

# Slavery in Material Agent Societies

Antônio Carlos da Rocha Costa<sup>1</sup>

<sup>1</sup>Programa de Pós-Graduação em Computação  
Universidade Federal do Rio Grande - FURG  
96.203-900 Rio Grande, RS, Brazil.

ac.rocha.costa@gmail.com

**Abstract.** *This paper formally characterizes slavery systems in material agent societies. The concepts of master-slave social relationship, master-slave economic relationship, slavery-based economic system, slavery-supporting legal system, and slavery-based material agent society are formally defined. A case study formally revisits North & Thomas' model stating the objective conditions that justify the rational choice between slavery-based economic systems and free labor-based economic systems, in material agent societies.*

**Keywords:** *material agent societies, slavery, slavery-based economic system, rational choice between slavery and free labor.*

**Resumo.** *Este artigo caracteriza formalmente sistemas de escravatura em sociedades de agentes materiais. Os conceitos de relacionamento social mestre-escravo, relacionamento econômico mestre-escravo, sistema econômico baseado em escravatura, sistema jurídico endossador de escravatura e sociedade de agentes materiais baseada em escravatura são formalmente definidos. Um estudo de caso revisita formalmente o modelo de North & Thomas que estabelece as condições objetivas que justificam a escolha racional entre sistemas econômicos baseados em escravidão e sistemas econômicos baseados em trabalho livre, em sociedades de agentes materiais.*

**Palavras-chave:** *sociedade de agentes materiais, escravidão, sistema econômico baseado em escravidão, escolha racional entre escravidão e trabalho livre.*

## 1. Introduction

In this paper, we introduce a conceptual framework supporting the formal modeling of *slavery* in material agent societies<sup>1</sup>.

The *social* and *economic master-slave relationships* that characterize *slavery systems*, the *economic systems* that arise from them, and the *legal systems* that support them, are informally explained and formally defined. A formal general definition is given of *slavery-based material agent societies*.

So called *chattel slavery* is assumed as the basic form of slavery, and is formally characterized. A case study formally revisits North & Thomas' model stating the objective conditions for the rational choice between *slavery-based economic systems* and *free labor-based economic systems*, in material agent societies.

The aim of the present paper, in line with the work that we have been doing concerning *agent societies* in general [Costa 2017c, Costa 2019], and *material agent societies*

---

<sup>1</sup>See, e.g., [Wikipedia 2018] for a general account of the history and variety of slavery.

in particular [Costa 2017b, Costa 2018], is to provide an *agent-based semantical model* for formal social and political theories of *slavery-based societies*. However, we investigate no single theory of slavery-based societies, regarding such applicability. In particular, we do not go beyond the general level of detail of the concepts mentioned above.

## 2. Material Agent Societies and Elementary Economic Exchanges

We take a general *agent society* to be any multiagent systems with a full-fledged social organization [Costa 2017c].

As in [Costa 2017b] and [Costa 2018], we say that an agent is a *material agent* whenever that agent has a *material body*, that is, a body that requires *energy* for its operation. And we call *material agent society* any agent society whose agents are all material agents.

We call *energy producer* any material agent that is capable of producing *energy objects*, with which energy is distributed for consumption in the society. All the other material agents of that society are said to be *energy consumers*.

Formally, we denote:

- *EnergProd*: the set of material agents that are *energy producers*;
- *EnergCons*: the set of material agents that are *energy consumers*.

Two types of *objects* are assumed to be exchangeable in an *elementary economic exchange* of two material agents:

- *EnergObj*, the type of the so-called *energy objects*, that is, objects that are carriers of the energy needed by the material agents for their functioning;
- *Chip*, the type of the so-called *chips*, that is, valuable objects that the material agents may be interested to acquire, possibly by exchanging some of the *energy objects* they have in their possession;

so that, in general, an *elementary economic exchange* is constituted by an exchange, between two material agents, of one or more *energy objects* for one or more *chips*.

