CBIE 2023
CONGRESSO BRASILEIRO DE INFORMÁTICA NA EDUCAÇÃO
Uma escola para o futuro: Tecnologia e conectividade a serviço da educação

Exploring the Relationship of Mathematical Reasoning and Computational Thinking

Thiago Diniz Ferreira[1], Jucelio Soares dos Santos[1], Rosângela de Araújo Medeiros[1]
Wilkerson L. Andrade[2], João Brunet[2], Monilly Ramos Araujo Melo[2]

[1]State University of Paraíba (UEPB), Patos, Brazil

[2]Federal University of Campina Grande (UFCG), Campina Grande, Brazil

thiago.diniz@aluno.uepb.edu.br, {jucelio,rosangelamedeiros}@servidor.uepb.edu.br

{wilkerson, joao.arthur}@computacao.ufcg.edu.br, monillyramos@gmail.com

Abstract. *We investigated the relationship between Mathematical Reasoning and Computational Thinking in high school students. We used Screening Programming and Bebras as assessment tools to estimate the participants' Mathematical Reasoning and Computational Thinking proficiency. Through the statistical analysis, we observed a robust and moderate correlation between these skills in students. Our findings suggest that the development of one skill can contribute to the improvement of the other and underscores the importance of considering these skills in an integrated manner during students' education. This study's findings provide relevant insights for planning instruction and learning strategies in the field of Computer Science while also fostering future research in diverse educational contexts.*

1. Introduction

Mathematical Reasoning and Computational Thinking are two fundamental skills in the contemporary world, with applications in various fields of knowledge. Mathematical Reasoning involves the capacity to deal with numbers, patterns, and abstract relationships [Morais et al. 2018], while Computational Thinking utilizes concepts and techniques from Computer Science to address problems and challenges [Santana et al. 2021].

Although research often addresses these two concepts separately, there is evidence that they are interconnected and can be mutually beneficial. Mathematical Reasoning can provide a solid foundation for Computational Thinking, developing analysis, logic, and abstraction skills. Computational Thinking can offer tools and strategies that enhance mathematical problem-solving, making it more efficient and systematic [Santana et al. 2021].

However, these skills are often isolated in school curricula and educational practices, leading to gaps in students' education [Resnick and Rosenbaum 2013]. This artificial separation hinders the transfer of knowledge and the practical application of Mathematical and Computational concepts and limits the development of broader cognitive skills [Wing 2021].

Considering this context, this research investigates the relationship between Mathematical Reasoning and Computational Thinking in high school students. Therefore, we utilized the Screening Programming [Andrade 2022, Dantas 2022], an instrument based on logic and abstract reasoning questions, to estimate the participants' Mathematical Reasoning ability. Additionally, we administered the Bebras [Dagienė and Sentance 2016], a recognized instrument for assessing skills in algorithms, abstraction, and logical Thinking, to estimate the participants' Computational Thinking.

This research provides theoretical and practical insights for developing pedagogical strategies integrating Mathematical Reasoning and Computational Thinking. Understanding the relationship between these skills is essential for comprehensive education and preparing individuals for the contemporary world's demands. Moreover, this investigation can contribute to shaping more prepared and adaptable professionals capable of applying logical-mathematical Thinking alongside the computational skills required to tackle the challenges of the 21st century [Grover and Pea 2018].

The paper investigates the relationship between mathematical reasoning and computational thinking. Section 2 presents previous studies highlighting the importance of mathematical foundations and shared skills, such as logic, abstraction, and problem-solving, in developing computational thinking. In section 3, we detail the adopted methodology, including the rigorous selection of participants and using correlation as a metric. Section 4 presents the results, revealing a moderately strong correlation between mathematical reasoning and computational thinking, highlighting the need for integrated educational approaches. In section 5, we report the conclusions and suggestions for future research, how to expand the sample, and investigate other variables that may influence this relationship.

2. Related Work

The relationship between Mathematical Reasoning and Computational Thinking has been widely explored in the literature, with several studies highlighting the intrinsic connection between these two skills. This section discusses the key findings from previous research that support the correlation between Mathematical Reasoning and Computational Thinking.

Mathematical foundations serve as a fundamental basis for the development of Computational Thinking skills [Silva 2018]. Computational Thinking involves the ability to solve problems logically and algorithmic, which aligns closely with the core principles of mathematics. Understanding patterns, performing calculations, and thinking abstractly are essential mathematical fundamentals that contribute to developing Computational Thinking abilities. Individuals with strong Mathematical Reasoning skills tend to possess a solid foundation that aids in acquiring and enhancing Computational Thinking skills.

