforward from Proposition 4.6. □

Proposition 5.7. *Let $\Box A_I - IFS$, $\Diamond A_I - IFS$ and $\triangle A_I$ be operators given in Equations (9a), (9b) and (11), respectively. The following expression holds:*

$$\mathsf{C}_I(\overline{\Box(\triangle A_I)}, \overline{\Diamond(\triangle A_I)}) = \frac{2}{3}(C_1(\triangle A_I, \Diamond(\triangle A_I)) \quad (28)$$

Proof. Straightforward from Proposition 4.7. □

6. MADM - MEDICAL DIAGNOSIS

Previous analytical expressions of modal operator A-CC are applied in developing a method to medical diagnosis (MADM-MD) which is adapted from [Xu 2006]) related to a medical knowledge base, providing a proper diagnosis D = {*Viral fever (VF) , Malaria (Ma) , Typhoid (Ty) , Stomach problem (SP) , Chest problem (CP)*} for a patient with the given symptoms S = {*temperature (T), headache (H), stomach pain (SPa), cough (C), chest pain (CPa)*} described in terms of A-IFSs. One methodology is applied where uses necessity, possibility modal operators and the operator $\triangle$ in MADM-MD.

The necessity modal operator ($\Box$) in the Eq. (9a) together with the operator ($\triangle$) and $\Box(\triangle T1)$ are applied to values from Table 1 (T1) in [Xu 2006], resulting in values of Table 1. In addition, each symptom is described by its related membership and non-membership degrees. The possibility modal operator ($\Diamond$) in the Eq. (9b) along with operator ($\triangle$) and $\Diamond(\triangle T2)$ are applied to values of Table 2 (T2) in [Xu 2006], and symptom results are described in Table 2. In this case study, the set of patients is given as follows: P = {*Al, Bob, Joe, Ted*}. So, we need to seek a diagnosis for each patient p_i, for $i = 1,2,3,4$.

Thus, the A-CC in Eq. (12) is calculated considering Tables 1 and 2, deriving a diagnosis for each patient. The method is performed applying the A-CC between the operators $\Box(\triangle T1)$ and $\Diamond(\triangle T2)$. Such results are listed in Table 3.

Based on the arguments in Table 3, for the method the diagnosis is given as follows: Al suffers from Viral fever, Bob from a stomach problem, Joe from Typhoid, and Ted from Viral ferver. Additionally, one can observe that in Xu [Xu 2006], the diagnosis is the same in three patients (Bob, Joe and Ted) and it is differs in other (Al), in Bertei [Bertei and Reiser 2019]) the diagnosis is similar in Al, Bob and Joe and differs in Ted. The difference in the results are justified by use distinct methodologies.

Table 1. Symptoms characteristic for the diagnoses

	VF	*Ma*	*Ty*	*SP*	*CP*
T	(0.4,0.6)	(0.7,0.3)	(0.21,0.79)	(0.03,0.97)	(0.02,0.98)
H	(0.15,0.85)	(0.08,0.92)	(0.54,0.46)	(0.12,0.88)	(0,1)
SPa	(0.03,0.97)	(0,1)	(0.06,0.94)	(0.8,0.2)	(0.04,0.96)
C	(0.28,0.72)	(0.7,0.3)	(0.08,0.92)	(0.06,0.94)	(0.04,0.96)
CPa	(0.03,0.97)	(0.02,0.98)	(0.01,0.99)	(0.06,0.94)	(0.72,0.28)

Table 2. Symptoms characteristic for the patient

	T	*H*	*SPa*	*C*	*CPa*
Al	(0.98,0.02)	(0.96,0.04)	(0.36,0.64)	(0.96,0.04)	(0.46,0.54)
Bob	(0.2,0.8)	(0.76,0.24)	(0.96,0.04)	(0.37,0.63)	(0.28,0.72)
Joe	(0.98,0.02)	(0.98,0.02)	(0.4,0.6)	(0.44,0.56)	(0.5,0.5)
Ted	(0.96,0.04)	(0.8,0.2)	(0.72,0.28)	(0.94,0.06)	(0.72,0.28)

Table 3. Resulting A-CC of symptoms for each patient

	VF	*Ma*	*Ty*	*SP*	*CP*
Al	**0,562**	0,486	0,399	-0,459	-0,360
Bob	-0,385	-0,422	0,188	**0,545**	-0,265
Joe	0,357	0,143	**0,542**	-0,314	-0,233
Ted	**0,652**	0,651	0,094	-0,373	-0,345

7. Conclusion

In this article, the analytical expressions of A-CC were related to modal operators as necessity and possibility along with the their composition with the *A*-model operator. The case study presents an alternative to evaluate patients, providing a diagnosis for each patient based on the interpretation provided by A-IFM compositions.