## 3. Masters and Slaves in Chattel Slavery

*Chattel slavery* is slavery where *slaves* are considered personal belongings of their *masters*, which can dispose of them as they wish. Other forms of slavery also exist (see, e.g., [Wikipedia 2018]).

In this paper, we take into account only *chattel slavery*. However, we make an informal use of the term *property*, that is, we use property to mean both *informal*, non-legally supported ownership of objects, as well as *formal*, legally supported ownership of objects, *slaves* being a particular type of ownable objects.

Given a material agent society *MatAgSoc*, we formally define<sup>2</sup>

- *MatObj*: the set of *material objects* of *MatAgSoc*;
- $MatAg \in \wp(MatObj)$ : the set of *material agents* of *MatAgSoc*;
- $Master \in \wp(MatAg)$ : the set of *masters* of *MatAgSoc*;

---

<sup>2</sup> $\wp(X)$  is the powerset of set *X*.

- $Slave \in \wp(MatAg)$ : the set of *slaves* of  $MatAgSoc$ .

For simplicity, we take that:

- $Master \cap Slave = \emptyset$ : no master is a slave, and vice-versa;
- $Master \in \wp(EnergCons)$ : all masters are *energy consumers*;
- $Slave \in \wp(EnergProd)$ : all slaves are *energy producers*.

In the following sections, we introduce formal accounts of diverse aspects of the *property relationship* between masters and slaves.

#### 4. Master-Slave Property Relationship

We call *system of master-slave property relationship* is the system of actions, norms and commands that empower the set of masters of a *slave-based material agent society*, maintaining slaves in their slavery condition.

Formally, we characterize the *property relationship* between masters and slaves in the following way:

- $owns \subseteq Master \times Slave$ , the *property relation* between masters and slaves, so that  $owns(master_i, slave_j)$  means that master  $master_i$  owns  $slave_j$ .

The property relationship is supposed to allow that  $master_i$  do any of the following actions on  $slave_j$ , whenever it happens that  $owns(master_i, slave_j)$ :

- $sell(slave_j, master_k)$
- $lend(slave_j, master_k)$
- $borrow(slave_j, master_k)$
- $lend(slave_j, master_k)$
- $rent(slave_j, master_k)$
- $kill(slave_j)$
- $free(slave_j)$
- $command(slave_j, cmd)$
- $punish(slave_j, cmd)$

meaning that:

- $master_i$  can *sell*, *lend*, *borrow*, *lend* and *rent*  $slave_j$  to any other master  $master_k$ , besides *killing* or *freeing* it, and *commanding* it to perform any command  $cmd$  and *punishing* it for the way he performed such command.

In accordance with the possibility of  $master_i$  performing  $sell(slave_j, master_k)$ , we take that master  $master_k$  can *buy* slave  $slave_j$  from master  $master_i$ . That is, we also have, as possible action:

- $buy(slave_j, master_i)$

In addition, the following (formal or informal) obligation is taken to be valid for slaves:

- $mustexec(slave_j, cmd, master_i)$

meaning that:

- $slave_j$  is supposed to peremptorily execute any command  $cmd$  issued by  $master_i$ .

For simplicity, we omit here any reference to the *conditions* under which those *actions* and *commands* may be effective, such as the explicit *connection* between *commands* and possible *punishments*. But, see Section 8 for some of the *legal* types of such conditions.

Other legal forms of *acquisition* of slaves (such as by having them *born* from parents that are already slaves, or by *capturing* them in certain specified conditions) are also considered in Section 8.