Logic and abstraction play crucial roles in Mathematical Reasoning, and Computational Thinking [Wing 2006, Brennan and Resnick 2012, Grover and Pea 2013]. Mathematical Reasoning requires the identification of relationships and patterns, hypothesis formulation, and making inferences. Similarly, Computational Thinking involves decomposing complex problems into smaller steps, identifying suitable algorithms and problem-solving strategies, and abstracting concepts to generalize solutions. The commonalities in logic and abstraction between Mathematical Reasoning and Computational Thinking contribute to the observed correlation between the two skills. Problem-solving is a shared characteristic of Mathematical Reasoning, and Computational Thinking [Guzdial 2015].

Mathematical problems often require the application of mathematical concepts and methods to find solutions. Similarly, Computational Thinking involves formulating and properly defining problems, identifying efficient approaches, and implementing solutions using algorithms and data structures. Mathematical Reasoning and Computational Thinking necessitate the ability to analyze, plan, and execute strategies to solve complex problems effectively.

Applying modeling and simulation further strengthens the relationship between Mathematical Reasoning and Computational Thinking [Sturgill 2019, Lesh and Doerr 2003]. Mathematical Reasoning involves formulating mathematical models to represent real-world phenomena and performing calculations to obtain results and interpretations. Similarly, Computational Thinking can be applied in creating computational models, implementing simulations, and analyzing the obtained results. The ability to apply mathematical concepts practically and utilize computational tools reinforces the correlation between Mathematical Reasoning and Computational Thinking.

The existing body of work emphasizes the strong connection between Mathematical Reasoning and Computational Thinking. The overlap in mathematical foundations, logic and abstraction skills, problem-solving approaches, and modeling and simulation applications underscores these two cognitive abilities' inter-dependency and mutual reinforcement. Understanding and further exploring this relationship can contribute to developing educational strategies and interventions to foster Mathematical Reasoning and Computational Thinking skills in individuals.

3. Methodology

In this section, we present the study planning to verify the relationship between Mathematical Reasoning and Computational Thinking of high school students.

3.1. Metric

In this study, we used correlation as a metric to assess the relationship between Mathematical Reasoning and Computational Thinking in the participants. Correlation is a widely used statistical measure to quantify the relationship between two variables. We determined this relationship's existence, direction, and strength by analyzing this correlation. This approach allowed us to examine if performance in one area is related to performance in the other and if improvement in one skill can positively influence the development of the other. Using correlation as a metric in this study was essential to explore the relationship between Mathematical Reasoning and Computational Thinking, providing valuable insights for understanding and developing these skills in the participants.

3.2. Participant Selection

The selection process of participants was rigorous, aiming to guarantee the representativeness and validity of the results obtained in this study. We established inclusion criteria and followed them rigorously during the selection of participants. In total, we included 50 participants who met the following inclusion criteria:

- **Signing of Informed Assent or Informed Consent Form:** All selected participants were adequately informed about the study's objectives and procedures.

Those of legal age signed the Informed Consent Form, while participants under the age of majority signed the Informed Assent Form. These forms ensured that participants were aware of their involvement in the study and consented to participate voluntarily;
- **Enrollment in the final year of High School:** The selected participants were duly enrolled in the final year of High School in a public school in Patos, Paraíba, Brazil;
- **Absence of sensory, cognitive, auditory, or visual limitations:** It was necessary to ensure that participants did not have sensory, cognitive, auditory, or visual limitations that could interfere with their ability to respond to the assessment instruments and fully participate in the study. This restriction guarantees that the obtained results are representative of participants without limitations.

3.3. Threat Analysis

We considered several factors that could pose threats and directly influence this study's conclusions, including:

- Issues related to misinterpretation of the questions;
- Research participants may feel intimidated or uncomfortable while taking the tests. We implemented guidelines from the research ethics committee to minimize this potential constraint. The Research Ethics Committee for Human Subjects at the Federal University of Campina Grande and the State University of Paraíba approved this research (Protocols: 23933919.4.0000.5182 — 23933919.4.3001.5187). We included in this study only the participants who signed the Free and Informed Consent Term or the Free and Informed Consent Term;
- The Screening Programming and Bebras instruments were virtually corrected to mitigate potential human errors;
- Like any empirical research, this work presents threats to validity. The number of study participants does not allow for generalization of the results;
- We considered the sample because it allows for forming a probabilistic database based on statistical probability, measurement axioms, and the instrument's purpose. This database should have centralized control of the application.

