Elementary Economic Systems in Material Agent Societies

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Abstract. *This paper formally characterizes the* elementary economic systems *of material agent societies, on the bases of the notions of (individual and group)* elementary economic behavior*,* elementary economic exchange *and* elementary economic process*. The* equilibrium *of an elementary economic system is defined in terms of the equilibrium of the set of group elementary economic processes that constitute such system. A case study illustrates the proposed concepts.*

Resumo. *Este artigo caracteriza formalmente os* sistemas econômicos elementares *das sociedades de agentes materiais, tendo por base as noções de* comportamento econômico elementar*,* troca econômica elementar *e* processo econômico elementar *(tanto individuais como grupais). O* equilíbrio *de um sistema econômico elementar é definido em termos do equilíbrio do conjunto dos processos econômicos elementares grupais que constitui tal sistema. Um estudo de caso ilustra os conceitos propostos.*

1. Introduction

The concept of *energy systems* of material agent societies, introduced in [Costa 2017b], assumes that energy is produced by, and distributed to, the material agents of material agent societies in the form of *energy objects*. That formalization, however, does not specify any particular way producers and consumers can interact to do that.

In the present paper, we focus on one particular mode of interactive distribution of energy objects, namely, *elementary economic processes*. We say that the set of elementary economic processes of a material agent society constitutes the *elementary economic system* of that society.

To allow for elementary economic processes to occur in a material agent society, we require that the material agents be capable of producing other types of objects, besides energy objects, so that different kinds of objects can be exchanged for each other, in the *elementary economic exchanges* that constitute those elementary economic processes.

For simplicity, in this paper we consider just one other type of objects, besides energy objects. We call them *chips*. Elementary economic processes are assumed, thus, to involve just the exchange of *energy objects* for *chips*, and vice-versa.

2. Material Agent Societies and their Energy Systems

We first summarize the concepts of *material agent*, *material agent society* and *energy systems* of material agent societies as these concepts were introduced in [Costa 2017b].

We say that an agent is a *material agent* whenever that agent has a *material body*, that is, a body that requires *energy* for its operation. We call *material agent society* any agent society whose agents are all material agents.

We consider here only material agent societies organized around an *energy system*, i.e., a particular social subsystem capable of *producing* and *distributing* energy objects within the society in a way that guarantees its *energetic autonomy* [Costa 2017b]. We call *energy producer* any material agent that participates in the operation of that energy system. As in [Costa 2017b], we assume that all producers are *energetically self-sufficient*, that is, are capable of producing all the energy they need for their own operation. The other material agents of the society, are said to be *energy consumers*.

3. Elementary Economic Behaviors and Exchanges

We take George Homans' model of social behaviors and exchanges [Homans 1961] as the operational model on the basis of which we define *elementary economic behaviors and exchanges*. This way, our *elementary economical model* builds on the assumption that any material agent mag is capable of performing the following two types of actions:

- *deliver an object to another agent* at a time t, denoted $deliver^t(mag, obj, mag')$, where mag is the deliverer, and mag' is the receiver, of obj;
- *receive an object from another agent* at a time t, denoted $receive^t(mag, obj, mag')$, where mag is the receiver, and mag' is the deliverer, of obj.

Besides, we assume that each material agent is capable, for each type of existent object, to account for the *sum total of objects* of that type that it has sent or received at each time.

Homans' model is heavily based on Burrhus Skinner's notion of *operant conditioning* [Skinner 1991]. Thus, a central feature of our model is a *reinforcement process* between *sequences* of actions of receiving objects from another material agent ($receive$ actions), and *sequences* of actions of delivering objects of other types to that material agent, in return ($deliver$ actions). More precisely, reinforcement processes operate between the *temporal rates* of those two sequences, that is, between the *temporal rate* at which the receiving operations are performed, and the *temporal rate* at which the delivering operations are performed.

4. Individual Elementary Economic Behaviors and Exchanges Formally Defined

4.1. Terms for Denoting Individual Elementary Economic Behaviors

The following *variables* range over the following sets (variables may appear in expressions with various types of decorations):

- mag, ranging over the set $\mathbf{Mag}$ of material agents;
- beh, ranging over the set $\mathbf{Beh}$ of behaviors of material agents;
- $afval$, ranging over the set $\mathbf{AffVal}$ of affective values of material agents;
- $exch$, ranging over the set $\mathbf{Exch}$ of exchanges between material agents ;
- obj, ranging over the set $\mathbf{Obj}$ of objects that may be exchanged by material agents;
- $p, q, \ldots$, ranging over the set $\mathbf{Sit}$ of social situations in a material agent society;
- t and τ, ranging over the set $\mathrm{T} = \{0, 1, \ldots\}$ of time instants.

Behaviors and affective values may be assigned to material agents:

- $mag.beh$: behavior of a definite material agent;
- $mag.afval$: affective value of a definite material agent.

The *rates of performance* of a behavior, or of an exchange process, is denoted by the operator "$\langle\ \rangle$" applied to that behavior or exchange process: $\langle beh\rangle$ or $\langle exch\rangle$.