## 5. Master-Slave Economic Exchanges

Let time be given by the linearly ordered set  $T = \{0, 1, 2, \dots\}$ , ranged over by variable  $t$ . For the purpose of the present paper, we call *elementary economic exchange* any exchange of the form (cf. [Costa 2018]):

$$e2exch^t = \langle mag/obj \rangle^t \xrightarrow{c} \langle mag'/obj' \rangle^t$$

meaning that material agents  $mag$  and  $mag'$  exchange objects  $obj$  and  $obj'$ , a pair of such objects at each time  $t$ , under the assumption that  $mag$  provides operational condition  $c$  for  $mag'$  to produce and delivery  $obj'$ , and  $mag'$  provides operational condition  $c'$  for  $mag$  to produce and delivery  $obj$ .

We call *master-slave elementary economic exchange* any elementary economic exchange of the form:

$$mse2exch^t = \langle master/\perp \rangle^t \xrightarrow{c} \langle slave/obj \rangle^t$$

meaning that, at each  $t$ , the slave  $slave$  sends an object  $obj$  to his master  $master$ , without receiving no object in exchange ( $\perp$  is the *null* object), while the master  $master$  is required to provide condition  $c$  for  $slave$  to produce and delivery  $obj$ , and the slave  $slave$  is not required to provide any condition ( $\perp$ ) for the master to produce and delivery nothing ( $\perp$ ).

Notice that in any *master-slave elementary economic exchange* like  $mse2exch^t$ , masters accumulate a set of received objects, up to time  $t$ , in the form:

$$accumobj[master/mse2proc^t] = \{obj^0, obj^1, \dots, obj^t\}$$

while slaves accumulate nothing, because we take that a set of *null* objects is an empty set. That is:

$$accumobj[slave/mse2proc^t] = \{\perp^0, \perp^1, \dots, \perp^t\} = \emptyset$$

In general, *masters* are allowed to have a *group of slaves* with more than one slave in it. In such a case, the *master-slave group elementary economic exchange* that the *master* and the *group of slaves* perform has the form:

$$msge2exch^t = \langle master/\perp \rangle^t \xrightarrow{c} \langle Slave/Obj \rangle^t$$

where:

- *Slave* is the *group of slaves* that belong to master  $master$ ;
- *Obj* is the *set of objects* that the set of slaves  $Slave$  produce and deliver, at each time, to their master  $master$ .

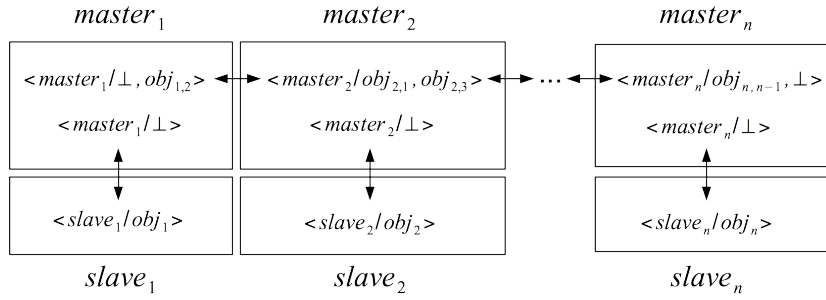


Figure 1. Sketch of a slavery-based elementary economic process.

## 6. Slavery-Based Elementary Economic Processes

In general, *individual elementary economic processes* have the form (see [Costa 2018]):

$$ie2proc^t = \langle mag_1/\perp, obj_{1,2} \rangle^t \xrightarrow{c_{2,1}} \langle mag_2/obj_{2,1}, obj_{2,3} \rangle^t \xrightarrow{c_{3,2}} \dots \xrightarrow{c_{n,n-1}} \langle mag_n/obj_{n,n-1}, \perp \rangle^t$$

where:

- $mag_i$  is the  $i$ -th material agent participating in  $ie2proc^t$ ;
- each  $\langle mag_i/obj_{i,i-1}, obj_{i,i+1} \rangle^t \xrightarrow{c_{i+1,i}} \langle mag_{i+1}/obj_{i+1,i}, obj_{i+1,i+2} \rangle^t$  is an *elementary economic exchange*;
- $obj_{i,k}$  is the object that  $mag_i$  produces and deliveries to its  $k$ -th partner, for  $k \in \{i-1, i+1\}$ ;
- $mag_1$  has no left partner, so  $obj_{1,0} = \perp$ ;
- $mag_n$  has no right partner, so  $obj_{n,n+1} = \perp$ .