3.4. Research Execution

To verify the correlation between the variables, we adopted the following steps:

- We administered the Screening Programming to measure the Mathematical Reasoning ability. At a separate time, we administered the Bebras to measure Computational Thinking ability. Participants were given up to 2 hours on each occasion to respond to the questions present in the instrument;
- We estimated the participants' abilities in the instruments using the eirt macro, adopting the 3ML through the maximum likelihood marginal estimation;
- Finally, we correlated the estimated Computational Thinking ability of the participants from the Bebras instrument with the Mathematical Reasoning ability in the Screening Programming instrument.

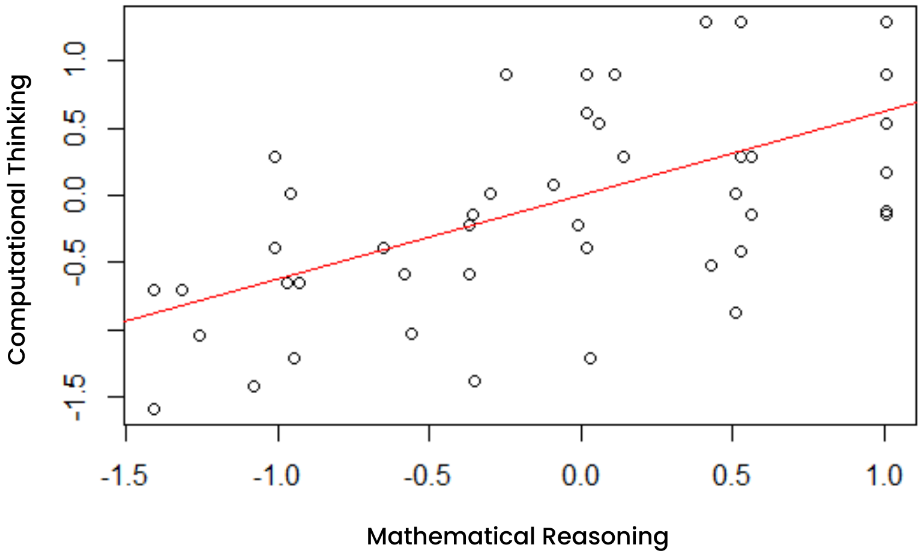
4. Analysis and Results

In this section, we present and discuss the results of this research, which aims to investigate the relationship between Computational Thinking and Mathematical Reasoning. Thus, we aim to answer the following research question:

- **RQ.** Is there a significant correlation between Mathematical Reasoning skills and Computational Thinking in high school students?

In this analysis, we investigated the relationship between Mathematical Reasoning and Computational Thinking skills in a group of high school students. To visualize this relationship, we generated a scatter plot that allows us to observe the behavior of these skills and identify possible correlations. We used the R software to perform this analysis and create the plot using the data collected from a representative sample of students. Figure 1 illustrates the scatter plot to identify the relationship between the students' Computational Thinking skills and Mathematical Reasoning.

Figure 1. Scatter Plot between the variables Computational Thinking and Mathematical Reasoning.



On the plot's horizontal axis, we have each student's Mathematical Reasoning skill scores; on the vertical axis, we have the Computational Thinking scores. Each point on the plot represents an individual student, and their position on the plot is determined by their scores in these skills. By observing the pattern of points on the scatter plot, we can notice a general upward trend. As the Mathematical Reasoning skill score increases, the Computational Thinking score also tends to increase. This positive relationship suggests a correlation between the two skills.

Furthermore, it is possible to observe that the dispersion of points around the upward trend is relatively small. This analysis indicates consistency in the relationship between Mathematical Reasoning and Computational Thinking skills in the students. Most points cluster near the trend line, reinforcing the presence of a moderately strong correlation between the skills.

Based on the scatter plot, we can conclude that Mathematical Reasoning and Computational Thinking skills exhibit a moderately strong correlation among participants. The upward pattern of the points indicates that the development of one skill is related to the improvement of the other. This analysis reinforces the importance of considering these skills in an integrated manner in the instruction and learning process, aiming for a more comprehensive and well-rounded education for the students.

Additionally, we performed a normality test on the data distribution using the *shapiro.test()* function in the R language. We present the normality test results for the ability estimates between the observed scenarios in Table 1.

Table 1. Test of Normality of Estimates of Abilities Between Computational Thinking and Mathematical Reasoning.