4.2. Basic formulas

Basic formulas are of one of the following forms:

(a) statements about *rates of performances* of behaviors or exchanges, or about *tendencies of variation* in such rates, thus:
  - $\langle beh\rangle\top$: behavior has a *high* rate of performance;
  - $\langle beh\rangle\bowtie$: behavior has a *medium* rate of performance;
  - $\langle beh\rangle\bot$: behavior has a *low* rate of performance;
  - $\langle beh\rangle\uparrow$: behavior has an increasing rate of performance;
  - $\langle beh\rangle\downarrow$: behavior has a decreasing rate of performance;

(b) statements about affective assessments of rates of performances of actions, namely, $\langle mag_1.beh\rangle_p[mag_2.afval]_q$, whose meaning is that the *rate* of the behavior beh of the material agent mag_1 has a *defined value*, in situation p, and that such rate of behavior is evaluated with affective value $afval$ by the material agent mag_2, in situation q. If $p = q$, one may write: $\langle mag.beh\rangle[mag.afval]_p$. All the components of the formula are optional, except for the $\langle beh\rangle$ component.

The following illustrate some of the formal variations of such formulas:

- definiteness of the rate of a behavior :
  - any formula in the set $\{\langle beh\rangle, \langle beh_i\rangle\}_{i\in\mathbb{N}}$, meaning that beh (or $\langle beh_i\rangle$) has a defined rate of performance;
- situated definiteness of the rate of a behavior:
  - any formula in the set $\{\langle beh\rangle_p, \langle beh_i\rangle_p\}_{i,p\in\mathbb{N}}$, meaning that beh (or beh_i) has a defined rate of performance in situation p;
- situated definiteness of the rate of behavior of a particular material agent:
  - any formula in the set $\{\langle mag_i.beh_j\rangle_p\}_{i,n,p\in\mathbb{N}}$, meaning that beh_j of mag_i has a defined rate of performance in situation p;
- affective evaluation by an implicitly specified material agent:
  - any formula in the set $\{\langle mag.beh\rangle[afval], \langle mag_i.beh_j\rangle[afval_k]\}_{i,j,k\in\mathbb{N}}$, meaning that: (a) $\langle mag.beh\rangle$ (or $\langle mag_i.beh_j\rangle$) has a defined rate of performance, and (b) it is evaluated with affective value $afval$ (or $afval_k$) by an implicitly specified evaluator material agent.

4.3. Compound formulas

(a) The *propositional composition* of formulas is given by the usual *propositional operators* $(\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow, \ldots)$, assumed to have their usual precedence degrees.

The meaning of $\neg\langle beh\rangle$ is that it is false that variable $\langle beh\rangle$ has a defined value, i.e., it is false that the behavior beh is not in execution. The formula $\neg\langle beh\rangle[afval]$ should be read as $\neg(\langle beh\rangle[afval])$, that is, the operator of affective evaluation has precedence over $\neg$. The same happens with the other propositional operators, so that to express the simultaneous affective evaluation of two behaviors one should write $(\langle beh_i\rangle \wedge \langle beh_j\rangle)[afval]$.

(b) *Functional dependence* compound formulas express *monotonic qualitative functional dependences* between two rates of performance of behaviors. The *functional dependence* compound formulas have the basic forms:

$$\langle mag_i.beh_i\rangle \nearrow \langle mag_j.beh_j\rangle \quad \text{and} \quad \langle mag_i.beh_i\rangle \searrow \langle mag_j.beh_j\rangle$$

The meaning of $\langle mag_i.beh_i\rangle \nearrow \langle mag_j.beh_j\rangle$ is that: (a) the rates of performance of beh_i of material agent mag_i, and of beh_j of mag_j, are both defined, and (b) the rate of beh_j *monotonically increases* with the rate of beh_i. Analogously, the meaning of $\langle mag_i.beh_i\rangle \searrow \langle mag_j.beh_j\rangle$ is that the functional dependence between the two rates of behaviors if *monotonically decreasing*.

Possible decorations of the *functional dependence* compound formulas with situation indexes are as follows: $\langle mag_i.beh_i\rangle \nearrow_p \langle mag_j.beh_j\rangle$ and $\langle mag_i.beh_i\rangle \searrow_p \langle mag_j.beh_j\rangle$. Notice that, since the *functional dependence* has to be determined in a single situation, the *situation indexes* of the two behaviors have to refer to the same situation.

There is no notion of *functional dependence* between affective evaluations of defined values of variables, given the assumption of *evaluation autonomy* of the material agents. So the only way *functional dependence* compound formulas may be decorated with affective evaluation operations is by indicating that the *whole functional dependence* is affectively evaluated, in the general forms: $\big[\langle mag_i.beh_i\rangle \nearrow_p \langle mag_j.beh_j\rangle\big]\,[mag_k.afval_k]$ and $\big[\langle mag_i.beh_i\rangle \searrow_p \langle mag_j.beh_j\rangle\big]\,[mag_k.afval_k]$, meaning that the indicated *functional dependences* are evaluated by material agent mag_k with affective value $afval_k$.

