An abstract-argumentation-based approach for the portfolio selection problem

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Abstract. *A portfolio problem is about selecting one or several out of a set of possible items, considering some constraints, and where outcomes are determined by a form of aggregating the properties of the items selected. In this paper, we aim to study the use of abstract argumentation theory for a simplified version of the portfolio selection problem, where the conflicts among the arguments (items) can be seen as constraints and the value of each argument (item) can determine the set of acceptable (selected) ones. An example is provided to demonstrate the performance of our approach.*

1. Introduction

The portfolio selection problem is a classical problem in finance that was introduced in [Markowitz 1952]. In general terms, it may be defined as a problem which involves: (i) selecting of one or several out of a set of possible items, (ii) under some constraints, which limit the possibility to select items, and (iii) where outcomes are determined by some aggregation of properties of the selected items [Lopes and Almeida 2013].

This problem occurs in many application areas, the most well-known examples include financial portfolio problems [Zopounidis 1999], project portfolios [Vetschera and De Almeida 2012] [Lopes and Almeida 2013], combination problems in chemistry [Nikolić et al. 2009], or land usage planning [Higgins et al. 2008]. Even, some problems related to agents exhibit similar characteristics, for instance, goals selection [Tinnemeier et al. 2007], plans selection [Nunes and Luck 2014], or the coalition formation problem [Sandholm and Lesser 1995].

Abstract argumentation is basically conceptualized in terms of arguments and attacks among them, along with the distinction among acceptable and unacceptable arguments. The responsible ones in charge of solving such inherent conflicts between arguments are the **semantics**. In abstract argumentation theory several semantics of **acceptability** were defined [Dung 1995][Baroni and Giacomin 2009]. These produce none, one or several acceptable sets of arguments, called **extensions**, which contain a set of consistent arguments.

Argumentation has already been applied in the portfolio problem in [Pendaraki and Spanoudakis 2012] and [Cruz-Reyes et al. 2014]. However, both works use structured argumentation and logical arguments in order to make decision about including and item or not. This work is based on abstract argumentation, which does not consider logical arguments. We think that depending on the kind of portfolio and constraints, our approach is more useful and appropriate than the structured one, since it is less complex and less expensive computationally.

The goal of this paper is to study the application of abstract argumentation theory to a simplified portfolio selection problem. We call it a simplified version, because we will cover problems with only one constraint. Table 1 shows the relation between the main concepts of the portfolio selection problem and the abstract argumentation theory. Both begin with a set of elements $\mathcal{I}$ and the exit is a subset $\mathcal{E} \subseteq \mathcal{I}$, which is obtained taking into account (i) some constraints – for example, that the set has no conflicting elements– and (ii) an aggregation method that maximizes the benefit of the portfolio, for example, using the values of the arguments to defeat those less valuable ones.

Table 1. Comparing the portfolio problem with abstract argumentation

PORTFOLIO PROBLEM	*ABSTRACT ARGUMENTATION*
a set of items	a set of arguments
constraints	conflicts/attacks
aggregation property	value of arguments
a subset of items	a subset of acceptable arguments

In the next section, an illustrating example is presented. In Section 3, we define the main concepts to apply abstract argumentation theory in the portfolio problem. Section 4 is devoted to the application of our approach to the illustrative example. Finally, we present some conclusions and point to possible future work in Section 5.

2. Illustrating example

In this section, we present a motivating example that is used afterwards to explicate our proposal. This is a generic example that will help to understand what kinds of problems can be solved by using our approach.

Consider the problem of a wedding planner in selecting the guests for the ceremony, when there exist antipathies (**constraints**) among some or all the candidate guests (**items**). Let us suppose that the bride and the groom made a list of guests without considering possible conflicts among them, let us call it, the list of candidate guests. *Vicky* is a wedding planner agent, who has to make the final selection taking into account the following kinds of conflicts:

- There are candidate guests with mutual antipathy. In other words, a guest A is not willing to attend the wedding if guest B goes and vice-versa.
- There are antipathies only from one guest to another. For example, a guest C is not willing to go to the wedding if guest D goes, but guest D has no problem with guest C.

Moreover, *Vicky* received a list with the **preference order** of the couple with respect to the guests. For example, if guest A is the boss of the groom and guest B is a friend of the bride, the former option will be more preferred than the latter one.

This scenario leads the following question: *how can agent Vicky select the final list of guests considering both the constraints and the preferences of the couple?*

3. Our approach

In this section, we present a framework for solving the simplified portfolio problem by using abstract argumentation. More precisely, given a set of conflicting elements, we are going to use abstract argumentation theory for deciding the subset of them that will make up the portfolio.