In the general case of *slavery-based elementary economic processes*, we have the form:

$$mse2proc^t = \langle master_1/\perp, obj_{1,2} \rangle^t \xrightarrow{c_{2,1}} \langle master_2/obj_{2,1}, obj_{2,3} \rangle^t \xrightarrow{c_{3,2}} \dots \xrightarrow{c_{n,n-1}} \langle master_n/obj_{n,n-1}, \perp \rangle^t$$

where one can notice that only *masters* participate in the society's economic processes, *slaves* being restricted to private economic exchanges with their *masters*, as sketched in Figure 1. Notice that a *master* participates in *two* elementary economic exchanges (one local, with his *slave*, the other global, with other *masters*), while a *slave* participates only in *one* elementary economic exchange, the local exchange with his *master*.

Notice also that this model of slavery-based elementary economic processes naturally extends to the cases where the masters may have *groups of slaves*, instead of just individual slaves.

## 7. Slavery-Based Elementary Economic Systems

Let the term *elementary economic material agent* denote material agent of the population of a material agent society that can participate in an elementary economic exchange. We define the general notion of *elementary economic system* of a material agent society as follows (cf. [Costa 2018]):

**Definition 7.1** The elementary economic system  $E2Sys$  of a material agent society  $MAgSoc$  is a time-indexed structure:

$$E2Sys_{MAgSoc}^t = (E2Ag^t, Obj_s^t, E2Beh^t, E2Exch^t, E2Proc^t)$$

where, for each time  $t$ :

- $E2Ag^t$  is the set of elementary economic material agents, which participate in the elementary economic processes of  $MAgSoc$ , at the time  $t$ ;
- $Obj^t$  is the set of objects that the elementary economic agents can exchange during the performance of their elementary economic exchanges, at that time;
- $E2Beh^t$  is the set of elementary economic behaviors that the elementary economic agents can perform during the performance of the elementary economic exchanges, at that time;
- $E2Exch^t$  is the set of elementary economic exchanges that the elementary economic agents can perform during the performance of the elementary economic processes, at that time;
- $E2Proc^t$  is the set of elementary economic processes that the elementary economic agents can perform in  $MAgSoc$ , at that time.

In the case of a slavery-based elementary economic system, we have that, for any time  $t$ :

- $E2Ag^t = Master^t \cup Slave^t$ , meaning that the set of elementary economic material agents of  $E2Sys^t_{MAgSoc}$  is partitioned into *masters* and *slaves*;
- $Obj^t = MasterSlaveObj^t \cup MasterMasterObj^t$ , meaning that the objects exchanged between elementary economic agents are either of the *master-slave* type or of the *master-master* type, with (see [Costa 2018]):
  - $MasterSlaveObj^t \in \wp(EnergObj)$ , that is, objects exchanged between masters and slaves (in fact, just from slaves to masters) are *energy objects*, resulting from the labor of the slaves;
  - $MasterMasterObj^t \in \wp(EnergObj) \cup \wp(Chip)$ , that is, objects exchanged between masters are either *energy objects* or *chips*, with chips exchanged in return for energy objects;
- $E2Beh = MasterBeh^t \cup SlaveBeh^t$ , that is, both masters' and slaves' elementary economic behaviors may participate in the elementary economic exchanges of  $E2Sys^t_{MAgSoc}$ ;
- $E2Exch = MasterSlaveExch^t \cup MasterMasterExch^t$ , that is, the elementary economic exchanges are either of the *master-slave* type or of the *master-master* type;
- $E2Proc = MasterSlaveProc^t \cup MasterMasterProc^t$ , that is, the elementary economic processes are either of the *master-slave* type or of the *master-master* type.