(c) The *ordering* of rates of performance of behaviors, and of results of affective evaluations of rates of behaviors, is given by the partial order relation "$\preccurlyeq$". We assume that "$\preccurlyeq$" operates uniformly on *affective values* and on *rates of performances of behaviors*, thus: $afval_1 \preccurlyeq afval_2$ and $\langle beh_1\rangle \preccurlyeq \langle beh_2\rangle$.

For any behavior beh it holds that its *lowest* rate of performance (given by $\bot$), *highest* rate of performance (given by $\top$), and *medium* rate of performance (given by $\bowtie$) are ordered by the partial ordering relation "$\preccurlyeq$" as: $\bot \preccurlyeq \bowtie \preccurlyeq \top$. For the affective values resulting from the affective evaluation of rates of behavior, we extend the use of "$\preccurlyeq$":

$$\langle mag_1.beh_1\rangle[mag_2.afval_2]_p \preccurlyeq \langle mag_3.a_3\rangle[mag_4.afval_4]_p \Leftrightarrow$$
$$\langle mag_1.beh_1\rangle[mag_2.afval_2]_p \;\wedge\; \langle mag_3.a_3\rangle[mag_4.afval_4]_p \;\wedge\; afval_2 \preccurlyeq afval_4$$

4.4. The Individual Behavioral Conditioning Rules

The fundamental relation between behaviors, according to Skinner [Skinner 1991] is the relation of *operant conditioning*, that is, the relation according to which an spontaneous behavior (an *operant*) is led to be performed in the context of, and in accordance to, a certain *conditioner*. The conditioning occurs because the *conditioner* behavior is assumed to represent a combination of events that is relevant for the internal functioning of the *operant agent*, so that it is capable of influencing (positively or negatively) the way that agent performs the *operant* behavior that is being conditioned.

The *operant conditioning rules* that we introduce below, attempt to formally capture some of the fundamental operant conditioning propositions that Homans took from

Skinner's behavioral psychology [Skinner 1991], and that he adopted as the behavioristic foundation of his social exchange theory [Homans 1961].

In Homans' conceptualization, the *operant* is a behavior that some agent directs toward another agent, and the *conditioner* is some reaction that the latter directs toward the former. In our *economic interpretation* of Homans' concept of social behavior, we limit both the *operant* and the *conditioner* behaviors to be sequences of actions of *delivering* and *receiving* objects (more precisely, *energy objects* and *chips*).

Skinner's basic rule, fully adopted by Homans, is: *an operant is conditioned by a conditioner whenever it happens that an increase or decrease in the rate of performance of the conditioner impacts the rate of performance of the operant behavior.*

The usual interpretation of this proposition is that the operant agent evaluates (positively or negatively) the variation (increase or decrease) in the rate of performance of the conditioner behavior, and reacts by varying accordingly the rate of performance of the operant behavior. We formally present this interpretation by the set of formulas:

$$\langle mag_1.receive(mag_1, obj_1, mag_2)\uparrow\rangle[mag_1.+] \rightsquigarrow \langle mag_1.deliver(mag_1, obj_2, mag_2)\uparrow\rangle$$

$$\langle mag_1.receive(mag_1, obj_1, mag_2)_\downarrow\rangle[mag_1.+] \rightsquigarrow \langle mag_1.deliver(mag_1, obj_2, mag_2)\uparrow\rangle$$

$$\langle mag_1.receive(mag_1, obj_1, mag_2)\uparrow\rangle[mag_1.-] \rightsquigarrow \langle mag_1.deliver(mag_1, obj_2, mag_2)_\downarrow\rangle$$

$$\langle mag_1.receive(mag_1, obj_1, mag_2)\uparrow\rangle[mag_1.-] \rightsquigarrow \langle mag_1.deliver(mag_1, obj_2, mag_2)_\downarrow\rangle$$

where the "$\rightsquigarrow$" symbol denotes the impact relation between the (positive or negative) evaluation, by mag_1, of the variation (increase or decrease) in the rate of reception of the *conditioner object* obj_1 from mag_2, and the consequent variation (increase or decrease) in the rate of performance of the delivery of the object obj_1 to mag_2, by mag_1.

We may use the following abbreviations for the increase or decrease in the rate of the conditioned operant:

$$\langle mag_1.receive(mag_1, obj_2, mag_2)\rangle \twoheadrightarrow^{+} \langle mag_1.deliver(mag_1, obj_1, mag_2)\rangle$$

$$\langle mag_1.receive(mag_1, obj_2, mag_2)\rangle \twoheadrightarrow^{-} \langle mag_1.deliver(mag_1, obj_1, mag_2)\rangle$$

The following are the *conditioning rules for individual elementary economic behaviors* that we establish on the basis of Homans' interpretation of Skinner's basic rule. They are characterized by the type of the conditioning acting on the operant behavior ("+" or "$-$") and by the level of activity of the conditioner ("$\top$", "$\bowtie$" or "$\bot$"):