In abstract argumentation [Dung 1995], an argumentation framework ($\mathcal{AF}$) is composed by a set of arguments and a binary attack relation defined over the set. Such $\mathcal{AF}$ can also be represented by a directed graph where the nodes represent the arguments and the edges the attacks. In summary, an argumentation framework is a set of elements with conflicting relations among them. Comparing with the portfolio problem, we can notice that it has similar characteristics, the set of elements would be the initial set of items and the conflicting relation can be seen as a kind of constraint. Formally:

Definition 3.1. (Portfolio-problem framework) An argumentation-like framework for dealing with the portfolio problem is a triple $\mathcal{PF} = \langle \mathcal{I}, \mathcal{R}, \texttt{PREF} \rangle$, where:

- $\mathcal{I}$ is a set of items, where each item has a conflict with one or more of the other items,
- $\mathcal{R} \subseteq \mathcal{I} \times \mathcal{I}$ is a binary relation of constraint. There is a constraint between $it_i, it_j \in \mathcal{I}$ when $(it_i, it_j) \in \mathcal{R}$ [1]. Let us call **unilateral attack** when only $(it_i, it_j) \in \mathcal{R}$ and **bilateral attack** when also $(it_j, it_i) \in \mathcal{R}$,
- PREF is a function that returns the preference value of each item.

As we said before, extensions are sets of consistent items (i.e. there is no unilateral nor bilateral attacks). Following definitions are the base for defining semantics, which produce one or more extensions.

Definition 3.2. (Conflict-freeness, Defense) (Adapted from [Dung 1995]) Let $\mathcal{PF} = \langle \mathcal{I}, \mathcal{R}, \texttt{PREF} \rangle$ be a portfolio problem framework and $\mathcal{S} \subseteq \mathcal{I}$ a set of items.

- $\mathcal{S}$ is called conflict-free iff $\nexists it_i, it_j \in \mathcal{S}$ such that $(it_i, it_j) \in \mathcal{R}$.
- $\mathcal{S}$ defends an item it_i iff $\forall it_j \in \mathcal{I}$ such that $(it_j, it_i) \in \mathcal{R}$, there exists a $it_k \in \mathcal{S}$ such that $(it_k, it_j) \in \mathcal{R}$.

Different acceptability semantics have been proposed in [Dung 1995], however, the preference relation among the items is not considered; whence these cannot be used in this work. Otherwise, in [Amgoud and Vesic 2011], the authors propose a generalized form of the grounded and the preferred semantics called **pref-grounded** and **pref-preferred** respectively. These new semantics take into account the preference relation and are used in our approach. Due to lack of space we present an adapted definition only of the first one. We have chosen it because it is unique and it always exist.

Before presenting the definition of the pref-grounded semantics, it is important to mention that it is based on the notion of **strong defense**, which is based on Definition 3.2 considering the preference relations. Figure 1 shows the four possible cases where set $\mathcal{E}$strongly defends item it_i from the attack of item $it_j \in \mathcal{E}'$, even when it is more preferable.

[1] In other words, it_i attacks it_j. For example, it_i is not willing to attend the wedding if it_j does.

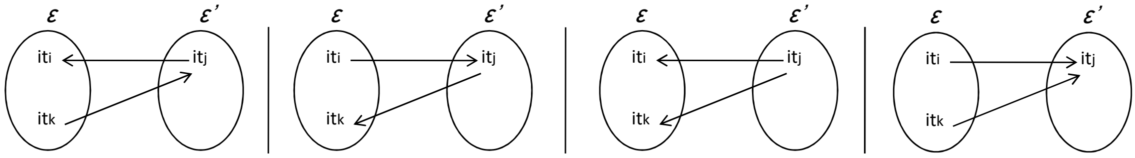

Figure 1. The four cases of strong defense. Let $\texttt{PREF}(it_k) > \texttt{PREF}(it_j) > \texttt{PREF}(it_i)$**.**

Definition 3.3. (Pref-grounded semantics) (Adapted from [Amgoud and Vesic 2011]) Let $\mathcal{PF} = \langle \mathcal{I}, \mathcal{R}, \texttt{PREF} \rangle$ be a portfolio problem framework and $\mathcal{E}, \mathcal{E}'$ be two subsets of $\mathcal{I}$. It holds that $\mathcal{E}$ dominates $\mathcal{E}'$ iff:

- $\mathcal{E}$ is a conflict-free set of $\mathcal{PF}$ and $\mathcal{E}'$ is not, or
- $\forall it_i \in \mathcal{E}$, it holds that $\mathcal{E}$ strongly defends it_i from attacks of set $\mathcal{E}'$.

The maximal set $\mathcal{E}$ is the pref-grounded extension of $\mathcal{PF}$[2], and therefore, $\forall it_i \in \mathcal{E}$, it_i is considered **acceptable**.

So far, the definitions related to the application of the abstract argumentation theory to resolve a simple version of the portfolio problem have been presented; however, we have not talked about their relation with agents. Obviously the main relation -in this work- occurs when agents are in charge of resolving this problem (for instance, agent $Vicky$), in which case, they may use this approach to find the best portfolio, taking into account the preference among the conflicting elements. However, we think that this approach may also be used in other problems related to agents. For example:

- An agent may use it for solving the problem that arises when there are incompatibilities among its goals. In this case, goals would be the elements in conflict and the preference can be given by the importance value of each of them. This approach would resolve this problem and return a set of goals without incompatibilities.