Notice that no particular requirement is established concerning the types of *conditions* that may be imposed on the elementary economic exchanges.

## 8. Slavery-Supporting Legal Systems

### 8.1. Legal Systems of Agent Societies

We define the general notion of *legal system* situated in an agent society  $AgSoc$  as follows (cf. [Costa 2015]):

**Definition 8.1** A legal system is a time-indexed structure:

$$LegalSys^t_{AgSoc} = (LOrd^t, LOrg^t, RLFact^t, LegalOps)$$

where:

- $LOrd^t$  is the legal order at time  $t$ ;
- $LOrg^t$  is the system of legal organs at time  $t$ ;
- $RLFact^t$  is the record of legal facts at time  $t$ ;
- $LegalOps$  is the set of legal operations, like:
  - $createlnorm$ , the creation of legal norms;
  - $deroglnrm$ , the derogation of legal norms;
  - $createlauth$ , the creation of authorizations to perform legal operations;
  - $cancellauth$ , the cancellation of such authorizations;
  - $recordlfct$ , the recording of a legal fact in the record of legal facts;
  - $deletelfct$ , the deletion of one such record.

with:

- $LegalSys_{AgSoc}^t$  supposed to contain a public record of legal facts, to be freely accessed by the agents of the society;
- $LOrd^t$  and  $LOrg^t$  such that  $LOrd^0 \neq \emptyset \neq LOrg^0$ .

## 8.2. Legal Systems of Slave-Based Societies

In the context of slavery-based material agent societies endowed with legal systems, we are particularly interested in the types of legal norms that support the masters in their maintenance of the slave-based social and economic relationships.

Clearly, the most fundamental of such legal norms are:

- $owns(master_i, slave_k) \Rightarrow Auth(master_i, command(master_i, slave_j, anyact))$ , which legally authorizes master  $master_i$  to *command* that slave  $slave_j$  does any act the master wishes;
- $owns(master_i, slave_k) \Rightarrow Auth(Master, punish(Master, Slave, AnyAct))$ , which legally authorizes master  $master_i$  to *punish* slave  $slave_j$  for not doing properly any act the master has commanded it to do.

Typically, the following conditional legal authorization is also formally adopted by the legal systems of slave-based material agent societies, so that the initial condition of being a slave, and which is its initial master, is established:

- $owns(master_i, slave_k) \wedge mother(slave_k, slave_j) \Rightarrow Auth(master_i, owns(master_i, slave_j))$ , which legally authorizes master  $master_i$  to *own* slave  $slave_j$  if the mother of slave  $slave_j$  is itself owned by master  $master_i$ .

Legal norms as the above ones, are sufficient for material agent societies whose only means to produce slaves is through their parental reproduction. Some material agent societies, however, adopt the legal procedure of allowing slaves to be produced by their capture from other societies (either in the context of war between the two societies, or in the context of commercial exploitation of the second society by the first one).

That type of legal norm may be formally sketched as follows:

- $authorized(mag_i, slavecapture) \wedge captured(mag_j, mag_i) \Rightarrow Auth(mag_i, owns(mag_i, mat_j))$ , which states that if material agent  $mag_i$  is legally authorized to capture slaves, and it happened that  $mag_i$  capture  $mag_j$  then  $mag_i$  is legally authorized to own  $mag_j$  as a slave, effectively making of  $mag_i$  a *master* and of  $mag_j$  a *slave*.