Further work considers the A-CC analysis based on the Atanassov's interval-valued intuitionistic fuzzy sets, including the study of main connectives together with their dual and conjugation operators.

References

Atanassov, K. (1983). Intuitionistic fuzzy sets. vii itkr session. sofia. *Centr. Sci.-Techn. Library of Bulg. Acad. of Sci.*, 1697(84).

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets Systems*, 20(1):87–96.

Atanassov, K. T. (1999). *Intuitionistic Fuzzy Sets: Theory and Applications*. Studies in Fuzziness and Soft Computing. Physica-Verlag HD, Heidelberg, Germany, Germany.

Atanassov, K. T. (2012). *On Intuitionistic Fuzzy Sets Theory*, volume 283 of *Studies in Fuzziness and Soft Computing*. Springer.

Bertei, A. and Reiser, R. (2018). Correlation coefficient analysis performed on duality and conjugate modal-level operators. In *2018 IEEE International Conference on Fuzzy Systems*.

Bertei, A. and Reiser, R. (2019). Correlation coefficient of modal level operators: An application to medical diagnosis. In *11th International Joint Conference on Computational Intelligence (FCTA 2019)*, pages 1–9.

Bertei, A., Reiser, R. H. S., and Foss, L. (2021). *Correlation Analysis Via Intuitionistic Fuzzy Modal and Aggregation Operators*, pages 163–190. Springer International Publishing, Cham.

Bertei, A., Zanotelli, R., Cardoso, W., Reiser, R., Foss, L., and Bedregal, B. (2016). Correlation coefficient analysis based on fuzzy negations and representable automorphisms. In *2016 IEEE International Conference on Fuzzy Systems*, pages 127–132.

Bustince, H., Barrenechea, E., and Mohedano, V. (2004). Intuicionistic fuzzy implication operators - an expression and main properties. *Uncertainty, Fuzziness and Knowledge-Based Systems*, 12:387–406.

Bustince, H., Burillo, P., and Soria, F. (2003). Automorphisms, negations and implication operators. *Fuzzy Sets and Systems*, 134(2):209 – 229.

Bustince, H., Kacprzyk, J., and Mohedanoi, V. (2000). Tntuitionistic fuzzy generators, application to intuitionistic fuzzy complementation. *Fuzzy Sets and Syst*, 114:485–504.

Dencheva, K. (2004). Extension of intuitionistic fuzzy modal operators (⊞) and (⊠). In *II International IEEE Conference*, volume 3, pages 21–22.

Deschrijver, G., Cornelis, C., and Kerre, E. (2004). On the representation of intuitionistic fuzzy t-norms and t-conorms. *IEEE Transactions Fuzzy Systems*, 1(12):45–61.

Meng, F., Wang, C., Chen, X., and Zhang, Q. (2016). Correlation coefficients of interval-valued hesitant fuzzy sets and their application based on the shapley function. *International Journal of Intelligence Systems*, 31(1):17–43.

Reiser, R., Visintin, L., Benítez, I., and Bedregal, B. (2013). Correlations from conjugate and dual intuitionistic fuzzy triangular norms and conorms. In *2013 Joint IFSA World Congress and NAFIPS Annual Meeting*, pages 1394–1399.

Szmidt, E. and Kacprzyk, J. (2012). A new approach to principal component analysis for intuitionistic fuzzy data sets. In et al., S. G., editor, *CCIS – IPMU 2012*, volume 298, pages 529–538. Springer.

Szmidt, E., Kacprzyk, J., and Bujnowski, P. (2012). Correlation between intuitionistic fuzzy sets: Some conceptual and numerical extensions. In *2012 IEEE International Conference on Fuzzy Systems*. p.1-7.

Xu, Z. (2006). On correlation measures of intuitionistic fuzzy sets. In *Intelligence Data Engineering and Automated Learning – IDEAL 2006*, pages 16–24.

Zadeh, L. (1965). Fuzzy sets. *Information Control*, 8(3):338–353.

Zadeh, L. (1975). The concept of a linguistic variable and its application to approximate reasoning. *Information Sciences*, 8(3):199 – 249.