Null Hypothesis	p-value
The estimate of Computational Thinking skill by Bebras does not follow a normal distribution.	0.09013
The estimate of Mathematical Reasoning skill by Screening Programming does not follow a normal distribution.	0.003244

As the data for the estimation of Mathematical Reasoning skills does not follow a normal distribution, we used the Spearman correlation test. The Spearman correlation test measures the relationship between two ordinal or non-parametric variables. This test is an alternative to the Pearson correlation test, which assumes that the variables are normally distributed. We performed the Spearman correlation test using the cor.test() function in R. The basic syntax is cor.test(x, y, method = “spearman”). Where x and y are the data vectors corresponding to the variables we want to test, the method = “spearman” specifies that we want to calculate the Spearman correlation.

After running Spearman’s correlation test in R, we get output that includes much information. In our specific case, the test result was 0.6635959. This value is known as the Spearman correlation coefficient (or Spearman score) and ranges from -1 to 1. The interpretation of the Spearman correlation coefficient is similar to that of the Pearson correlation coefficient, but it considers the monotonic relationships between variables rather than linear ones.

If Spearman’s correlation coefficient is close to 1, this indicates a strong positive correlation between the two variables, meaning that as one variable increases, the other also tends to increase. If the coefficient is close to -1, this indicates a strong negative correlation between the two variables, meaning that the other tends to decrease as one variable increases. If the coefficient is close to 0, this indicates a weak or zero correlation between the two variables, which means there is no apparent systematic relationship between them.

In our case, the Spearman correlation coefficient obtained was 0.6635959. This positive value indicates a moderately strong correlation between Mathematical Reasoning and Computational Thinking skills. As the Mathematical Reasoning skill increases, the Computational Thinking skill also tends to increase, and vice versa. However, it is essential to remember that Spearman’s correlation does not imply causality. It is impossible to state that one skill causes the other based only on the observed correlation.

We can explain the moderately strong correlation between Mathematical Reasoning skills and Computational Thinking for a few reasons, including:

- **Mathematical foundations:** Computational Thinking involves the ability to solve problems logically and algorithmically. These skills are intrinsically linked to mathematical fundamentals, such as understanding patterns, the ability to perform calculations, and the ability to think abstractly. Therefore, individuals with good Mathematical Reasoning generally have a solid foundation to develop Computational Thinking skills [Silva 2018];
- **Logic and abstraction:** Mathematical Reasoning and Computational Thinking require logic and abstraction skills. Mathematical Reasoning requires identifying relationships and patterns, formulating hypotheses, and making inferences. Likewise, Computational Thinking involves decomposing complex problems into smaller steps, identifying algorithms and resolution strategies, and the ability to abstract concepts and generalize solutions. These similarities in terms of logic and abstraction contribute to the correlation between the two skills [Wing 2006, Brennan and Resnick 2012, Grover and Pea 2013];
- **Problem-Solving :** Mathematical Reasoning and Computational Thinking are essential problem-solving skills. Mathematical problems often involve applying concepts and methods to find solutions. At the same time, Computational Thinking requires the ability to properly formulate problems, identify efficient approaches, and implement solutions using algorithms and data structures. Both skills share the ability to analyze, plan and execute strategies to solve complex problems [Guzdial 2015];
- **Modeling and simulation:** Mathematical Reasoning and Computational Thinking are used in modeling and simulation situations. Mathematical Reasoning involves formulating mathematical models to represent real-world phenomena and performing calculations to obtain results and interpretations. Likewise, we can use Computational Thinking in the creation of computational models, in the implementation of simulations, and the analysis of the obtained results. The ability to apply mathematical concepts practically and use computational tools strengthens the correlation between the two skills [Sturgill 2019, Lesh and Doerr 2003].

In summary, the moderately strong correlation between Mathematical Reasoning and Computational Thinking to conceptual, logical, and problem-solving similarities between these two areas. Both skills are fundamental to developing analytical, abstract, and critical skills needed in Mathematics, Computer Science, and related fields.

5. Final Considerations and Suggestions for Future Work

Based on the operational methodology adopted and the research questions investigated, this study provided valuable insights into the relationship between mathematical Reasoning and computational Thinking. The results contribute to a deeper understanding of these skills and highlight the importance of considering them in an integrated way in students' academic and professional training.

The estimation of the mathematical reasoning ability of the participants through Screening Programming demonstrated the students' proficiency level in this specific ability. At the same time, applying Bebras to estimate computational Thinking allowed identify how students demonstrate skills in algorithms, abstraction, and logical Thinking. The correlation between mathematical Reasoning and computational thinking skills revealed a moderately strong correlation between these two skills in study participants, suggesting that developing skills in one of these areas can improve the other. Therefore, it is essential to consider these skills integrated into the teaching and learning process, aiming at a more complete and comprehensive training of students.