## 9. Slavery-Based Material Agent Societies

A detailed formal account of the architecture of material agent societies that are organized on the basis of slavery is out of the scope of the present paper. Here, we can only leave implicit the amount of details that would have to be provided in order to properly instantiate, as a *slavery-based material agent society*, the following general structure of an *agent society* (see [Costa 2017c]):

$$\bullet \text{ AgSoc} = (\text{Pop}, \text{Org}, \text{MEnv}, \text{SEnv}, \text{IMP}, \text{ACC})$$

where:

- *Pop* is the *population* of *AgSoc*;
- *Org* is the *organizational structure* of *AgSoc*;
- *MEnv* is the *material environment* of *AgSoc*;
- *SEnv* is the *symbolic environment* of *AgSoc*, where the society's system of *legal norms* is embedded;
- *IMP* is the collection of *implementation relations* between *Pop* and *Org*;
- *ACC* is the collection of *access relations* between *Org* and the environments *MEnv* and *SEnv*.

Moreover, considering the case of slavery-based material agent societies that *do not* produce slaves by capturing them in another society, but that, besides producing them by parental reproduction, also *import* them from *slave capturing societies*, the full account of the details of that slavery-based society would require placing it in an *inter-societal context* (see [Costa 2017a]), which is also out of the scope of the present paper.

## 10. A Case Study: North & Thomas' Model of Rational Choice between Slavery and Free Work in Material Agent Societies

Douglass C. North and Robert Paul Thomas develop in [North and Thomas 1971] an institutional dynamical model for the rise and fall of manorial systems, that encompasses both *serfdom* and *slavery*. They contrast *serfdom* and *slavery* by telling the first to be “*essentially a contractual arrangement where labor services were exchanged for the public good of protection and justice*” (p.778), with a “*contractual relationship which could be changed only by both parties*” (p.779), while in the second a slave “*has no legal control over decision-making with respect either to his labor or to his income stream*” (p.779).

Clearly, by basing the distinction on the notion of *contract*, North & Thomas' model presupposes the existence of some sort of (formal or informal) *legal system* that is effective in the society and capable of enforcing the compliance with valid contracts.

The core of their rational model for the choice between *slavery-based* and *free labor-based* economic system is the following:

“*Slavery was always more profitable than free labor <..> when the following conditions existed: (1) a market economy, (2) profitable opportunities to produce those types of economic activities where the costs of supervision to reduce shirking were low, and (3) where the costs of enforcing property rights in human beings were low.*” (p.779)

Notice that:



1. Condition (1) means the possibility of freely *selling* and *buying* slaves.
2. Let  $costsuperv(slave_j, master_i)$  denote the *cost of the supervision*, for master  $master_i$ , of the operation of the slave  $slave_j$ . Analogously, denote the cost of the corresponding supervision, concerning the free laborer  $freelaborer_k$ , by  $costsuperv(freelaborer_k, master_i)$ . Then, condition (2) means that it is rational to adopt a *slavery-based economic system*, instead of an *free labor-based economic system*, in a given society, if and only if, for most of the masters ( $mstr$ ) it holds that:

$$\sum_{j=1}^{j=n} costsuperv(slave_j, mstr) \leq \sum_{k=1}^{k=m} costsuperv(freelaborer_k, mstr)$$

where:

- $n$  is the *average number of slaves* owned by most of the masters, in the alternative of the *slavery-based economic system*;
  - $m$  is the *average number of free laborers* hired by most of the masters, in the alternative of the *free labor-based economic system*.
3. Let  $costenforc(slave_j, master_i)$  denote the *cost*, for master  $master_i$ , of *enforcing property rights* in the slave  $slave_j$  (both regarding the slave itself and the other competing masters). Analogously, let  $costenforc(freelaborer_k, master_i)$  denote the cost, for master  $master_i$ , of the corresponding enforcement, concerning the *product* of the labor of the free laborer  $freelaborer_k$ . Then, condition (3) means that it is rational to adopt a *slavery-based economic system*, instead of an *free labor-based economic system*, in a given society, if and only if, for most of the masters ( $mstr$ ) it holds that:

$$\sum_{j=1}^{j=n} costenforc(slave_j, mstr) \leq \sum_{k=1}^{k=m} costenforc(freelaborer_k, mstr)$$

where:

- $n$  is the *average number of slaves* owned by most of the masters, in the alternative of the *slavery-based economic system*;
- $m$  is the *average number of free laborers* hired by most of the masters, in the alternative of the *free labor-based economic system*.