1. Rule $BC_{(+,\bowtie)}$: If mag_1 evaluates *positively* (+) the reception of obj_2 from mag_2, in return to mag_1 delivering obj_1 to mag_2, and mag_2 delivers obj_2 to mag_1 at *a regular temporal rate* ($\bowtie$) then: the more frequently mag_2 delivers obj_2 to mag_1, the more frequently will mag_1 deliver obj_1 to mag_2. Formally:

$$\frac{\langle mag_1.receive(mag_1, obj_2, mag_2)\rangle \twoheadrightarrow^{+} \langle mag_1.deliver(mag_1, obj_1, mag_2)\rangle \quad \langle mag_2.deliver(mag_2, obj_2, mag_1)\rangle\bowtie}{\langle mag_2.deliver(mag_2, obj_2, mag_1)\rangle \nearrow \langle mag_1.deliver(mag_1, obj_1, mag_2)\rangle}\ BC_{(+,\bowtie)}$$

2. Rule $BC_{(+,\top)}$: If mag_1 evaluates ***positively*** (+) the reception of obj_2 from mag_2, in return to mag_1 delivering obj_1 to mag_2, and mag_2 delivers obj_2 to mag_1 at a ***very high temporal rate*** ($\top$) then: the more frequently mag_2 delivers obj_2 to mag_1, the less frequently will mag_1 deliver obj_1 to mag_2. Formally:
$$\frac{\langle mag_1.receive(mag_1, obj_2, mag_2)\rangle \twoheadrightarrow^+ \langle mag_1.deliver(mag_1, obj_1, mag_2)\rangle \quad \langle mag_2.deliver(mag_2, obj_2, mag_1)\rangle\top}{\langle mag_2.deliver(mag_2, obj, mag_1)\rangle \searrow \langle mag_1.deliver(mag_1, beh, mag_2)\rangle} BC_{(+,\top)}$$

3. Rule $BC_{(+,\perp)}$: If mag_1 evaluates ***positively*** (+) the reception of obj_2 from mag_2, in return to mag_1 delivering obj_1 to mag_2, and mag_2 delivers obj_2 to mag_1 at a ***very low temporal rate*** ($\perp$) then: the more frequently mag_2 delivers obj_2 to mag_1, the more frequently will mag_1 deliver obj_1 to mag_2. Formally:
$$\frac{\langle mag_1.receive(mag_1, obj_2, mag_2)\rangle \twoheadrightarrow^+ \langle mag_1.deliver(mag_1, obj_1, mag_2)\rangle \quad \langle mag_2.deliver(mag_2, obj, mag_1)\rangle\perp}{\langle mag_2.deliver(mag_2, obj, mag_1)\rangle \nearrow \langle mag_1.deliver(mag_1, beh, mag_2)\rangle} BC_{(+,\perp)}$$

4. Rule BC_-: If mag_1 evaluates ***negatively*** (−) the reception of obj_2 from mag_2, in return to mag_1 delivering obj_1 to mag_2 then: the more frequently mag_2 delivers obj_2 to mag_1, the less frequently will mag_1 deliver obj_1 to mag_2. Formally:
$$\frac{\langle mag_1.receive(mag_1, obj_2, mag_2)\rangle \twoheadrightarrow^- \langle mag_1.deliver(mag_1, obj_1, mag_2)\rangle}{\langle mag_2.deliver(mag_2, obj, mag_1)\rangle \searrow \langle mag_1.deliver(mag_1, beh, mag_2)\rangle} BC_-$$

5. Rule $BC_\vee$: Any *increase* in frequency of a particular behavior beh_1 by mag entails by that very fact a *decrease* in the frequency of any alternative behavior beh_2 that mag can perform. Formally:
$$\frac{beh_i, beh_j \in \mathrm{beh}[mag]}{\langle mag.beh_i\rangle \searrow_{i\neq j} \langle mag.beh_j\rangle} BC_\vee$$
where $\mathrm{beh}[mag]$ denotes the set of behaviors that mag is capable of performing.

4.5. Terms for Denoting Individual Elementary Economic Exchanges

An *individual elementary economic exchange* is a *pair of operant conditionings* acting between two material agents, mag_1 and mag_1, so that the delivery of the object obj_1, by mag_1 to mag_2 acts as a *conditioner* to the delivery of the object obj_2 by mag_2 to mag_1, and vice-versa.

We say that an *individual elementary economic exchange* between mag_1 and mag_2, involving the exchange of the objects obj_1 and obj_2 between them, is performed in the *doubly positive mode of exchange* if and only if:

1. obj_1 is an *energy object* and obj_2 is a *chip*, or vice-versa;
2. $\langle mag_1.receive(mag_1, obj_2, mag_2)\rangle \twoheadrightarrow^+ \langle mag_1.deliver(mag_1, obj_1, mag_2)\rangle$
3. $\langle mag_2.receive(mag_2, obj_1, mag_1)\rangle \twoheadrightarrow^+ \langle mag_2.deliver(mag_2, obj_2, mag_1)\rangle$

We denote such a doubly positively reinforced *individual elementary economic exchange* by: $\langle mag_1/obj_1\rangle \;{}_{+}\!\rightleftarrows^{+} \langle mag_2/obj_2\rangle$.