Based on the results and conclusions of this investigation, we present some suggestions for future work:

- Expand the sample of participants and consider different levels of education, such as graduate and basic education, to assess the relationship between mathematical Reasoning and computational Thinking at different stages of training;
- Investigate the influence of additional variables, such as prior programming experience or exposure to advanced mathematical concepts, on the relationship between mathematical Reasoning and computational Thinking;
- Explore teaching and learning strategies that more effectively integrate mathematical Reasoning and computational Thinking, aiming to promote an interdisciplinary approach and the transfer of knowledge between these two areas;
- Investigate the impact of the interconnection between mathematical Reasoning and computational Thinking in developing broader cognitive skills, such as solving complex problems, creativity, and the capacity for continuous learning.

These suggestions can contribute to expanding knowledge about the relationship between mathematical Reasoning and computational Thinking and providing relevant insights for planning and improving teaching and learning strategies in this area. Furthermore, these investigations can benefit students and professionals involved in Computer Science education.

References

[Andrade 2022] Andrade, P. H. (2022). Análise da usabilidade de um sistema web na mensuração de habilidades cognitivas preditoras em novatos em programação. In *Trabalho de Conclusão de Curso, Monografia (Bacharelado em Computação) – Centro de Ciências e Tecnologias, Universidade Estadual da Paraíba.*

[Brennan and Resnick 2012] Brennan, K. and Resnick, M. (2012). New frameworks for studying and assessing the development of computational thinking. In *Proceedings of the 2012 annual meeting of the American educational research association, Vancouver, Canada*, volume 1, page 25.

[Dagienė and Sentance 2016] Dagienė, V. and Sentance, S. (2016). It's computational thinking! bebras tasks in the curriculum. In *Informatics in Schools: Improvement of Informatics Knowledge and Perception: 9th International Conference on Informatics in Schools: Situation, Evolution, and Perspectives, ISSEP 2016, Münster, Germany, October 13-15, 2016, Proceedings 9*, pages 28–39. Springer.

[Dantas 2022] Dantas, D. M. (2022). Análise da confiabilidade de um instrumento na mensuração de habilidades cognitivas preditoras em novatos em programação. In *Trabalho de Conclusão de Curso, Monografia (Bacharelado em Computação) – Centro de Ciências e Tecnologias, Universidade Estadual da Paraíba.*

[Grover and Pea 2013] Grover, S. and Pea, R. (2013). Computational thinking in k–12: A review of the state of the field. *Educational researcher*, 42(1):38–43.

[Grover and Pea 2018] Grover, S. and Pea, R. (2018). Computational thinking: A competency whose time has come. *Computer science education: Perspectives on teaching and learning in school*, 19(1):19–38.

[Guzdial 2015] Guzdial, M. (2015). Learner-centered design of computing education: Research on computing for everyone. *Synthesis Lectures on Human-Centered Informatics*, 8(6):1–165.

[Lesh and Doerr 2003] Lesh, R. A. and Doerr, H. M. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Routledge.

[Morais et al. 2018] Morais, C., Serrazina, L., and Ponte, J. P. (2018). Mathematical reasoning fostered by (fostering) transformations of rational number representations. *Acta Scientiae*, 20(4).

[Resnick and Rosenbaum 2013] Resnick, M. and Rosenbaum, E. (2013). Designing for tinkerability. In *Design, make, play*, pages 163–181. Routledge.

[Santana et al. 2021] Santana, B. L., Chavez, C. v. F. G., and Bittencourt, R. A. (2021). Uma definição operacional para pensamento computacional. In *Anais do Simpósio Brasileiro de Educação em Computação*, pages 93–103. SBC.

[Silva 2018] Silva, E. C. d. (2018). Pensamento computacional e a formação de conceitos matemáticos nos anos finais do ensino fundamental: uma possibilidade com kits de robótica.

[Sturgill 2019] Sturgill, R. (2019). Mathematical modeling: Issues and challenges in mathematics education and teaching. *Editorial Team*, 11(5):71.

[Wing 2006] Wing, J. M. (2006). Computational thinking. *Communications of the ACM*, 49(3):33–35.

[Wing 2021] Wing, J. M. (2021). Pensamento computacional. *Educação e Matemática*, (162):2–4.