- It can also be applied in multi-agent systems, more specifically in coalition formation. In this case, agents can be seen as elements of the main set, which have conflicts among them and each agent has a reputation value.

4. Applying the approach to the illustrative example

Now, let us show how our approach solves the problem presented in Section 2.

Let $\mathcal{G} = \{g_1, ..., g_{10}\}$ be the list of candidate guests and $\mathcal{R}_\mathcal{G} = \{(g_1, g_4), (g_2, g_5), (g_5, g_2), (g_3, g_6), (g_9, g_6), (g_4, g_8)\}$ be the conflict relation among the guests. This can be translated into a $\mathcal{PF}$ represented by a directed graph (see Figure 2(a)), where nodes represent the guests and edges the conflict relation among them. For example, the pair (g_1, g_4) is an unilateral attack and means that *guest 1* is not willing to go to the ceremony if *guest 4* does. On the other hand, pairs (g_2, g_5) and (g_5, g_2) represent a bilateral attack since *guest 2* does not want to attend to the wedding if *guest 5* does and vice-versa.

For the sake of simplicity, let us suppose that the preference of the couple is related to the number of the guest, hence $\texttt{PREF}_\mathcal{G}(g_1) < \texttt{PREF}_\mathcal{G}(g_2) < .... < \texttt{PREF}_\mathcal{G}(g_{10}))$. Thus, *guest 10* is the most preferred guest and *guest 1* the least one.

[2] More details about this semantics can be found in [Amgoud and Vesic 2011].

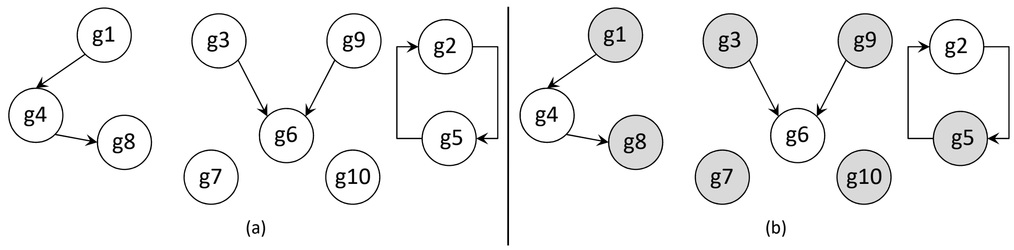

Figure 2. (a) Graph representation of the conflicts among guests. (b) Grey filled nodes represent the pref-grounded extension and hence the acceptable elements, in other words, the final guests list.

Thus, we can say that the portfolio problem framework of agent $Vicky$ for determining the final guests list is $\mathcal{PF}_{\mathcal{G}} = \langle \mathcal{G}, \mathcal{R}_{\mathcal{G}}, \texttt{PREF}_{\mathcal{G}} \rangle$. The following step is to calculate the pref-grounded extension of $\mathcal{PF}_{\mathcal{G}}$. In order to calculate the conflict-free sets of $\mathcal{PF}_{\mathcal{G}}$, we use ConArg [Bistarelli and Santini 2011], a computational tool for modeling and solving argumentation frameworks. The following are two of the three hundred free-conflict sets returned by ConArg. We have chosen the sets with the greatest number of elements since one of the conditions to be a pref-grounded extension is the maximality of the set. Let us analize each one:

(1) $\{g_1, g_2, g_3, g_7, g_8, g_9, g_{10}\}$: This set cannot be a pref-grounded extension because item g_2 has a bilateral attack to g_5, but g_2 is less preferred than g_5 and it does not have any other item that strongly defends it. Therefore, g_2 cannot be part or the final guests list.

(2) $\{g_1, g_3, g_5, g_7, g_8, g_9, g_{10}\}$: This set includes g_1, which unilateral attack to g_4 fails[3] since g_4 is more preferable. There also exists g_8, which is unilaterally attacked by g_4 as well. This attack also fails since g_8 is more preferable than g_4. Considering the four cases of strong defense, we can say that g_8 strongly defends g_1. Therefore, this set is the pref-grounded extension since it is conflict-free, it strongly defends all its items and it is the maximal set with the characteristics previously mentioned. Figure 2(b) shows the pref-grounded extension of $\mathcal{PF}_{\mathcal{G}}$.

In the case that none of the sets with the greatest number of elements is the pref-grounded extension, the other sets with less number of elements have to be analized.

5. Conclusions and future work

We want to begin this section answering the question of Section 2. $Vicky$ can use abstract argumentation theory to obtain the final guests list, fulfilling the constraint that this has no conflicts among its elements and taking into account the preferences.

We presented an example to show how this theory is applied to a simplified problem of portfolio selection. Next step is to evaluate it and compare with other techniques. We also want to use other semantics and study in what cases their application will be suitable. Finally, we want to study other kind of relation among the elements namely the support. For example, a given guest can say that he/she only attends to the ceremony if another guest confirms his attendance.

[3] We use the term "fail´´ because the attacked item is more preferable than the attacker one. Therefore, the attacker item needs another one that strongly defends it to be considered part of the pref-grounded extension.

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