The *temporal evolution* of any individual elementary economic exchange may lead the material agents involved in it to change the way they evaluate the conditioners they receive from their partners. Variations in the rate of reception of conditioners may also lead the operant agent to change such evaluations. As a result, the following are also *modes of exchange* that may occur during the performance of an individual elementary economic exchange:

- *mixed* modes of exchange:
  $\langle mag_1/obj_1\rangle \;{}_{-}\!\rightleftarrows^{+}\; \langle mag_2/obj_2\rangle$ and $\langle mag_1/obj_1\rangle \;{}_{+}\!\rightleftarrows^{-}\; \langle mag_2/obj_2\rangle$;
- *doubly negative* mode of exchange:
  $\langle mag_1/obj_1\rangle \;{}_{-}\!\rightleftarrows^{-}\; \langle mag_2/obj_2\rangle$.

4.6. Equilibrium of Individual Elementary Economic Exchanges

We say that the individual elementary economic exchange:

$$ie2exch = \langle mag_1/obj_1\rangle \;{}_{+}\!\rightleftarrows^{+}\; \langle mag_2/obj_2\rangle$$

which is performed at a time τ, is *equilibrated* at that time if and only if it holds, for each of the behaviors of mag_1 and mag_2, that they are performed at a *medium rate* at that time, that is: $\langle mag_1/obj_1\rangle^{\tau} \bowtie$ and $\langle mag_2/obj_2\rangle^{\tau} \bowtie$.

We denote by $\mathrm{equil}[ie2exch]^{\tau}$ the fact that the individual elementary economic exchange $ie2exch$ is equilibrated at the time τ.

5. Individual Elementary Economic Processes

5.1. Sequential Composition of Operant Conditionings of Individual Elementary Economic Behaviors

Operant conditionings between individual elementary economic behaviors can be *sequentially composed*, in the sense that a behavior beh_2 that is conditioned by a behavior beh_1 can, itself, condition a behavior beh_3. In such situation, one can say that behavior beh_1 also conditions behavior beh_3.

The following are the rules defining such *sequential compositions*, considering the possible positive or negative conditionings that the behaviors may have on each other.

$$\frac{\langle beh_1\rangle \twoheadrightarrow^{+} \langle beh_2\rangle \qquad \langle beh_2\rangle \twoheadrightarrow^{+} \langle beh_3\rangle}{\langle beh_1\rangle \twoheadrightarrow^{+} \langle beh_3\rangle}\; BehSC_1$$

$$\frac{\langle beh_1\rangle \twoheadrightarrow^{-} \langle beh_2\rangle \qquad \langle beh_2\rangle \twoheadrightarrow^{-} \langle beh_3\rangle}{\langle beh_1\rangle \twoheadrightarrow^{+} \langle beh_3\rangle}\; BehSC_2$$

$$\frac{\langle beh_1\rangle \twoheadrightarrow^{+} \langle beh_2\rangle \qquad \langle beh_2\rangle \twoheadrightarrow^{-} \langle beh_3\rangle}{\langle beh_1\rangle \twoheadrightarrow^{-} \langle beh_3\rangle}\; BehSC_3$$

$$\frac{\langle beh_1\rangle \twoheadrightarrow^{-} \langle beh_2\rangle \qquad \langle beh_2\rangle \twoheadrightarrow^{+} \langle beh_3\rangle}{\langle beh_1\rangle \twoheadrightarrow^{-} \langle beh_3\rangle}\; BehSC_4$$

5.2. Sequential Composition of Individual Elementary Economic Exchanges

Individual elementary economic exchanges can also be sequentially composed. The following is a *sample* of the rules defining such sequential compositions:

$$\frac{\langle mag_1/obj_1\rangle \;{}_{+}{\rightleftarrows}^{+}\; \langle mag_2/obj_2\rangle \qquad \langle mag_2/obj_2\rangle \;{}_{+}{\rightleftarrows}^{+}\; \langle mag_3/obj_3\rangle}{\langle mag_1/obj_1\rangle \;{}_{+}{\rightleftarrows}^{+}\; \langle mag_3/obj_3\rangle}$$

$$\frac{\langle mag_1/obj_1\rangle \;{}_{-}{\rightleftarrows}^{-}\; \langle mag_2/obj_2\rangle \qquad \langle mag_2/obj_2\rangle \;{}_{-}{\rightleftarrows}^{-}\; \langle mag_3/obj_3\rangle}{\langle mag_1/obj_1\rangle \;{}_{+}{\rightleftarrows}^{+}\; \langle mag_3/obj_3\rangle}$$

$$\frac{\langle mag_1/obj_1\rangle \;{}_{-}{\rightleftarrows}^{+}\; \langle mag_2/obj_2\rangle \qquad \langle mag_2/obj_2\rangle \;{}_{-}{\rightleftarrows}^{+}\; \langle mag_3/obj_3\rangle}{\langle mag_1/obj_1\rangle \;{}_{+}{\rightleftarrows}^{+}\; \langle mag_3/obj_3\rangle}$$

$$\frac{\langle mag_1/obj_1\rangle \;{}_{-}{\rightleftarrows}^{+}\; \langle mag_2/obj_2\rangle \qquad \langle mag_2/obj_2\rangle \;{}_{+}{\rightleftarrows}^{-}\; \langle mag_3/obj_3\rangle}{\langle mag_1/obj_1\rangle \;{}_{-}{\rightleftarrows}^{-}\; \langle mag_3/obj_3\rangle}$$