In summary, one can see, by this brief account of the elements of North & Thomas' model, that a rational choice is possible between a *slavery-based economic system* and a *free labor-based economic system*, in any given *material agent society*, at any time of its history.

## 11. Discussion

This paper introduced a formal model for *slavery* in material agent societies. Clearly, we adopted the term *slave* to refer to material agents that can be bought and sold in a material agent society, while we adopted the term *free laborer* to material agents that can be *hired* for the realization of specified jobs.

The basic elements of North and Thomas' [North and Thomas 1971] model for the rational choice between *slavery-based* and *free labor-based* economic systems was

briefly reviewed. One sees that the analysis of their model opens the possibility for a *mixed* type of economic systems of material agent societies, namely, that which combines *slave material agents* and *free laborer material agents*.

Two criteria arise for a rational choice between *slavery* and *free laboring*, in such mixed situations:

- first, a choice at the level of the *type of economic activity*: choose between *slavery* and *free laboring* according to the costs of *work supervision* and *property rights enforcement* peculiar to each activity;
- second, a choice at the level of the *particular situation of the master*: choose between *slavery* and *free laboring* according to the costs of *work supervision* and *property rights enforcement* for each particular master.

A situation where the full combination of all such possibilities are adopted would certainly introduce extra complexity in the legal system of the society, because legal provisions would have to be established for each such possibility, including different legal norms applying to the same master, in accordance with the particular type of economic relation he has with each of his workers.

A more sensible choice would be that the material agent society chooses, for each *type of economic activity*, either *slavery* or *free labor*. In such case, the legal norms of the legal system, concerning the way work is performed in the material agent society, could be specialized for the different types of economic activities.

Finally, notice that even if the legal system of the society adopts only free labor based economic activities, it may be rational for some particular economic activity, or for some particular master, to establish a slavery-based form of economic exchange with its workers, giving rise to *illegal* slavery forms of economic activities in the society.

## 12. Conclusion

This paper introduced the notion of *slavery* in material agent societies. Formal account was given of its basic aspects. North & Thomas' model for the rational choice between *slavery-based* and *free labor-based* elementary economic systems was shown to be applicable to material agent societies.

The particular possibility of material agent societies with *mixed modes* of economic system (slavery + free labor) was discussed.

Future work, in this line of investigation, could explore the *details of the legal systems* regulating slavery and free labor, characterizing the *basic differences* between the legal norms that apply to each of them.

## References

- Costa, A. C. R. (2015). Situated legal systems and their operational semantics. *Artificial Intelligence & Law*, 43(1):43–102.
- Costa, A. C. R. (2017a). Ecosystems as agent societies, landscapes as multi-societal agent systems. In Adamatti, D. F., editor, *Multiagent Based Simulations Applied to Biological and Environmental Systems*, pages 25–43. IGI Global, Hershey.

- Costa, A. C. R. (2017b). Energy systems in material agent societies. *RITA - Revista de Informática Teórica e Aplicada*, 24(2):130–144.
- Costa, A. C. R. (2017c). SML - a society modeling language. Technical report, Tutorial presented at WESAAC 2017, São Paulo. Available on <http://wesaac.c3.furg.br>.
- Costa, A. C. R. (2018). Elementary economic systems in material agent societies. In *Anais do WESAAC 2018*, pages 12–24, Fortaleza. UECE.
- Costa, A. C. R. (2019). *A Variational Basis for the Regulation and Structuration Mechanisms of Agent Societies*. Springer, Cham.
- North, D. C. and Thomas, R. P. (1971). The rise and fall of manorial systems: A theoretical model. *The Journal of Economic History*, 31(4):777–803.
- Wikipedia (2018). *Slavery*. Available at <https://en.wikipedia.org/wiki/Slavery>.