5.3. Individual Elementary Economic Processes Formally Defined

We call *individual elementary economic process* any finite, non-null, time-indexed sequence of compositions of individual elementary economic exchanges, of the form:

$$ie2iproc^t = \langle mag_1/obj_1\rangle^t \;{}_{c_{2,1}}{\rightleftarrows}^{c_{1,2}}\; \langle mag_2/obj_2\rangle^t \;{}_{c_{3,2}}{\rightleftarrows}^{c_{2,3}}\; \cdots \;{}_{c_{n,n-1}}{\rightleftarrows}^{c_{n-1,n}}\; \langle mag_n/obj_n\rangle^t$$

where each $\langle mag_i/obj_i\rangle^t \;{}_{c_{j,i}}{\rightleftarrows}^{c_{i,j}}\; \langle mag_j/obj_j\rangle^t$ is an individual elementary economic exchange that may occur at the time t, the *conditioning signs* are $c_{i,j} \in \{+,-\}$, there is *no cycle* in the process (that is, $mag_i/obj_i \neq mag_j/obj_j$, for every i and j), and $n \geq 2$ is the *length* of the individual elementary economic process.

5.4. Equilibrium of Individual Elementary Economic Processes

The equilibrium of an *individual elementary economic process*, at a given time, is given by the fact that each of the exchanges that compose it are equilibrated at that time. That is, the elementary economic process:

$$ie2iproc^t = \langle mag_1/obj_1\rangle^t \;{}_{c_{2,1}}{\rightleftarrows}^{c_{1,2}}\; \langle mag_2/obj_2\rangle^t \;{}_{c_{3,2}}{\rightleftarrows}^{c_{2,3}}\; \cdots \;{}_{c_{n,n-1}}{\rightleftarrows}^{c_{n-1,n}}\; \langle mag_n/obj_n\rangle^t$$

is equilibrated at the time τ (with $0 \leq \tau \leq t$) if and only if each of the individual elementary economic exchanges that constitute it is equilibrated, that is:

$$\forall \langle mag_i/obj_i\rangle^t \in ie2iproc^t \, (\text{equil}[\langle mag_i/obj_i\rangle]^\tau)$$

We denote by $\text{equil}[ie2proc]^\tau$ the fact that $ie2proc^t$ is equilibrated at the time τ.

6. Group Elementary Economic Behaviors and Exchanges

6.1. Group Elementary Economic Behaviors and Exchanges Formally Defined

The general form of individual elementary economic exchange introduced in Sect. 5, namely, $\langle mag_1/obj_1\rangle \;{}_{c_{2,1}}{\rightleftarrows}^{c_{1,2}}\; \langle mag_2/obj_2\rangle$, is constituted by a combination of elementary economic behaviors performed by two *individual* material agents (mag_1 and mag_2).

We can naturally extend the concept of elementary economic behavior to *groups* of material agents, considering that each such group operates as a *unity*, collectively delivering objects to another group, and collectively being reinforced by the reception of objects from that other group.

Let Mag and Mag' be two such *groups*, and Obj the *set of objects* that they may exchange between them. The operation $deliver^t(Mag, Obj, Mag')$ indicates, then, the delivery of that set of objects by one group to the other, and similarly for the operation $receive^t(Mag, Obj, Mag')$.

A *group elementary economic exchange* between Mag_1 and Mag_2, exchanging sets of objects Obj_1 and Obj_2, is denoted by: $\langle Mag_1/Obj_1\rangle\ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \langle Mag_2/Obj_2\rangle$.

6.2. Equilibrium of Group Elementary Economic Exchanges

We say that the group elementary economic exchange:

$$ge2exch = \langle Mag_1/Obj_1\rangle\ {}_{+}\!\rightleftarrows^{+} \langle Mag_2/Obj_2\rangle$$

which is performed at a time τ, is *equilibrated* at that time if and only if it holds, for each of the behaviors of Mag_1 and Mag_2, that they are performed at a *medium rate* at that time, that is, $\langle Mag_1/obj_1\rangle^\tau \bowtie$ and $\langle Mag_2/obj_2\rangle^\tau \bowtie$.

We denote by $\mathrm{equil}[ge2exch]^\tau$ the fact that the group elementary economic exchange $ge2exch^t$ is equilibrated at the time τ.

7. Group Elementary Economic Processes

7.1. Group Elementary Economic Processes Formally Defined

A *group elementary economic process* is a time-indexed sequence of group elementary economic exchanges, of the form:

$$ge2proc^t = \langle Mag_1/Obj_1\rangle^t\ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \langle Mag_2/Obj_2\rangle^t\ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \dots\ {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}} \langle Mag_n/Obj_n\rangle^t$$

7.2. Equilibrium of Group Elementary Economic Processes

We say that the *group elementary economic process*:

$$ge2proc^t = \langle Mag_1/Obj_1\rangle^t\ {}_{c_{2,1}}\!\rightleftarrows^{c_{1,2}} \langle Mag_2/Obj_2\rangle^t\ {}_{c_{3,2}}\!\rightleftarrows^{c_{2,3}} \dots\ {}_{c_{n,n-1}}\!\rightleftarrows^{c_{n-1,n}} \langle Mag_n/Obj_n\rangle^t$$

is *equilibrated* at the time τ (with $0 \le \tau \le t$), if and only if each of the group elementary economic exchanges that constitute it is equilibrated, that is:

$$\forall \langle Mag_i/Obj_i\rangle^t \in ge2iproc^t\ (\mathrm{equil}[\langle Mag_i/Obj_i\rangle]^\tau)$$

We denote by $\mathrm{equil}[ge2proc]^\tau$ the fact that $ge2proc^t$ is equilibrated at the time τ.

8. Elementary Economic Systems

8.1. Elementary Economic Systems Formally Defined

Formally, we define:

Definition 8.1 *The* elementary economic system *EES of a material agent society $MAgSoc$ whose population of material agents is Pop, is a time-indexed structure:*

$$EES^t_{MAgSoc} = (Groups^t, Objs^t, GE2Beh^t, GE2Exch^t, GE2Proc^t)$$

where[1]*:*

[1] $\wp(X)$ denotes the powerset of the set X.

- $Groups^t \subseteq \wp(Pop)$ *is the family of* groups of material agents *of* Pop *that can participate in the elementary economic group processes of* $GE2Proc^t$*;*
- $Objs^t \subseteq \wp(\mathbf{Obj})$ *is the family of* sets of objects *that the groups of material agents of* $Groups^t$ *can exchange between them during the performance of the group elementary economic processes of* $GE2Proc^t$*;*
- $GE2Beh^t$ *is the set of* group elementary economic behaviors *that the groups of material agents of* $Groups^t$ *can perform during the performance of the group elementary economic processes of* $GE2Proc^t$*;*
- $E2GExch^t$ *is the set of* group elementary economic exchanges *that the groups of material agents of* $Groups^t$ *can perform during the performance of the group elementary economic processes of* $GE2Proc^t$*;*
- $GE2Proc^t$ *is the set of* group elementary economic processes *that the groups of material agents of* $Groups^t$ *can perform in* $MAgSoc$ *at the time* t*.*

8.2. Equilibrium of Elementary Economic Systems

We say that the elementary economic system:

$$EES^t_{MAgSoc} = (Groups^t, Objs^t, GE2Beh^t, GE2Exch^t, GE2Proc^t)$$

is *equilibrated* at the time τ (with $0 \leq \tau \leq t$) if and only if each of the group elementary economic process that constitute it is equilibrated at that time, that is:

$$\forall ge2proc \in GE2Proc^t \, (\text{equil}[ge2proc]^\tau)$$

We denote by $\text{equil}[EES_{MAgSoc}]^\tau$ the fact that the elementary economic system EES^t_{MAgSoc} is equilibrated at the time τ.

9. Ecosystems: A Case Study in Elementary Economical Analysis

We provide here elements for the *elementary economical analysis of ecosystems*. More precisely, we consider: (a) *ecosystems* as material agent societies; and (b) the *interactional structure* that constitute the operational part of the organizational structure of an ecosystem as an *elementary economical system*.

The casting of ecosystems as agent societies was introduced in [Costa 2017a]. The proposal for considering the *interactional structure* of an ecosystem as an *elementary economical system* is presented here for the first time.

Figure 1 pictures a general model for the interactional structure of ecosystems. We base on it the elementary economical analysis that follows.

As in [Costa 2017a], we state that an *ecosystem* is a material agent society $EcoSys^t = (Pop^t, Org^t, MEnv^t)$ where, for each time t:

- Pop^t is the *populational structure* of the society, with:
- Org^t is the *organizational structure* of the society;
- $MEnv^t$ is the society's *material environment*.

so that, regarding Fig. 1, we have:

- $Pop^t = (Plants^t, Herbivores^t, Carnivores^t, Detrivores^t)$;
- $Org^t = (O_2^t, CO_2^t, Water^t, Nutrients^t, EatenBy^t, SolarEnergy^t)$

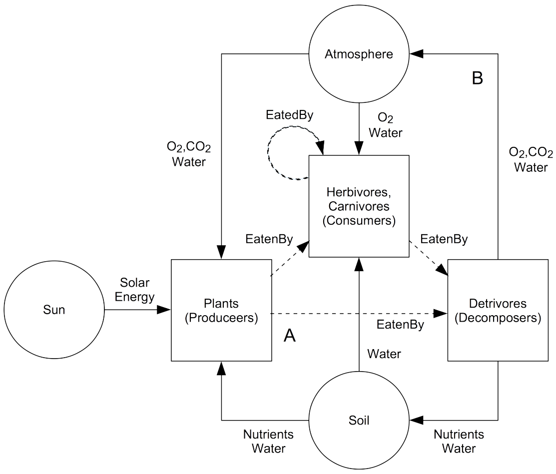

Figure 1. The general interactional structure of ecosystems, based on [Pidwirny 2009].

- $MEnv^t = (Atmosphere^t, Soil^t, Sun^t)$
with:
- $O_2^t \subseteq (Detrivores^t \times Atmosphere^t) \cup (Atmosphere^t \times Plants^t)$
$\cup (Atmosphere^t \times Herbivores^t) \cup (Atmosphere^t \times Carnivores^t))$
- $CO_2 \subseteq (Detrivores^t \times Atmosphere^t) \cup (Atmosphere^t \times Plants^t)$
- $Water^t \subseteq (Detrivores^t \times Atmosphere^t) \cup (Atmosphere^t \times Plants^t)$
$\cup (Atmosphere^t \times Herbivores^t) \cup (Atmosphere^t \times Carnivores^t))$
$\cup (Detrivores^t \times Soil^t) \cup (Soil^t \times Plants^t))$
$\cup (Soil^t \times Herbivores^t) \cup (Soil^t \times Carnivores^t))$
- $Nutrients^t \subseteq (Detrivores^t \times Soil^t) \cup (Soil^t \times Plants^t)$
- $EatenBy^t \subseteq (Herbivores^t \times Detrivores^t) \cup (Carnivores^t \times Detrivores^t)$
$\cup (Plants^t \times Detrivores^t) \cup (Plants^t \times Herbivores^t)$
$\cup (Plants^t \times Carnivores^t) \cup (Carnivores^t \times Carnivores^t)$
- $SolarEnergy^t \subseteq Sun^t \times Plant^t$

Clearly:

- $Detrivores$, $Carnivores$, $Herbivores$ and $Plants$ constitute the *populational groups* of the ecosystems;
- the Sun and the $Soil$ are the source of all *energy* consumed by the populational groups of ecosystems;
- the $Atmosphere$ and the $Soil$ operate as *transportation means* for the O_2, CO_2, $Water$ and $Nutrients$ that the populational groups exchange between them;
- individuals of a populational group eating individuals of another populational group is also a means for the former group to receive energy from the latter one.

Also: (a) the set of exchanges operating in the ecosystem constitute a complex network, and (b) there are more than two types of objects being exchanged in the system.

So, from the economical point of view, such system surpasses the conditions stipulated by the concepts introduced in the present paper, and an elementary economical analysis of ecosystems can only partially account for it. The following are two of the *elementary economical processes* present in Fig. 1::

1. $ge2proc_A = \langle Detrivores/Soil/Water \rangle \ {}_{+}\rightleftarrows^{+} \ \langle Plants/Plants \rangle$

where $\langle Detrivores/Soil/Water \rangle$ denotes that $Detrivores$ deliver objects of type $Water$ through the $Soil$ to $Plants$, and $\langle Plants/Plants \rangle$ denotes that $Plants$ deliver objects of type $Plants$ directly to (be eaten by) $Detrivores$;

2. $ge2proc_B = \langle Detrivores/Atmosphere/O_2 \rangle \ {}_{+}\rightleftarrows^{+} \ \langle Carnivores/Carnivores \rangle$

which has a reading analogous to the previous one.

So, we can tentatively construe the interactional structure of ecosystems as elementary economic systems as:

$$EES^t_{MAgSoc} = (Groups^t, Objs^t, GE2Beh^t, GE2Exch^t, GE2Proc^t)$$

with:

- $Groups^t = Pop^t$, that is, all of the *populational groups* of the ecosystem participate in the elementary economic system;
- $O_2, Water \in Objs^t$, since they are *objects* exchanged by the populational groups;
- $deliver(Detrivores, Water, Carnivores)$,
  $deliver(Carnivores, Carnivores, Detrivores) \in GE2Beh^t$
  since they are *group elementary economic behaviors*;
- $\langle Detrivores^t.deliver(Detrivores, Water, Carnivores) \rangle$,
  ${}_{+}\rightleftarrows^{+} \ \langle Carnivores^t.deliver(Carnivores, Carnivores, Detrivores) \rangle \in GE2Exch^t$
  since it is a *group elementary economic exchange*;
- $ge2proc_A, ge2proc_B \in GE2Proc^t$, since they are *group elementary economic processes*.

10. Conclusion

The concepts introduced in the present paper seem to be the most elementary economical concepts that can fit the basic features of energy systems of material agent societies, as they were defined in [Costa 2017b].

The definition of *full-fledged economic systems* for material agent societies is reserved for future work. It will require the lifting of many of the constraints that are intrinsic to the elementary systems. For instance: (a) that operant conditioning be *direct* (i.e., due to the interacting partner), allowing for *indirect* operant conditionings, through chains of indirect partners; (b) the *absence of cycles* in economic processes, leading to complex networks of interlinked economic processes; (c) admission of *only two types of objects* in economic exchanges.

We remark that we have no knowledge of other reports dealing with the concepts introduced here, analogously to what happened in [Costa 2017b].

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